Part VIII: PID Control of Multi-rotor UAVs

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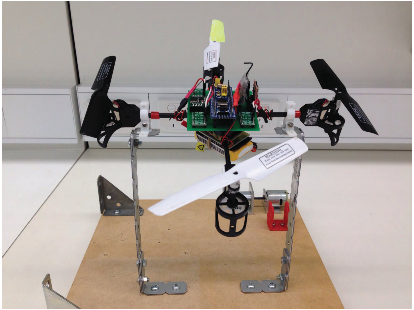
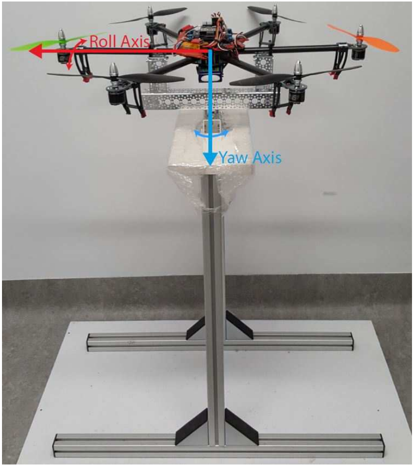
Outline

Outline

Multi-rotor UAVs



(a) Quadrotor



(b) Hexacopter

Figure 1: Unmanned aerial vehicles (multi-rotor)

Inertial Frame and Body Frame (i)

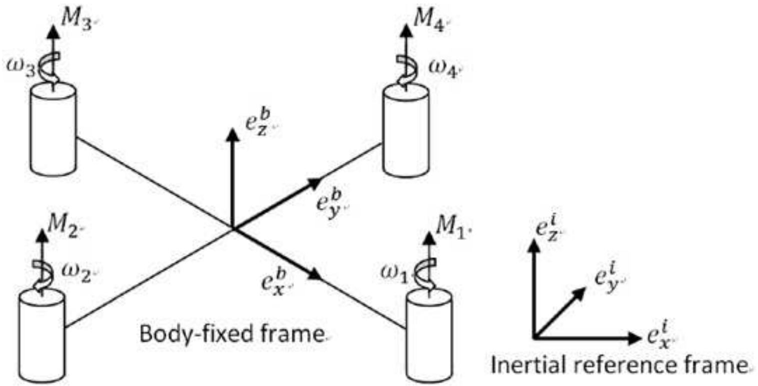


Figure 2: Inertial frame and body frame of the quadrotor

Inertial Frame and Body Frame (ii)

Origin of the body frame

- Figure 2 illustrates the framework used to find a quadrotor dynamics model.
- The origin of the body frame is in the mass center of the quadrotor and $z-$ axis is upwards.
- M_1, M_2, M_3, M_4 are four rotors with DC motors.

Euler Angles

Euler angles

The quadrotor's attitude is defined by three Euler angles, namely roll about $x-$axis, pitch about $y-$axis, and yaw about $z-$axis.

Notations

- Roll angle ϕ: about the x body axis;
- Pitch angle θ: about the y body axis;
- Yaw angle ψ: about the z body axis;
- The transformation sequence is $\psi \rightarrow \theta \rightarrow \phi$ in order to obtain the unique solution.

Dynamic Model

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} (I_{yy} - I_{zz})qr/I_{xx} \\ (I_{zz} - I_{xx})pr/I_{yy} \\ (I_{xx} - I_{yy})pq/I_{zz} \end{bmatrix} + \begin{bmatrix} 1/I_{xx} & 0 & 0 \\ 0 & 1/I_{yy} & 0 \\ 0 & 0 & 1/I_{zz} \end{bmatrix} \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \quad (1)$$

Notations

- I_{xx}, I_{yy} and I_{zz} are the moments of inertia for the three axes in x, y, z directions;
- p, q and r the body frame angular velocities in x, y, z directions;
- τ_x, τ_y, τ_z are the corresponding torques in x, y, z directions.
- The quadrotor is assumed to have symmetric structure with four arms aligned with the $x-$axis and $y-$axis, and as a result there is no interaction between the torques along the three axes.

Relationships between ϕ, θ, ψ and p, q, r

The relationships between the Euler angular velocities and the body frame angular velocities are described in the following differential equation:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & cos(\phi) & -sin(\phi) \\ 0 & sin(\phi)/\cos(\theta) & \cos(\phi)/cos(\theta) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}. \quad (2)$$

Outline

Attitude Control (i)

System Outputs

- For attitude control of quadrotor, the objective is to feedback control the three Euler angles so that they follow three reference signals (ϕ^*, θ^*, ψ^*).
- Therefore, the outputs of the control systems are the three Euler angles: ϕ, θ, ψ.

