Part V: Tuning Rules and Auto-tuners for PID Controllers

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Outline

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Ziegler-Nichols tuning rules

- There are two sets of Ziegler-Nichols tuning rules for PID controller.
    - One is based on oscillation testing of the plant;
    - The other is based on step response testing;
- Both tuning rules are only applicable to stable plants.

The procedure of oscillation testing

- In the plant testing, the controller is set to proportional mode without integrator and derivative action.
- The sign of K_c must be the same as the steady-state gain of the plant for the reason of introducing negative feedback in the control system.
- With the proportional closed-loop control, the feedback gain K_c is set to be a very small value in magnitude to begin the experiment.
- The value of K_c is gradually increased until the control signal $u(t)$ exhibits sustained oscillation (see Figure 1).

Oscillation data

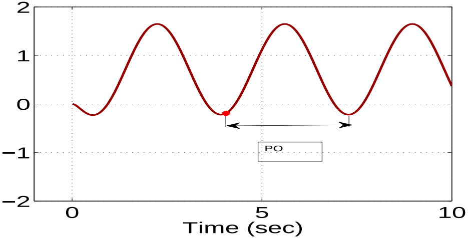


Figure 1: Sustained closed-loop oscillation

There are two parameters obtained from this test: the value of K_c that has caused the oscillation and the period of the oscillation. We denote this particular K_c as K_o and the period as P_o.

Z-N tuning rules

Table 1: Ziegler-Nichols tuning rule using oscillation testing data

	K_c	τ_I	τ_D
P	$0.5K_o$		
PI	$0.45K_o$	$\frac{P_o}{1.2}$	
PID	$0.60K_o$	$\frac{P_o}{2}$	$\frac{P_o}{8}$

Exclusion of Two Classes of Plants

This set of tuning rules can not be applied to

- First order stable plant with stable zero;
- Second order stable plant with stable zeros;

Why? Can you analyze your answers with illustrations of root-locus.

Example

Assume that a continuous-time plant has the Laplace transfer function

$$G(s) = \frac{s-2}{(s+1)(s+2)(s+3)} \tag{1}$$

Find the PI and PID controller parameters using Ziegler-Nichols tuning rule and simulate the closed-loop control systems.

Solution I

This system has a negative steady-state gain of $-\frac{1}{3}$, so the feedback control gain should be negative. Beginning the tuning process by setting $K_c = -1$ decreasing gradually to $K_c = -7.5$, the closed-loop control system exhibits sustained oscillation as shown in Figure 2. From this Figure, it reads the period of oscillation is 3.35.

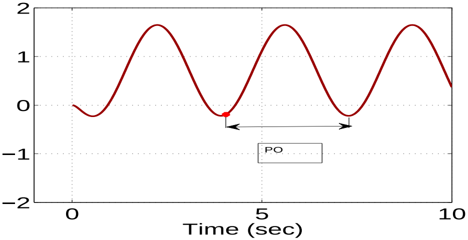
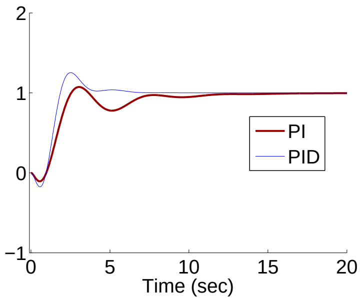


Figure 2: Sustained closed-loop oscillation

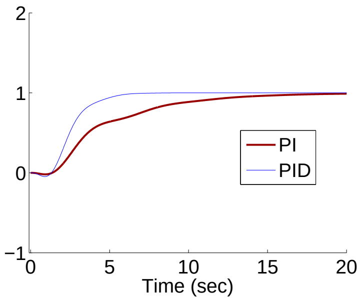
Solution II

Base on Table 1, the proportional gain for the PI controller is $K_c = 0.45 \times (-7.5) = -3.38$ and the integral time constant $\tau_I = \frac{3.35}{1.2} = 2.79$. The proportional gain for PID controller is $K_c = 0.6 \times (-7.5) = -4.5$, $\tau_I = \frac{3.35}{2} = 1.68$, and $\tau_D = \frac{3.35}{8} = 4.2$.

Solution III



(a) Original structures



(b) Alternative Structures

Figure 3: Comparison of closed-loop PI and PID control using Z-N rules

It is seen that with the derivative term, the closed-loop oscillation existed in the PI controller is reduced.

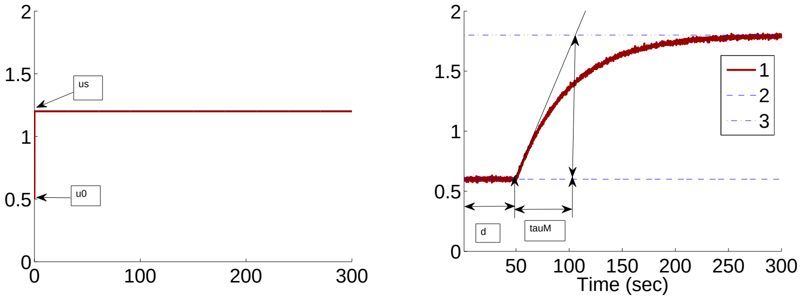
Outline

What is a reaction curve?

- Basically it is curve generated using a step response test.
- The plant step response test is performed in open-loop operation, suitable to stable plant only.
- When performing this test, the plant input signal $u(t)$ takes a step change from an initial constant value U_0 to a normal operation value, U_s, the measurement of the plant output signal $y(t)$ in response to the step input change gives us the plant step response test data or the reaction curve.
- The response test completes when the value of the output signal reaches a constant or the signal fluctuated around a constant value.

The parameters we needed for tuning

Time delay d, steady-state gain K_{ss} and time constant τ_M. We draw the steady-state response first and a line starting from the rising of the response with a maximum slope, which intersects with the steady-state line.



(a) Input signal (b) Output signal

Figure 4: Step response data. Key: line (1) the output response; line (2) steady-state output position before the response (Y_0); line (3) steady-state output position in completion of the response (Y_s).

