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NOT JUST FUN AND GAMES

Best known for inventing the game of Life, John H. Conway is adept at finding the theorems hidden in simple puzzles

By Mark Alpert

Stepping into John H. Conway's office at Princeton University is like stepping into a mathematician's playpen. Dozens of polyhedra made of colored cardboard hang from the ceiling like mirror balls at a discotheque. Dangling among them is a Klein bottle constructed from chicken wire. Several models of crystal lattices sit beside the window, and a pyramid of tennis balls rises from the floor. At the center of it all is Conway himself, leaning back in his chair, his face obscured by oversize glasses and a bushy, gray beard. The eclectic 61-year-old mathematician is clearly in his element.

"What's your date of birth?" he asks me soon after we shake hands

"April 19, 1961," I reply.

"Tuesday!" he shouts immediately.Then he corrects himself. "No, damn! Wednesday!" Slightly irritated by his error, he explains that long ago he devised an algorithm for determining the day of the week that any given date falls on. Called the Doomsday Rule, the algorithm is simple enough for Conway to do the calculations in his head. He can usually give the correct answer in under two seconds. To improve his speed, he practices his calendrical calculations on his computer, which is programmed to quiz him with random dates every time he logs on.

At this point, I begin to wonder why Princeton University is paying this man a salary. But over the past three decades Conway has made some of his greatest contributions to mathematical theory by analyzing simple puzzles. "It's impossible for me to go into the office and say, 'Today I'll write a theorem," Conway admits. "I usually have half a dozen things running through my head, including games and puzzles. And every so often, when I feel guilty, I'll work on something useful." Conway's useful work spans the gamut of mathematical disciplines, ranging from theorems about knots and sphere packing to the discovery of a whole new class of numbers--the aptly named surreal numbers.

Born in Liverpool, England, in 1937, Conway showed an early interest in mathematics. At the age of four, according to his mother, he began reciting the powers of two. Liverpool was being bombed by the German Luftwaffe at the time, and Conway has a lasting memory of one of the air raids. "While my father was carrying me to our backyard shelter one night, I happened to look up at the sky. There were spotlights overhead, and I saw the bombs falling from the planes. They were chained together and whirling around. It looked so beautiful, I said, 'Look, Daddy! That's so nice!"

Conway attended the University of Cambridge, where he studied number theory and logic and eventually joined the faculty of the mathematics department. In his spare time he became an avid backgammon player. "I used to play backgammon in the common room at Cambridge," Conway recalls. "My more sedate colleagues would come in occasionally for a cup of coffee or tea, but I'd be there all day long." Conway's career didn't really take off until the late 1960s, when he became intrigued by a theoretical lattice that extends into 24 dimensions. By contemplating this lattice, Conway discovered a new finite group, which is the set of symmetries of a geometric object. A cube, for example, has 24 symmetries--there are 24 ways to rotate it to an identical position. But the Conway group, as it became known, has more than 1018 symmetries, making it the largest finite group known at the time of its discovery. (It was later superseded by the so-called Monster group, which has more than 1053 symmetries.) Finding a new group is an extraordinarily difficult achievement, and Conway's colleagues soon began to hail him as a genius.

At about the same time, Conway was exploring the idea of the universal constructor, which was first studied by American mathematician John von Neumann in the 1940s. A universal constructor is a hypothetical machine that could build copies of itself--something that would be very useful for colonizing distant planets. Von Neumann created a mathematical model for such a machine, using a Cartesian grid--basically, an extended checkerboard--as his foundation. Conway simplified the model, and it became the now famous game of Life.

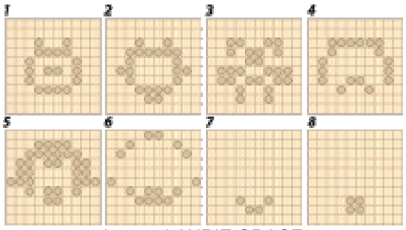

Image: LAURIE GRACE

CHESHIRE CAT, A LIFE PATTERN transforms into a grin (7) and finally a paw print (8).