Control variables or manipulated variables

The manipulated variables or the control signals are the three torques, τ_x, τ_y, τ_z, along the x, y and z directions.

Attitude Control (ii)

Intermittent variables

The body frame angular velocities p, q and r along the x, y and z directions are the intermittent variables.

Cascade control

- Because there are two sets of nonlinear dynamic equations, cascade control is a good choice for this nonlinear control problem.
- The body frame angular velocities p, q and r are the secondary variables because they are directly related to the manipulated variables τ_x, τ_y and τ_z.
- The three Euler angles, ϕ, θ, ψ are the primary variables to achieve the attitude control

Cascade Control of One Axis

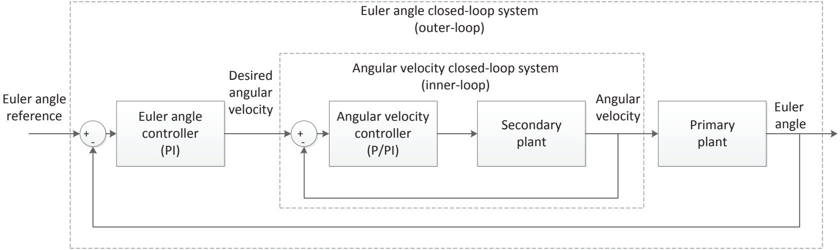


Figure 3: Cascade feedback control structure

Discussions of Cascade Control

- The dynamics of three axes are almost decoupled, thus PI controllers are designed for each axis separately.
- The inner-loop controller (also called secondary controller) is to control inner-loop (secondary) plant, where its reference signal is the desired angular velocity that is also the control signal generated from the outer-loop (primary) controller.
- For the cascade control system, the primary objective is to control the outer-loop (primary) plant to achieve desired closed-loop performance.

Outline

Linearization in Body Reference Frame

Operating condition

When the quadrotor is working in a normal operating condition with a balanced load, the angular velocities p, q and r have a steady-state operating condition at 0.

Linearized model

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 1/I_{xx} & 0 & 0 \\ 0 & 1/I_{yy} & 0 \\ 0 & 0 & 1/I_{zz} \end{bmatrix} \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \tag{3}$$

$$qr \approx q_0 r_0 + r_0(q - q_0) + q_0(r - r_0) = 0$$

where the steady-state values of $q_0 = r_0 = 0$.

Linearization between ϕ, θ, ψ and p, q, r

Operating conditions

The steady-state operating conditions for the three Euler angles are chosen to be zero ($\phi_0 = 0$, θ_0, ψ_0).

Linearized model

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{4}$$

Summary of the Models

- The dynamics models for a quadrotor UAV consist of two sets of integrators in a normal operating condition given by (3)-(4).
- The operating conditions are assumed to be zero, which are valid in a normal operation. If abnormality occurs, the linearized models may not be valid.

Outline

Actuators

- The torques τ_x, τ_y and τ_z in the body frame are the control signals.
- The four DC motors are the actuators that will realize the control signals calculated using the controllers.
- The DC motors have dynamics and we need to consider them in the control system design.
- The challenge is how to realize the calculated torques from the cascade control system using the DC motors.

The Actuator Dynamics (i)

In quadrotor control, the torques τ_x, τ_y and τ_z in the body frame are generated by the differences in rotor thrusts.

Rotor thrusts

The upward thrust produced by each rotor is

$$T_i = b_t \omega_i^2, \quad i = 1, 2, 3, 4.$$

The total thrust is, hence,

$$T = \sum T_i$$

where b_t is the thrust constant determined by air density, the length of the blade and the blade radius, ω_i is the *ith* rotor's angular speed.

The Actuator Dynamics (ii)

Torques (τ_x, τ_y)

The torques about quadrotor's $x-$axis and $y-$axis are

$$\tau_x = d_{mm}(T_4 - T_2) = d_{mm}b_t(\omega_4^2 - \omega_2^2) \tag{5}$$

$$\tau_y = d_{mm}(T_3 - T_1) = d_{mm}b_t(\omega_3^2 - \omega_1^2), \tag{6}$$

Torque τ_z

The torque applied to each propeller by the motor is opposed by aerodynamic drag and the total reaction torque about the z-axis is

$$\tau_z = k_d({\omega_1}^2 + {\omega_3}^2 - {\omega_2}^2 - {\omega_4}^2), \tag{7}$$

where d_{mm} is the distance from the motor to the mass center, and k_d is a drag constant determined by the same factors as b_t.