Ziegler-Nichols tuning rules with reaction curve

Table 2: Ziegler-Nichols tuning rules with reaction curve

	K_c	τ_I	τ_D
P	$\frac{\tau_M}{K_{ss}d}$		
PI	$0.9\frac{\tau_M}{K_{ss}d}$	$3d$	
PID	$1.2\frac{\tau_M}{K_{ss}d}$	$2d$	$0.5d$

Cohen-Coon tuning rules with reaction curve

Table 3: Cohen-Coon tuning rules with reaction curve

	K_c	τ_I	τ_D
P	$\frac{\tau_M}{K_{ss}d}(1+\frac{d}{3\tau_M})$		
PI	$\frac{\tau_M}{K_{ss}d}(0.9+\frac{d}{12\tau_M})$	$\frac{d(30\tau_M+3d)}{9\tau_M+20d}$	
PID	$\frac{\tau_M}{K_{ss}d}(\frac{4}{3}+\frac{d}{4\tau_M})$	$\frac{d(32\tau_M+6d)}{13\tau_M+8d}$	$\frac{4d\tau_M}{11\tau_M+2d}$

Wang-Cluett tuning rules with reaction curve

Table 4: Wang-Cluett tuning rules with reaction curve, $L = \frac{\tau_M}{d}$

	K_c	τ_I	τ_D
P	$\frac{0.13+0.51L}{K_{ss}}$		
PI	$\frac{0.13+0.51L}{K_{ss}}$	$\frac{d(0.25+0.96L)}{0.93+0.03L}$	
PID	$\frac{0.13+0.51L}{K_{ss}}$	$\frac{d(0.25+0.96L)}{0.93+0.03L}$	$\frac{d(-0.03+0.28L)}{0.25+L}$

Example

The unit step response of a continuous-time transfer function model

$$G(s) = \frac{0.5e^{-20s}}{30s+1} \tag{2}$$

is shown in Figure 5.

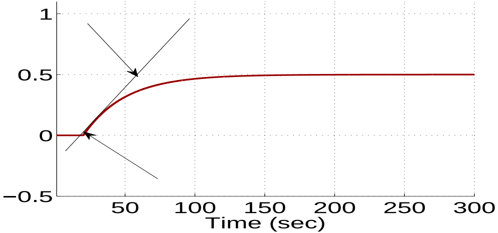


Figure 5: Unit step response

$t_1 = 21, Y_0 = -0.02; t_2 = 58, Y_s = 0.5.$ Find the PI controllers using the reaction curve based-tuning rules.

Solution I

$$K_{ss} = \frac{Y_s - Y_0}{U_s - U_0} \approx 0.5 \tag{3}$$

where $U_s - U_0$ is one since a unit step signal is used as the input. The time delay $d = t_1 = 21$, and the parameter $\tau_M = t_2 - t_1 = 58 - 21 = 37$.

Solution II

Table 5: PI controller parameters with reaction curve

	K_c	τ_I
Ziegler-Nichols	3.1714	63
Cohen-Coon	3.3381	32.7131
Wang-Cluett	2.0571	41.4811

Closed-loop response

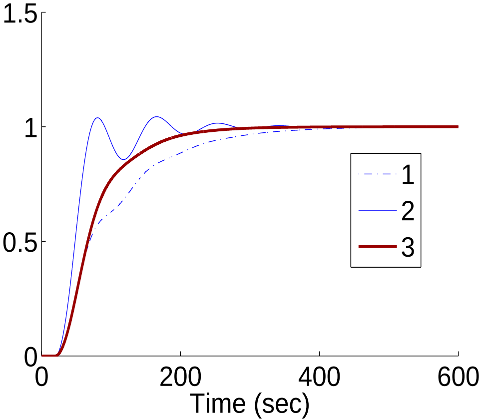


Figure 6: Closed-loop unit step response with PI controller. Key: line (1) Ziegler-Nichols tuning rule; line (2) Cohen-Coon tuning rule; line (3) Wang-Cluett tuning rule.

Example

A continuous-time plant has the transfer function

$$G(s) = \frac{0.5e^{-20s}}{(30s+1)^3} \quad (4)$$

The unit step response of this transfer function model is shown in Figure 7.

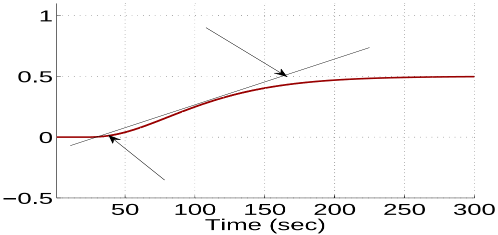


Figure 7: Unit step response

$t_1 = 36, Y_0 = -0.0022 \approx 0; t_2 = 164, Y_s = 0.4981 \approx 0.5.$

Solution I

The steady state gain $K_{ss} = \frac{Y_s - Y_0}{1} = 0.5$. The time delay is $d = t_1 = 36$ and the parameter $\tau_M = t_2 - t_1 = 164 - 36 = 128$.

Table 6: PI controller parameters with reaction curve

	K_c	τ_I
Ziegler-Nichols	6.4	108
Cohen-Coon	6.5667	75.9231
Wang-Cluett	3.8867	127.2154

Solution II

Both PI controllers from Ziegler-Nichols and Cohen-Coon tuning rules failed to produce a stable closed-loop system. However, the PI controller using Wang-Cluett tuning rule gives a stable closed-loop response.

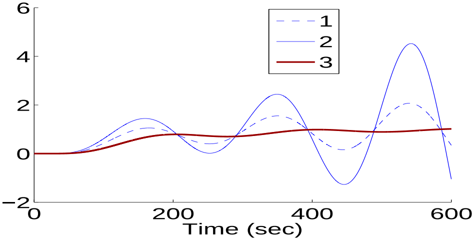


Figure 8: Closed-loop unit step response with PI controller. Key: line (1) Ziegler-Nichols tuning rule; line (2) Cohen-Coon tuning rule; line (3) Wang-Cluett tuning rule.

Solution III

Next, we will find the PID controller parameters using the reaction curve based methods.

Table 7: PID controller parameters with reaction curve

	K_c	τ_I	τ_D
Ziegler-Nichols	8.5333	72	18
Cohen-Coon	9.9815	79.5246	12.4541
Wang-Cluett	3.8867	127.2154	9.1340

Solution III

both PID controllers using Ziegler-Nichols and Cohen-Coon tuning rules are unable to produce a stable closed-loop control system, yet the PID controller using Wang-Cluett tuning rule produces a stable closed-loop system.

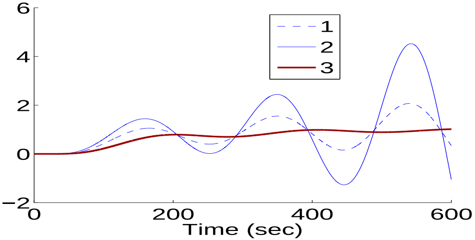


Figure 9: Closed-loop unit step response with PID controller. Key: line (1) Ziegler-Nichols tuning rule; line (2) Cohen-Coon tuning rule; line (3) Wang-Cluett tuning rule.

