Yanyan Wu,[1]
GE Global Research,
Schenectady, NY

Jami J. Shah

Joseph K. Davidson

Department of Mechanical
and Aerospace Engineering,
Arizona State University,
Tempe, AZ 85287-6106

Computer Modeling of Geometric Variations in Mechanical Parts and Assemblies

This paper reports on part of a project related to the development of a computer model for GD&T (Geometric Dimensioning and Tolerancing) to support tolerance specification, validation and tolerance analysis. The paper examines the basic elements involved in geometric variation and their interrelations. Logical tolerance classes are defined in terms of a target, a datum reference frame, and metric relations. ASME Y14.5 tolerance classes are mapped to these logical classes. The development of a data model for GD&T and its application in supporting design specification, validation, and tolerance analysis are discussed. [DOI: 10.1115/1.1572177]

1 Introduction

The desired requirements for a computer model (data structure) for GD&T are: (1) completeness, (2) compatibility, (3) computability, and (4) validity. Completeness implies that the model must have the ability to support all the information needed to define all tolerance classes, i.e., the ability to store all the tolerance information, including the information about the datum reference frame (DRF) and the precedence, etc. Compatibility implies the ability to be consistent with engineering practice, particularly with national and international standards, such as the ASME Y14.5 [1,2] and ISO 1100 [3]. Computability implies that the model must be understandable to computers without human interaction to enable GD&T reasoning and the tolerance analysis. The model must capture the semantics of geometric variations so it can be manipulated in order to answer questions of interest. Validity means that incorrect or illegal GD&T specifications should be detected and resolved. This includes over-, under-, or conflicting-dimensioning or tolerancing, or inadequate or improper controls. It would be even more attractive to make the model self-validating, i.e. a model that does not allow incorrect GD&T to be specified in the first place. It is also desirable that the underlying model not only enforce the hard rules specified in the standards, but also advise on good practice rules for practical design, manufacturing, and inspection experience.

In order to develop a computer model, one must have a clear interpretation of Y14.5. Because the Y14.5 standard has ambiguities, these must first be resolved. Each type of dimensional and geometric variation has different significance in engineering. The tolerance standards contain a classification of these variations. For the purpose of computer modeling, each tolerance class defined in the standard should be interpreted in terms of the controlled entity type, the nature of the geometric/dimensional variation, a datum reference frame if applicable, and metric relations involved. However, tolerance classes are not defined in this manner in the standards. Further, a tolerance specification should not only be technically correct, it should also be economically reasonable; it should be able to achieve the desired level of control without increasing the manufacturing and inspection cost over other alternatives. Some seemingly similar tolerance classes have very different consequences. This needs to be clearly understood when building a GD&T advisor to help designers specify tolerances.

[1]Corresponding author.

Contributed by the Computer Aided Product Development (CAPD) Committee for publication in the JOURNAL OF COMPUTING AND INFORMATION SCIENCE IN ENGINEERING. Manuscript received November 2002; Revised February 2003. Guest Editor: A. Desrochers.

2 Literature Review

In an effort to represent GD&T in computer systems, researchers have proposed various attribute models, offset models, parametric, kinematic, and DOF models.

Attribute models: The basic characteristic of attribute models is that a tolerance is directly stored as an attribute of either geometric entities or metric relations in CAD systems [4–10]. The common deficiency of these approaches is that they cannot do validation since GD&T semantics is not built into the model structure.

Offset models: In this approach, the maximal and minimal object volumes were obtained by offsetting the object by corresponding amounts on either side of the nominal boundary [11,12]. Offset models can only represent a composite tolerance zone; they cannot distinguish between effects of different tolerance types, nor interrelations among tolerance specifications.

Parametric models: Tolerances are modeled as ± variations of dimensional or shape parameters. Parameter values can be found by a set of simultaneous equations representing the constraints [13–15]. Most commercial CAT systems use this approach. The parametric equations can be used for point-to-point tolerance analysis rather than zone based analysis.