The Actuator Dynamics in Matrix Form

The relationship between torques, thrust and rotors' angular speed is given in the following matrix form:

$$\begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} = \begin{bmatrix} 1/4b_t & 0 & -1/2d_{mm}b_t & -1/4k_d \\ 1/4b_t & -1/2d_{mm}b_t & 0 & 1/4k_d \\ 1/4b_t & 0 & 1/2d_{mm}b_t & -1/4k_d \\ 1/4b_t & 1/2d_{mm}b_t & 0 & 1/4k_d \end{bmatrix} \begin{bmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \tag{8}$$

Because the altitude of the UAV is not controlled in this case, the total thrust is manually set by the operator.

Outline

Implementation of Control Signal

- From (8), once the manipulated variables T, τ_x, τ_y,τ_z are decided by the cascade feedback controllers, the velocities of motors will be uniquely determined.
- The rotors acting as the actuators in this UAV control application will implement the control actions determined by T, τ_x, τ_y, τ_z, through their reference signals ω_1, ω_2 ω_3 and ω_4.
- However, because the velocities of the DC motors are not measured and can not directly changed, they could not be chosen as the implementation of the control signals.
- Instead, the duty cycles of PWM signals regulating the DC voltages of the motors are used for the actual implementation of the control signals.

DC Motor Dynamics

- The DC motor dynamics are approximated by a first-order transfer function with time delay:

$$\frac{\Omega_i(s)}{V_i(s)} = \frac{r_{wv} e^{-d_m s}}{\epsilon_m s + 1}, \tag{9}$$

- $V_i(s)$ is the Laplace transform of the armature voltage to the *ith* motor, ϵ_m is the time constant, d_m is the time delay, and r_{wv} is the steady-state gain.

Duty Cycle of the Motor

How do we change the armature voltage?

- The armature voltage v_i is changed by manipulating the duty cycle of the PWM signal of each motor drive.
- The relationship between the motor armature voltage and the PWM duty cycle is

$$v_i = d_i V_{bat}, \tag{10}$$

where d_i is the PWM signal duty cycle of the *ith* DC motor drive and V_{bat} is the battery voltage assumed to be constant.

Actuator Dynamics

$$\frac{\Omega_i(s)}{D_{cycle-i}(s)} = \frac{V_{bat} r_{wv} e^{-d_m s}}{\epsilon_m s + 1} \tag{11}$$

$D_{cycle-i}(s)$ is Laplace transform of the PWM signal duty cycle of the *ith* DC motor drive.

Outline

Control System Configuration

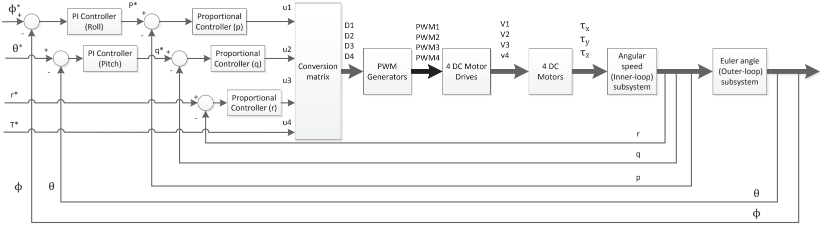


Figure 4: quadrotor closed-loop control system block diagram in cascade control

Discussions of Control Architecture

- Two PI controllers are deployed to control roll and pitch angles in order to eliminate steady-state errors.
- The yaw angle is not controlled because there is no yaw angle measurement due to the lack of digital compass on the quadrotor designed. Instead, the yaw angular velocity is controlled, as the angular velocity in the $z-$axis can be measured accurately by the gyroscope.
- The inner-loop control systems may have the choice of proportional control or PI control. Use of PI controllers will eliminate the steady-state errors but it slightly complicates the controller design and hardware implementation.