Outline

Automatic Tuning of PID Controllers

- Automatically find the mathematical model of the plant to be controlled;
  - identification experiment design to ensure the collection of input and output data contains useful information for controller design;
  - closed-loop system is required to be stable for safety of equipment during the experiments;
  - identification experiments need to be simple and easy to execute.
- Automatically determine the controller parameters with minimum human intervention.
- The auto-tuner presented here is derived based on an integrating with delay model, which is used to approximate a large class of integrating and stable systems as shown in the examples.

Outline

Auto-tuner Mechanism for Integrating Systems

Relay Feedback Control

- A proportional controller with known gain K_T is used to stabilize the integrating system;
- a relay feedback control system is deployed for the output of the closed-loop system.

Block diagram

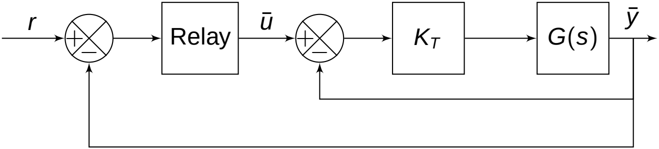


Figure 10: Relay feedback control system

The Input and Output Signals

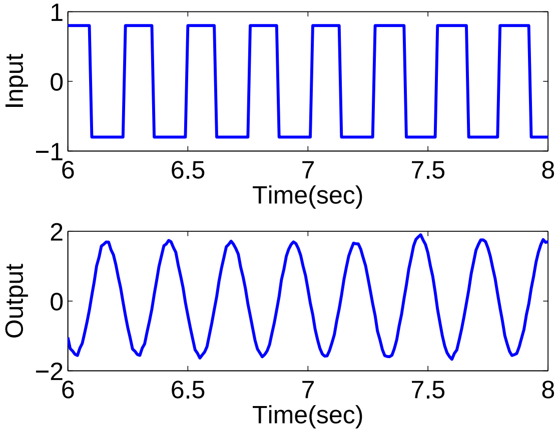


Figure 11: Relay feedback control signals: top figure input signal; bottom figure output signal.

Relay Control

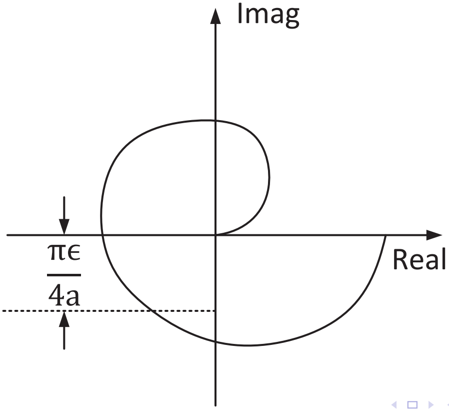
- Calculate the relay feedback error: $e(t_k) = r(t_k) - \bar{y}(t_k)$.
- If $|e(t_k)| \leq \epsilon$; then $\bar{u}(t_k) = \bar{u}(t_{k-1})$.
- If $|e(t_k)| > \epsilon$; then $\bar{u}(t_k) = u_{ss} + a \times sign(e(t_k))$.
- u_{ss} is the steady-state value of input signal chosen to produce symmetric oscillation.

Notations

- The reference signal $r(t)$ is a constant that represents the steady-state operation of the plant.
- ϵ is the hysteresis selected to avoid the possible random switches caused by the measurement noise and a is the amplitude of the relay.
- The signal $\bar{y}(t)$ represents the actual output measurement.

The Characteristics of Relay Control

- Assume that the period of the oscillation is T.
- The frequency of the periodic signal $\bar{u}(t)$, denoting by $\omega_1 = \frac{2\pi}{T}$, approximately corresponds to the frequency illustrated on the Nyquist curve shown in Figure 12.



Outline

Estimation of Open-loop Frequency Response

To estimate the open-loop frequency response, the first step is to estimate the closed-loop frequency response

$$T(j\omega_1) = \frac{K_T G(j\omega_1)}{1 + K_T G(j\omega_1)}$$

where $G(j\omega_1)$ is the open-loop frequency response at ω_1.

Estimation of $T(j\omega_1)$

- The pair of input and output signals corresponding to the relay feedback control system is used.
- The input signal equals the relay output signal:

$$u(t) = \bar{u}(t) - u_{ss} = a \times sign(e(t))$$

- The closed-loop output signal with steady-state removed becomes

$$y(t) = \bar{y}(t) - r(t) = -e(t)$$

Characteristics of Periodic Signals

- For a period T, the Fourier series expansion of the periodic input signal $u(t)$, is expressed as

$$u(t) = \frac{4a}{\pi}(\sin \frac{2\pi}{T}t + \frac{1}{3}\sin \frac{6\pi}{T}t + \frac{1}{5}\sin \frac{10\pi}{T}t + \ldots) \quad (5)$$

- By choosing sampling interval Δt and the number of samples within one period $N = \frac{T}{\Delta t}$, the discretized input signal $u(t)$ at sampling instant $t_k = k\Delta t$ becomes

$$u(k) = \frac{4a}{\pi}(\sin \frac{2\pi k}{N} + \frac{1}{3}\sin \frac{6\pi k}{N} + \frac{1}{5}\sin \frac{10\pi k}{N} + \ldots) \quad (6)$$

Estimation of $T(j\omega_1)$ using Fast Fourier Transform

- The simplest way to estimate the frequency response of the system under relay feedback is to use Fast Fourier Transform.
- Assuming that the data length is L, the Fourier transform of the input signal $u(k)$, $k = 1, 2, \ldots, L$, is

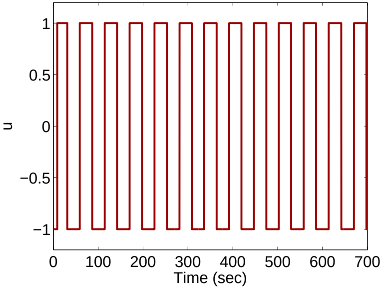
$$U(n) = \frac{1}{L}\sum_{k=1}^{L} u(k)e^{-j\frac{2\pi(k-1)(n-1)}{L}} \tag{7}$$

  and the corresponding Fourier transform of the output is

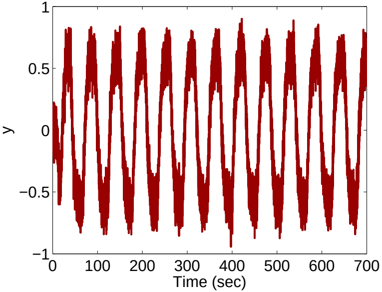
$$Y(n) = \frac{1}{L}\sum_{k=1}^{L} y(k)e^{-j\frac{2\pi(k-1)(n-1)}{L}} \tag{8}$$

  where $n = 1,\ 2,\ 3,\ \ldots,\ L$.
- From both (7) and (8), with the definition of Fourier transform, the corresponding discrete frequency ω_d is defined from 0 to $\frac{2\pi(L-1)}{L}$ with an incremental of $\frac{2\pi}{L}$.