In the game, you start with a pattern of checkers on the grid--these represent the "live" cells. You then remove each checker that has one or no neighboring checkers or four or more neighbors (these cells "die" from loneliness or overcrowding). Checkers with two or three neighbors remain on the board. In addition, new cells are "born"--a checker is added to each empty space that is adjacent to exactly three checkers. By applying these rules repeatedly, one can create an amazing variety of Life forms, including "gliders" and "spaceships" that steadily move across the grid.

Conway showed the game of Life to his friend Martin Gardner, the longtime author of *Scientific American*'s Mathematical Games column. Gardner described the game in his October 1970 column, and it was an immediate hit. Computer buffs wrote programs allowing them to create ever more complex Life forms. Even today, nearly 30 years after the game's introduction, Conway receives voluminous amounts of e-mail about Life. "The game made Conway instantly famous," Gardner comments. "But it also opened up a whole new field of mathematical research, the field of cellular automata."

Conway, though, moved on to other pursuits. Some of his Cambridge colleagues were skillful at the ancient game of Go, and as Conway watched them play he tried to develop a mathematical understanding of the game. He noticed that near the end of a typical Go match, when the board is covered with snaking lines of black and white stones, the game resembles the sum of several smaller games. Conway realized that certain games actually behave like numbers. This insight led him to formulate a new definition of numbers that included not only the familiar ones--the integers, the rational numbers, the real numbers and so on--but also the transfinite numbers, which represent the sizes of infinitely large sets.

Mathematicians have long known that there is more than one kind of infinity. For example, the set of all integers is infinitely large, but it is smaller than the set of all real numbers. Conway's definition encompassed all the transfinite numbers and, better still, allowed mathematicians to perform the full array of algebraic operations on them. It was a theoretical tour de force: by defining finite and transfinite numbers in the same way, Conway provided a simpler logical foundation for all numbers. Stanford University computer scientist Donald E. Knuth was so impressed by Conway's breakthrough that he wrote a quirky novella, called *Surreal Numbers*, that attempts to explain the theory. In the story, Conway is cast as God--there is a character named "C" whose voice booms out of the sky. Although the comparison may seem a little extreme, Conway acknowledges that he has a healthy ego. "After I make a discovery, my feelings are a bit of a mix," he says. "I admire the beauty of the thing I've discovered, how it all fits together. But I also admire my own skill at finding it."

Conway's interest in games culminated in 1982 with the publication of *Winning Ways for Your Mathematical Plays*, a two-volume work he wrote with Elwyn R. Berlekamp of the University of California at Berkeley and Richard K. Guy of the University of Calgary. The book has become the bible of recreational mathematics; it describes dozens of brain-teasing games, most of them invented by the authors, with outlandish names such as Toads-and-Frogs and Hackenbush Hotchpotch. But the main purpose of the book, Conway insists, is not entertainment. "The book is really more about theory than games," he says. "I'm much more interested in the theory behind a game than the game itself. I got the theory of surreal numbers from analyzing the game of Go, but I never really played the game." In fact, the only game Conway plays regularly is backgammon--a pastime that defies mathematical analysis because it involves the element of chance

Unfortunately, Conway's personal life has not been as orderly as his mathematical theorems. He has endured bouts of depression and a heart attack. In the mid-1980s Conway moved from Cambridge to Princeton, and since then much of his work has focused on geometry. He is currently exploring the symmetries of crystal lattices--which explains the presence of the lattice models in his office. He is also pursuing what he calls his "grandiose project," a rethinking of the fundamental axioms of set theory. Conway recognizes, however, that he is slowing down. "I used to go through these white-hot phases when I couldn't stop thinking about a problem," he admits. "But now those phases are not so common. It's been ages since I had one."

Among mathematicians, though, Conway's reputation is already assured. "It's hard to predict which of his many major achievements will most impress mathematicians of the future," says Martin Kruskal of Rutgers University, who has spent years investigating the surreal numbers that Conway discovered. Conway himself worries a little that his work on games and puzzles may overshadow his more significant accomplishments, such as the discovery of surreal numbers and the Conway group. But his career is strong evidence that playful thinking can often lead to serious mathematics. "Games usually aren't very deep," Conway muses. "But sometimes, something you thought was frivolous can turn out to be a deep structural problem. And that's what mathematicians are interested in."