Model for Inner-loop Controller Design (i)

Nonlinear model (input being velocity of motor)

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 1/I_{xx} & 0 & 0 \\ 0 & 1/I_{yy} & 0 \\ 0 & 0 & 1/I_{zz} \end{bmatrix} \begin{bmatrix} d_{mm} b_t (\omega_4^2 - \omega_2^2) \\ d_{mm} b_t (\omega_3^2 - \omega_1^2) \\ k_d (\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2) \end{bmatrix} \tag{12}$$

Linearization

From first-order Taylor series expansion, ω_i^2 can be approximated as follows:

$$\omega_i^2 \approx 2\omega_0 \omega_i - \omega_0^2 \tag{13}$$

where ω_0 is the motor's rated speed

Model for Inner-loop Controller Design (ii)

$$\dot{p} = \frac{2d_{mm}b_t\omega_0}{I_x}(\omega_4 - \omega_2) \quad (14)$$

$$\dot{q} = \frac{2d_{mm}b_t\omega_0}{I_y}(\omega_3 - \omega_1) \quad (15)$$

$$\dot{r} = \frac{4k_d\omega_0}{I_z}(\omega_1 + \omega_3 - \omega_4 - \omega_2) \quad (16)$$

Performing Laplace transform leads to

$$sP(s) = \frac{2d_{mm}b_t\omega_0}{I_x}(\Omega_4(s) - \Omega_2(s)) \quad (17)$$

$$sQ(s) = \frac{2d_{mm}b_t\omega_0}{I_y}(\Omega_3(s) - \Omega_1(s)) \quad (18)$$

$$sR(s) = \frac{4k_d\omega_0}{I_z}(\Omega_1(s) + \Omega_3(s) - \Omega_4(s) - \Omega_2(s)) \quad (19)$$

Model for Inner-loop Controller Design (iii)

Considering motor dynamics, and substituting equation (11) to equation (17)–(19) gives

$$sP(s) = \frac{2d_{mm}b_t\omega_0}{I_x}\frac{V_{bat}r_{wv}e^{-d_ms}}{\epsilon_m s+1}(D_{cycle-4}(s) - D_{cycle-2}(s)) \tag{20}$$

$$sQ(s) = \frac{2d_{mm}b_t\omega_0}{I_y}\frac{V_{bat}r_{wv}e^{-d_ms}}{\epsilon_m s+1}(D_{cycle-3}(s) - D_{cycle-1}(s)) \tag{21}$$

$$sR(s) = \frac{4k_d\omega_0}{I_z}\frac{V_{bat}r_{wv}e^{-d_ms}}{\epsilon_m s+1}(D_{cycle-1}(s) + D_{cycle-3}(s) - D_{cycle-4}(s) - D_{cycle-2}(s)) \tag{22}$$

- These are the relationships between the secondary variables and the duty cycles of the motors. Using PID feedback control, we can calculate the linear combinations of the duty cycles.
- However, we need to calculate duty cycle of each motor for the control purpose.

Calculation of Duty Cycle of Each Motor

Control signals from inner-loop controllers

Now the four control signals u_T, u_x, u_y and u_z are defined, and

$$\begin{aligned} u_T &= T^* \\ u_x &= d_{cycle-4} - d_{cycle-2} \\ u_y &= d_{cycle-3} - d_{cycle-1} \\ u_z &= d_{cycle-1} + d_{cycle-3} - d_{cycle-4} - d_{cycle-2} \end{aligned}$$

where T^* is the signal from user and u_x, u_y and u_z are the linear combinations of the duty cycles, which are calculated from the inner-loop controllers.

Calculation of duty cycles of each motor

Solving these four linear equations gives the calculations of the duty cycles:

$$\begin{bmatrix} d_{cycle-1} \\ d_{cycle-2} \\ d_{cycle-3} \\ d_{cycle-4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & -1 \\ 1 & -1 & 0 & 1 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_T \\ u_x \\ u_y \\ u_z \end{bmatrix} \tag{23}$$

PI Controller Design for Inner-loop (i)

$$G_p(s) = \frac{P(s)}{U_x(s)} = \frac{2d_{mm}b_t\omega_0}{I_x}\frac{V_{bat}r_{wv}e^{-d_m s}}{\epsilon_m s^2 + s} \tag{24}$$

$$G_q(s) = \frac{Q(s)}{U_y(s)} = \frac{2d_{mm}b_t\omega_0}{I_y}\frac{V_{bat}r_{wv}e^{-d_m s}}{\epsilon_m s^2 + s} \tag{25}$$

$$G_r(s) = \frac{R(s)}{U_z(s)} = \frac{4k_d\omega_0}{I_z}\frac{V_{bat}r_{wv}e^{-d_m s}}{\epsilon_m s^2 + s} \tag{26}$$

Three controllers can be designed independently for each channel based on the above transfer functions.