Example: Input and Output Data

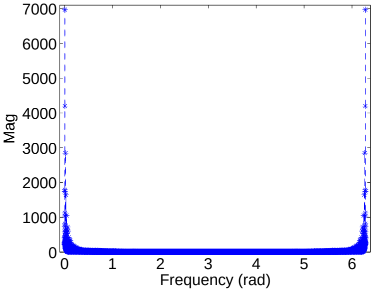


(a) Input data

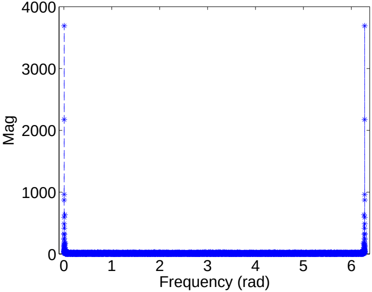


(b) Output data

Fourier Transform (1)

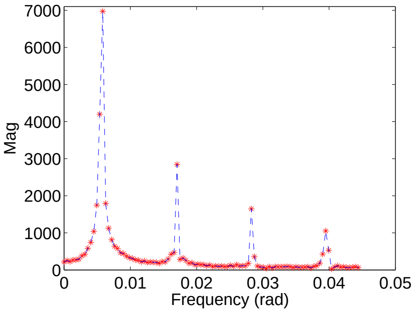


(c) Fourier transform $U(e^{j\omega_d})$

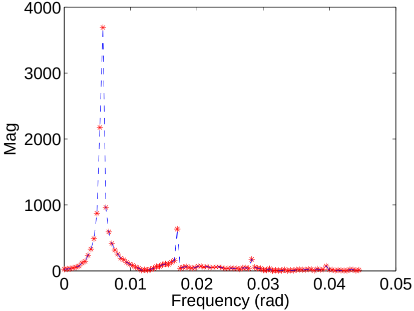


(d) Fourier transform $Y(e^{j\omega_d})$

Fourier Transform (2)



(e) Fourier transform $U(e^{j\omega_d})$, $0 \leq \omega_d \leq 0.045$



(f) Fourier transform $Y(e^{j\omega_d})$, $0 \leq \omega_d \leq 0.045$

Example (iii)

- Locating the fundamental frequency of the relay signal as the maximum value of $U(e^{j\omega_d})$, Identify the peaks of $U(e^{j\omega_d})$ as the 14*th* sample, which is the frequency at $\omega_d = \frac{2*\pi(14-1)}{L}$, $L = 14001$.
- The estimation of the frequency response of the system is then given by

$$T(14) = Y(14)/U(14) = -0.0040 - 0.5293i$$

- The second peak is identified at the 39*th* sample, which is the frequency at $\omega_d = \frac{2*\pi(39-1)}{L}$, $T = -0.1081 + 0.1950i$. The third peak is identified at 64*th* sample, which is the frequency at $\omega_d = \frac{2*\pi(64-1)}{L}$, $T = 0.1054 - 0.0151i$.

Comparative Results

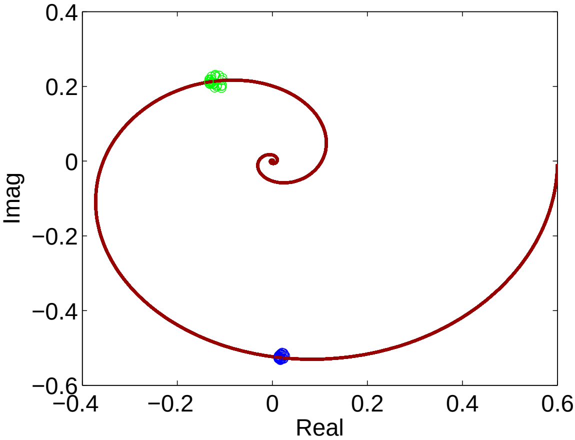


Figure 13: Comparison between the estimated frequency points with the actual frequency response.

Recursive Estimation of $T(j\omega_1)$ (i)

For a stable system with transfer function $T(z)$, in general, it has the z-transfer function model in frequency sampling filter form:

$$T(z) = \sum_{l=-\frac{N-1}{2}}^{\frac{N-1}{2}} T(e^{jl\omega_d})F^l(z), \quad (9)$$

where $F^l(z)$ is the lth frequency sampling filter given by

$$\begin{aligned} F^l(z) &= \frac{1}{N}\frac{1 - z^{-N}}{1 - e^{jl\omega_d}z^{-1}} \\ &= \frac{1}{N}(1 + e^{jl\omega_d}z^{-1} + ... + e^{j(N-1)l\omega_d}z^{-(N-1)}). \end{aligned}$$

Recursive Estimation of $T(j\omega)$ (ii)

Output is expressed as

$$y(k) = \sum_{l=-\frac{N-1}{2}}^{\frac{N-1}{2}} T(e^{jl\omega_d}) f^l(k) + v(k) \tag{10}$$

However,

$$f^l(k) = \begin{cases} 0, & \text{if } l = 0, \pm 2, \pm 4, \pm 6, \pm 8, \dots \\ \frac{2a}{j\pi |l|} e^{jl\omega_d k}, & \text{if } l = \pm 1, \pm 3, \pm 5, \pm 7, \pm 9, \dots \end{cases} \tag{11}$$

Recursive Estimation of $T(j\omega)$ (iii)

$$
\begin{aligned}
y(k) &= T(e^{j\omega_d})f^1(k) + T(e^{-j\omega_d})f^{-1}(k) + T(e^{j3\omega_d})f^3(k) + T(e^{-j3\omega_d})f^{-3}(k) \\
&+ T(e^{j5\omega_d})f^5(k) + T(e^{-j5\omega_d})f^{-5}(k) + \ldots + v(k) \qquad (12)
\end{aligned}
$$

Recursive Estimation of $T(j\omega)$ (iv)

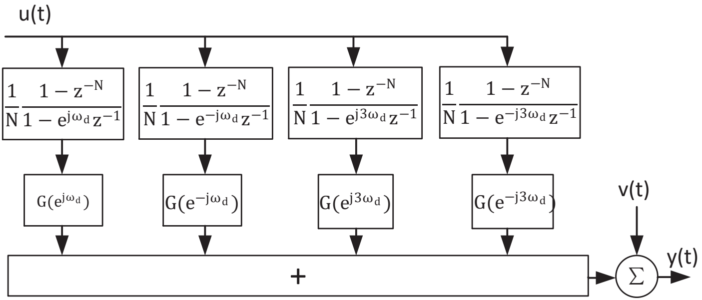


Figure 14: Block diagram of frequency sampling filter model using relay control.

Recursive Estimation of $T(j\omega)$ (v)

Define the complex parameter vector to be estimated as

$$\theta = [T(e^{j\omega_d}) \; T(e^{-j\omega_d}) \; T(e^{j3\omega_d}) \; T(e^{-j3\omega_d})]^{T*}$$

and its corresponding regressor vector as

$$\phi(k) = [f^1(k) \; f^{-1}(k) \; f^3(k) \; f^{-3}(k)]^{T*}$$

where A^{T*} denotes the complex conjugate transpose of A.

Recursive Estimation of $T(j\omega_1)$ (vi)

RLS

Here, a standard recursive least squares algorithm is written as

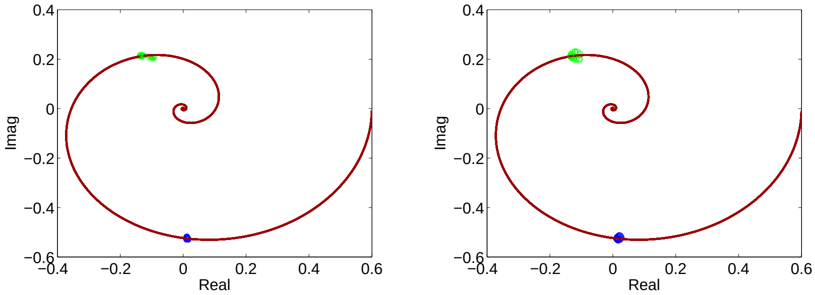
$$P(k-1) = P(k-2) - \frac{P(k-2)^T\phi(k)\phi(k)^T P(k-2)}{1+\phi(k)^T P(k-2)\phi(k)} \quad (13)$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k-1)\phi(k)(y(k) - \phi(k)^T\hat{\theta}(k-1)) \quad (14)$$

Initial conditions

$P(-1)$ and $\hat{\theta}(0)$ are the initial conditions selected for the recursive least squares algorithm. $\hat{\theta}(k)$ contains the estimated frequency response parameters.

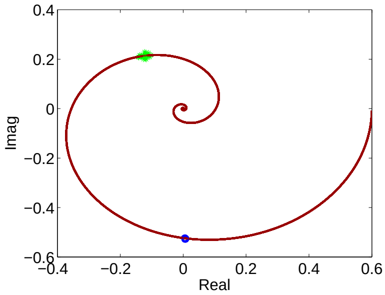
Comparative Studies-Long Data Length



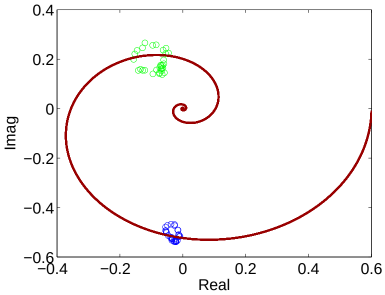
(a) FSF Estimation (b) FFT Estimation

Figure 15: Monte-Carlo simulation results with 31 random seeds and long experimental time ($T_{sim} = 800(sec)$). $G_p(j\omega)$ (solid line), o is the estimated values at $\omega_1 = \frac{2\pi}{N\Delta t}$ and $*$ is the estimated values at $\omega_3 = 3\omega_1$.

Comparative Studies-Short Data Length



(a) FSF Estimation



(b) FFT Estimation

Figure 16: Monte-Carlo simulation results with 31 random seeds and short experimental time ($T_{sim} = 200(sec)$). $G_p(j\omega)$ (solid line), o is the estimated values at $\omega_1 = \frac{2\pi}{N\Delta t}$ and $*$ is the estimated values at $\omega_3 = 3\omega_1$.

Open-loop Frequency Response

Discrete-time frequency response

$$G(e^{j\omega_d}) = \frac{1}{K_T} \frac{T(e^{j\omega_d})}{1 - T(e^{j\omega_d})} \quad (15)$$

Continuous-time Frequency Response

- The discrete-time frequency response $G(e^{j\omega_d})$ is a close approximation to its continuous-time frequency response under the assumption that the system operates in a fast sampling environment, where the equivalent continuous-time frequency is $\omega_1 = \frac{\omega_d}{\Delta t}$.
- Continuous-time frequency response

$$G_p(j\omega_1) \approx G(e^{j\omega_d})$$

Outline

Integrator Plus Time Delay Model

- For an integrating plus time delay system, a single frequency is sufficient to determine its gain K_p and time delay d.
- The approximate model of an integrating system is assumed to be of the following form:

$$G_p(s) = \frac{K_p e^{-ds}}{s} \tag{16}$$

Finding the Parameters (i)

- Letting the frequency response of the integrator plus delay model (16) be equal to the estimated $G_p(j\omega_1)$ leads to

$$\frac{K_p e^{-jd\omega_1}}{j\omega_1} = G_p(j\omega_1) \tag{17}$$

- Equating the magnitudes on both side of (17) gives

$$K_p = \omega_1 |G_p(j\omega_1)| \tag{18}$$

where $|e^{-jd\omega_1}| = 1$.