PI Controller Design for Inner-loop (ii)

- By ignoring fast pole, inner-loop plant transfer functions can be approximated by a integrator with a time delay:

$$G(s) = \frac{K}{s}e^{-ds} \tag{27}$$

where K is the steady-state gain and d_s is the time delay.

- A tuning rules based PI controller method design will be used to calculate K_c and τ_I.

$$\hat{K}_c = \frac{1}{0.5080\beta + 0.6208} \tag{28}$$

$$\hat{\tau}_I = 1.9885\beta + 1.2235 \tag{29}$$

where $\tau = \beta d$ is the desired closed-loop time constant, d is the time delay. β is a tuning factor to adjust the closed-loop response speed.

Design of Outer-loop PI Controllers

Recall the model for the outer-loop system:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{30}$$

These are integrating systems with unity gain. However, we need to consider the time delay from the actuators. So we model the outer-loop transfer functions using

$$G(s) = \frac{e^{-ds}}{s}$$

Then, we use the tuning rules to find the PI controllers for the outer-loop systems.

Summary: Attitude Control of Quadrotor

Output variables (primary variables)

The Euler angles ψ, θ and ϕ.

Manipulated variables for actuation

The duty cycles of the four motors are the signals for the actuators in quadrotor control.

Secondary variables

The body frame angular velocities p, q and r are the secondary variables for cascade control.

Linear models

Integrating with time delay models are used in both inner-loop and outer-loop PI controller design. Tuning rules for integrating with time delay systems are working well.

Outline

Components

Function	Model
DC motor drive	DRV8833 Dual Motor Driver Carrier
Sensor board	MPU6050
Micro processor	STM32F103C8T6
RC receiver	WFLY065
DC motor	820 Coreless Motor
RC transmitter	WFT06X-A
Data logger	SparkFun OpenLog

Table 1: quadrotor hardware list

Experimental Data

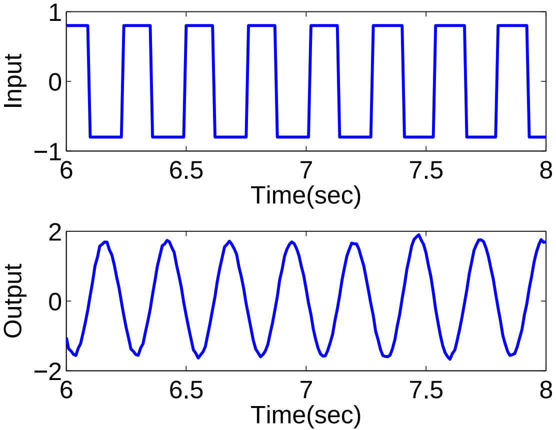


Figure 5: Relay feedback control signals from inner-loop system: top figure input signal; bottom figure output signal.

Closed-loop Control Results

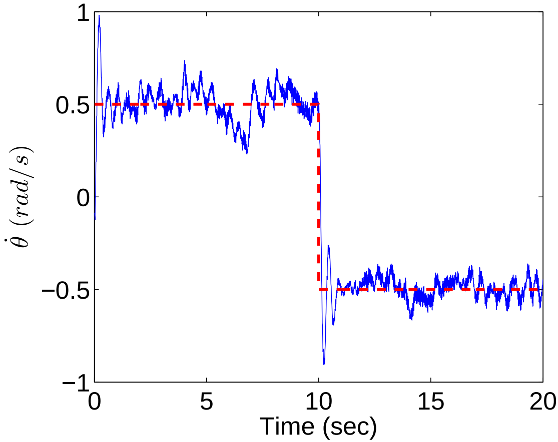


Figure 6: Inner-loop step response in closed-loop control. Dashed line: reference signal; solid line: output.

Auto-tuning of Primary PI Controller

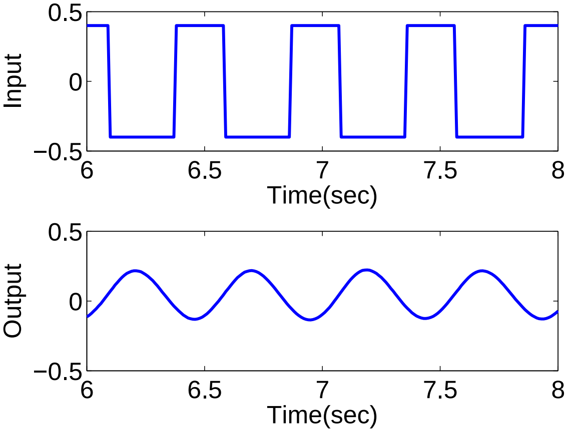


Figure 7: Relay feedback control signals from outer-loop system: top figure input signal; bottom figure output signal.