Finding the Parameters (ii)

- Additionally, from (17), the following relationship holds:

$$e^{-jd\omega_1} = \frac{j\omega_1 G_p(j\omega_1)}{K_p}$$

- This gives the estimate of time delay as

$$d = -\frac{1}{\omega_1} tan^{-1} \frac{Imag(jG_p(j\omega_1))}{Real(jG_p(j\omega_1))} \tag{19}$$

PID Controller Design

The parameter β is the scaling factor for the desired closed-loop time constant, which is defined as

$$
\begin{aligned}
\tau_{cl} &= \beta d \\
K_c &= \frac{\hat{K}_c}{dK_p} \\
\tau_I &= d\hat{\tau}_I \\
\tau_D &= d\hat{\tau}_D
\end{aligned}
$$

Normalized PID Parameters (i)

Table 8: Normalized PID controller parameters ($\xi = 0.707$)

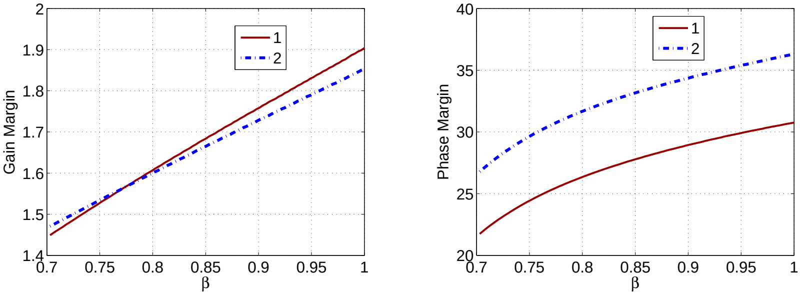
	$0.7 \le \beta \le 1$	$1 < \beta \le 11$
$\hat{K}_c$	$\frac{1}{0.3280\beta^2+0.0786\beta+0.6442}$	$\frac{1}{0.7184\beta+0.3661}$
$\hat{\tau}_I$	$-3.7845\beta^2 + 10.2044\beta - 4.0298$	$1.3970\beta + 1.2271$
$\hat{\tau}_D$	$\frac{1}{-1.9064\beta^2+6.1545\beta-1.5875}$	$\frac{1}{1.4275\beta+1.6450}$

Normalized PID Parameters (ii)

Table 9: Normalized PID controller parameters ($\xi = 1$)

	$0.7 \leq \beta \leq 1$	$1 < \beta \leq 11$
$\hat{K}_c$	$\frac{1}{0.3100\beta^2 - 0.0486\beta + 0.7853}$	$\frac{1}{0.5138\beta + 0.5909}$
$\hat{\tau}_I$	$-3.0205\beta^2 + 9.6838\beta - 3.8821$	$1.9886\beta + 1.2118$
$\hat{\tau}_D$	$\frac{1}{-1.7078\beta^2 + 5.1844\beta - 1.0555}$	$\frac{1}{1.0156\beta + 1.7550}$

Gain and Phase Margins: PID

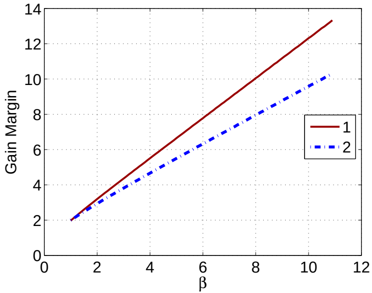


(a) Gain margin($0.7 \le \beta \le 1$)

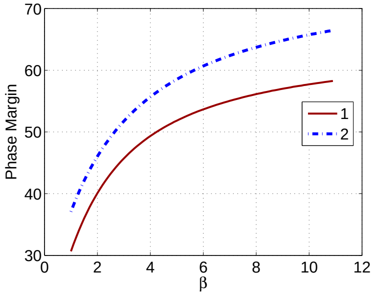
(b) Phase margin ($0.7 \le \beta \le 1$)

Figure 17: Calculated gain and phase margins for PID controllers. Key: (1) using Table 8 ($\xi = 0.707$); (2) using Table 9($\xi = 1$)

Gain and Phase Margins: PID



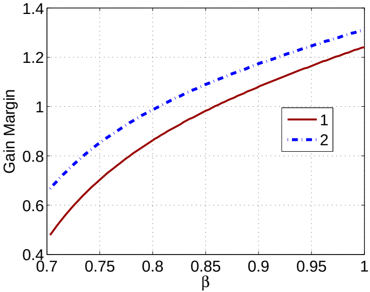
(a) Gain margin ($1 < \beta \leq 11$)



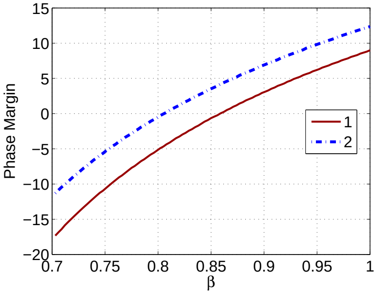
(b) Phase margin ($1 < \beta \leq 11$)

Figure 18: Calculated gain and phase margins for PID controllers. Key: (1) using Table 8 ($\xi = 0.707$); (2) using Table 9($\xi = 1$)

Gain and Phase Margins: PI



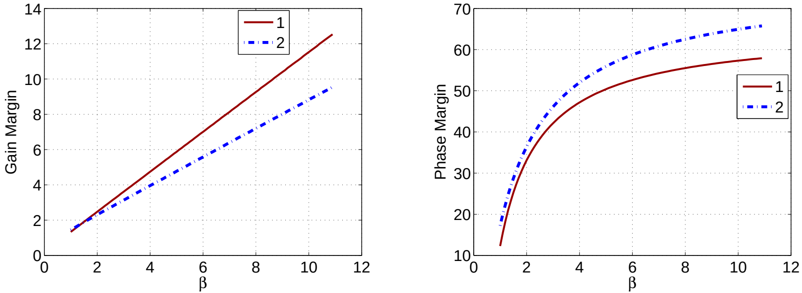
(a) Gain margin ($0.7 \leq \beta \leq 1$)



(b) Phase margin ($0.7 \leq \beta \leq 1$)

Figure 19: Calculated gain and phase margins for PI controllers. Key: (1) using Table 8 ($\xi = 0.707$); (2) using Table 9($\xi = 1$)

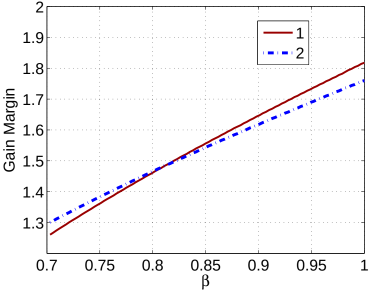
Gain and Phase Margins: PI



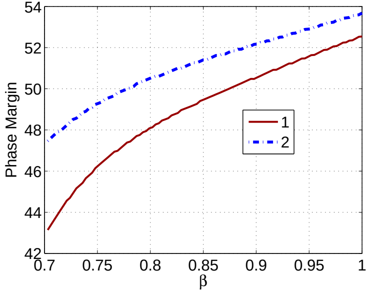
(a) Gain margin ($1 < \beta \leq 11$) (b) Phase margin ($1 < \beta \leq 11$)

Figure 20: Calculated gain and phase margins for PI controllers. Key: (1) using Table 8 ($\xi = 0.707$); (2) using Table 9($\xi = 1$)

Gain and Phase Margins: PD



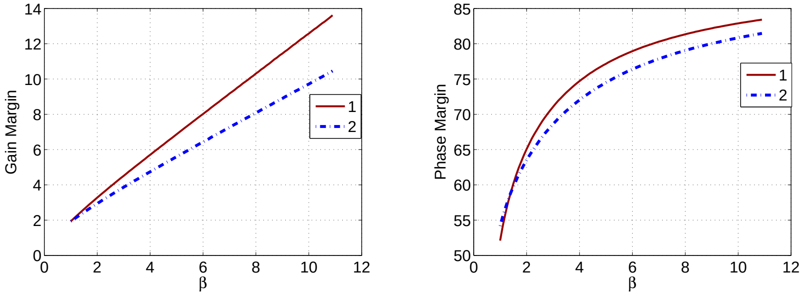
(a) Gain margin ($0.7 \leq \beta \leq 1$)



(b) Phase margin ($0.7 \leq \beta \leq 1$)

Figure 21: Calculated gain and phase margins for PD controllers. Key: (1) using Table 8 ($\xi = 0.707$); (2) using Table 9($\xi = 1$)

Gain and Phase Margins: PD



(a) Gain margin ($1 < \beta \leq 11$) (b) Phase margin ($1 < \beta \leq 11$)

Figure 22: Calculated gain and phase margins for PD controllers. Key: (1) using Table 8 ($\xi = 0.707$); (2) using Table 9($\xi = 1$)

Outline

Example 1

The transfer function for the secondary system is assumed to have the form:

$$G_1(s) = \frac{2e^{-3s}}{s(s+1)} \tag{20}$$

- The proportional controller used to stabilize the secondary system is selected to be $K_{T_1} = 0.04$.
- In the simulation, a zero mean white noise with standard deviation of 0.025 was added to the measured output.
- The relay amplitude is selected to be 1.75 and hysteresis is 0.2 to prevent the relay from the switching caused by the random noise.

Input and Output data

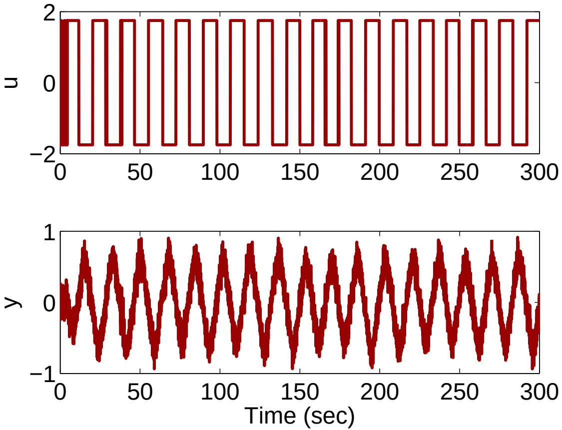


Figure 23: Relay feedback control signals from closed-loop system: top figure input signal; bottom figure output signal.

Estimation Result

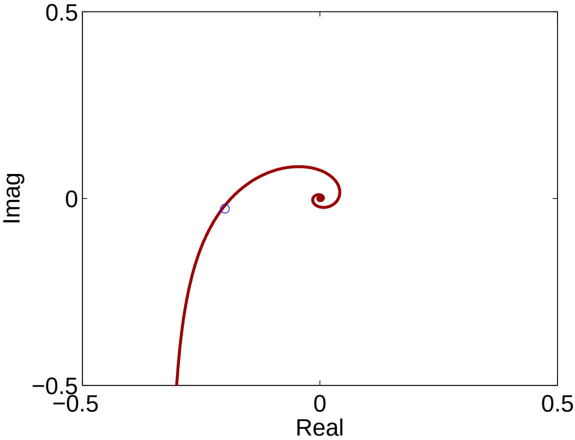


Figure 24: Nyquist loci with with $K_{T1} = 0.04$. on which o is the estimated value at $\omega_1 = \frac{2\pi}{T}$.

PID Controller Parameters

Model

With the frequency response value of the secondary system, the following integrator with delay model is calculated:

$$G_p(s) = \frac{1.7852e^{-4.0547s}}{s}$$

Controller

Choosing $\beta = 2$, which gives the desired closed-loop time constant about 8 second,

$$K_c = 0.0844; \ \tau_I = 21.0864; \tau_D = 1.0449$$

Nyquist plot

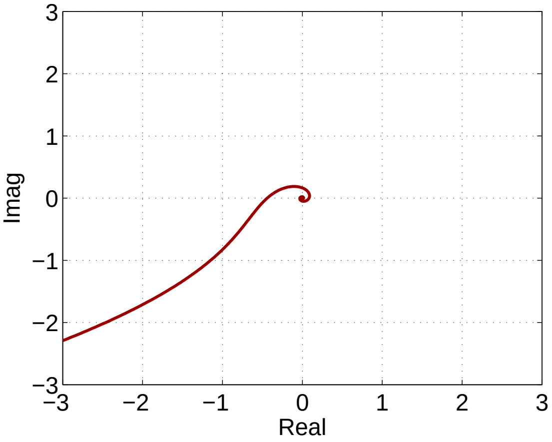


Figure 25: Nyquist curve with $C_1(j\omega)$ auto-tuned.

Closed-loop Response

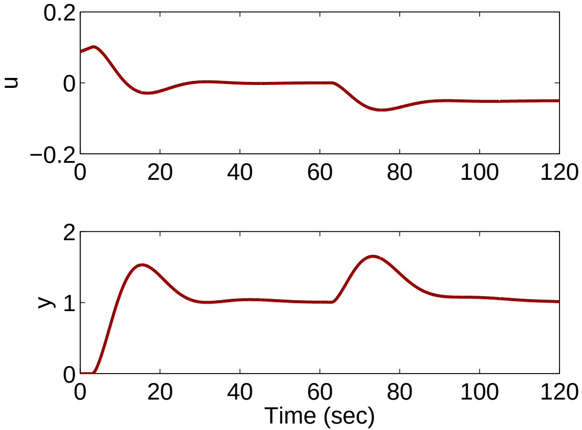


Figure 26: Closed-loop response using with the auto-tuned controller.