Closed-loop Control Results

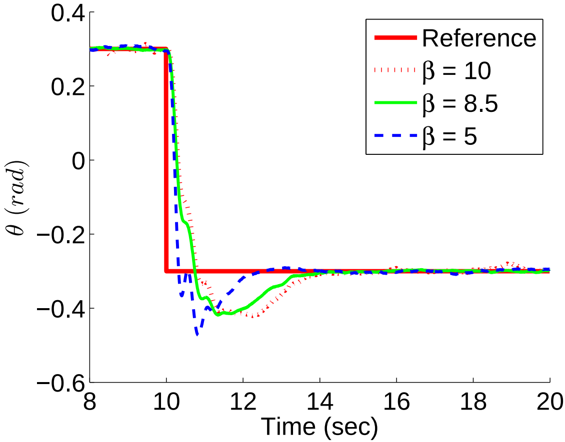


Figure 8: Comparative outer-loop step response in closed-loop control.

Outline

Components

Table 2: Flight controller and avionic components

Components	Descriptions
Airframe	Turnigy Talon Hexacopter
Microprocessor	ATMega2560
Inertial measurement unit	MPU6050
Electronic speed controllers	Turnigy 25A Speed Controller
Brushless DC motors	NTM Prop Drive 28-26 235W
Propellers	10x4.5 SF Props
RC Receiver	OrangeRX R815X 2.4Ghz receiver
RC Transmitter	Turnigy 9XR PRO transmitter
Datalogger	CleanFlight Blackbox Datalogger

Relay experimental data

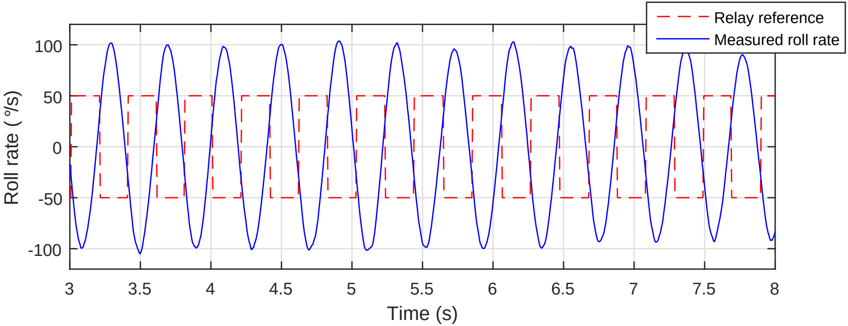


Figure 9: Inner loop relay test result. $K_T = 0.3$, $R_a = 50^\circ/s$, $\epsilon = 30^\circ/s$

Controller parameters

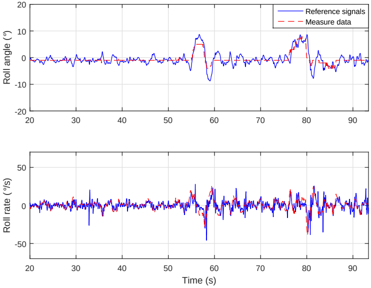
Controller parameters found for inner-loop

$K_c = 0.33, \tau_I = 0.26$ and $\tau_D = 0.03$

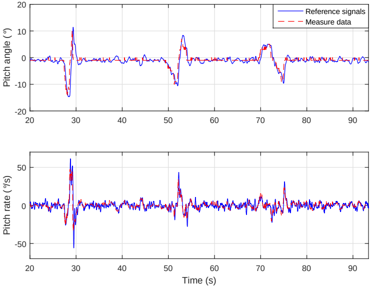
Controller parameters found for outer-loop

$K_c = 3.3$, $\tau_I = 0.63$ and $\tau_D = 0.013$.

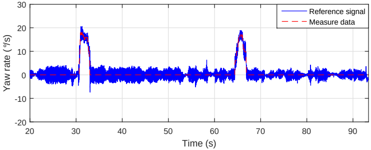
Outdoor flight testing



(a) Roll control:top-angle,bottom- rate



(b) Pitch control:top-angle,bottom- rate



(c) Yaw control results

Figure 11: Experimental testing results. Key- red dashed lines: the reference signals; blue solid lines: the measured data

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