Example 2 (i)

- Assume that a second order system with time delay is described by the transfer function

$$G(s) = \frac{e^{-3s}}{(8s+1)(6s+1)}$$

- Choose the proportional feedback control gain $K_T = 0.6$, and relay amplitude of 1.75 and hysteresis of 0.2.
- Find the PID controller parameters for $\beta = 1.5$ and $\xi = 0.707$.

Example 2 (ii)

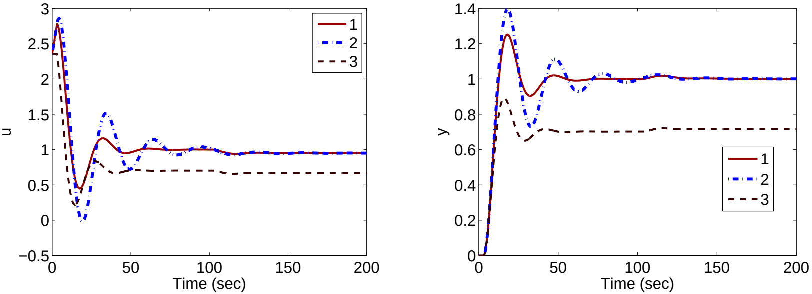
- The auto-tuner found the $G(j\omega_1) = -0.2195 - j0.0637$ where $\omega_1 = 0.2513$.
- With this information, the integrator plus time delay model becomes

$$G_p(s) = \frac{0.0575e^{-5.1263s}}{s}$$

- By choosing $\beta = 1.5$, the PID controller parameters are found as

$$K_c = 2.3519; \;\; \tau_I = 17.0325; \;\; \tau_D = 1.3555$$

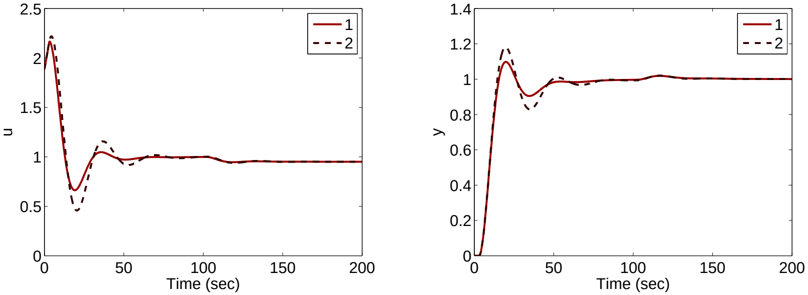
Example (2) Simulation Results



(a) Control signal (b) Output

Figure 27: Comparison of closed-loop performance for three types of controllers ($\beta = 1.5$, $\xi = 0.707$). Key: (1)PID control response; (2)PI control response; (2)PD control response

Example (2) Reducing β



(a) Control signal (b) Output

Figure 28: Comparison of closed-loop performance for two types of controllers ($\beta = 2$, $\xi = 0.707$). Key: (1)PID control response; (2)PI control response

Example (3)(i)

- Consider the system with transfer function

$$G(s) = \frac{3e^{-3s}}{(2s+1)^4}$$

- Use auto-tuner to find the PID controller parameters for this system.
- $\beta = 1$ and $\xi = 0.707$ are selected for fast disturbance rejection.
- Feedback control gain $K_T = 0.2$, and relay amplitude of 1.75 and hysteresis of 0.2 are used in the simulation.

Example (3) (ii)

- The estimated frequency is

$$G_p(j\omega_1) = -1.6274 - j0.1490$$

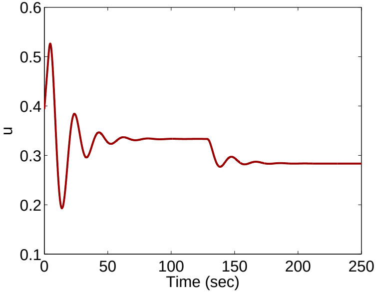
- The integrator plus delay model is

$$G(s) = \frac{0.2175e^{-5.0272s}}{s}$$

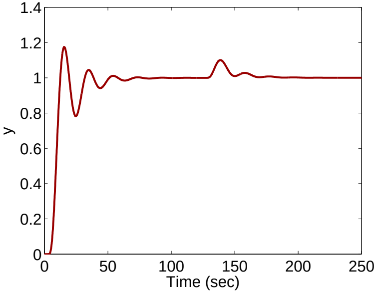
- With $\beta = 1$ and $\xi = 0.707$, the following PID controller parameters are found using the tuning rules:

$$K_c = 0.3936; \;\; \tau_I = 12.0156; \;\; \tau_D = 1.8895$$

Example (3) Simulation Results



(a) Control signal



(b) Output

Figure 29: Closed-loop response ($\beta = 1$, $\xi = 0.707$)

Example (4) (i)

- Consider the system with transfer function

$$G(s) = \frac{(-s+1)e^{-s}}{(3s+1)(2s+1)}$$

- Use auto-tuner to find the PID controller parameters for this system.
- $\beta = 1$ and $\xi = 0.707$ are selected for fast disturbance rejection.
- Feedback control gain $K_T = 0.2$, and relay amplitude of 1.75 and hysteresis of 0.2 are used in the simulation.

Example (4) (ii)

- The estimated frequency response is

$$G_p(j\omega_1) = -0.4073 - j0.2325$$

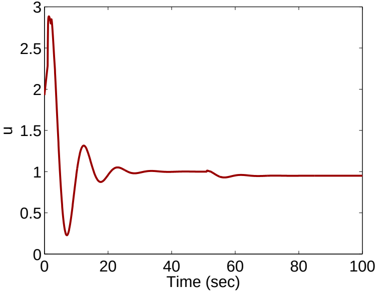
- The integrator plus delay model is

$$G(s) = \frac{0.2175e^{-2.2689s}}{s}$$

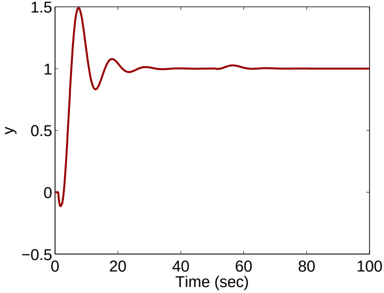
- The PID controller parameters are

$$K_c = 1.9286; \quad \tau_I = 5.4229; \quad \tau_D = 0.8528$$

Example (4) Simulation Results



(a) Control signal



(b) Output

Figure 30: Closed-loop response ($\beta = 1$, $\xi = 0.707$)