Kinematic models: Bodies are modeled in terms of links and joints. A kinematic link is used between a tolerance zone and its datum features [16–18]. Tolerance analysis is based on vector additions. The first order partial derivative of analyzed dimension with respect to its component dimensions in terms of a transformation matrix was employed for tolerance analysis. Both the parametric model and kinematic model can represent all the tolerance classes, but not all the information involved in GD&T can be stored. Datum systems cannot be validated and the analysis is point based rather than zone based.

DOF models treat geometric entities (points, lines, planes) as if they were rigid bodies with degrees of freedom (DOFs) [19–21]. Geometric relations (angular and linear) are treated as constraints on DOFs. Y14.5 tolerance classes are characterized by how each DOF of each entity is controlled. Technologically and Topologically Related Surfaces (TTRS) models bear many similarities to DOF models [22,23]. Later researchers have tried to express Y14.5 tolerance classes in terms of TTRS but this is not fully achieved. Although mathematically elegant, TTRS models are indifferent to Y14.5 Rule #1, floating zones, effects of bonus and shift, form tolerance, or datum precedence. DOF models facilitate the validation of DRF and tolerance types.

3 Morphology of Geometric Variations

3.1 Basic Elements of GD&T. There are three major elements involved in GD&T: geometric entities, metric relations, and geometric variations (allowed variation in shape and size), as shown in Fig. 1. Geometric entities involved in GD&T include vertices, edges, faces, and the symmetry elements of a face (a feature of size). A feature of size could be a cylindrical face, a spherical face, or a slot. Each feature of size has a size dimension. A face can be a planar face, or a free form surface. A free form surface is a face other than a planar face or a feature of size. Metric (nominal) relations are used to control the size and the shape of a part. Metric relations may be linear, orientation, shape/form, or mixed. Linear dimensions include distance/location and size (radius or diameter, width). A shape/form relation defines the intrinsic shape of a geometric entity. A shape/form relation includes one or several linear, and orientation metric relations. An orientation relation could be an angle, a perpendicular relation, or a parallel relation. A mixed dimension is a combination of both linear and angular relation, e.g., concentric relation, coincident, or tangent relation. Metric relations could be unary or binary and a size relation can be unary or binary. A shape relation is always unary. Metric relations between geometric entities are often referred to as nominal dimensions and shape definitions. Since it is not possible to produce entire batches of parts to exact dimensions under normal manufacturing conditions, tolerances are specified to relax slightly these nominal dimensions/relations by indicating the limits of acceptable variations. Each tolerance class can relax its corresponding type of metric relation (size, location, orientation, form). There are also GDT classes that control multiple variations simultaneously (concentricity, runout). A profile tolerance can belong to any of four types. Tolerances can be specified to geometric entities independently or with respect to other geometric entities (datums) depending on unary/binary relation type.

3.2 Metric Relations in GD&T and DOF. The nominal shape, size, location of every geometric entity must be fixed with respect to all other geometric entities on a part. The ways in which the shape, size, location of a geometric entity can vary will be referred to as Degrees Of Freedom (DOF). Metric relations constrain the DOFs of geometric entities with respect to each other. Kinematic DOF defines the independent ways in which an object's geometry can change. If one thinks of geometric entities as rigid bodies, the spatial displacements can be resolved into six kinematic DOFs. Those kinematic DOFs are divided into three translational degree of freedom (TDOF) along the X, Y, Z axis of the coordinate system and three rotational degree of freedom (RDOF) around the X, Y, Z axes of the coordinate system. Because of dimensionality, shape and symmetry of an object, it may be translationaly invariant in some directions and rotationally invariant in some orientations, i.e. it does not change the shape, size, or location when it is displaced in an invariant direction. Thus the total number of DOF required to locate an object is less than six. For example, a point has three TDOFs along the X, Y, Z axes of the coordinate system. A line has two TDOFs along two directions, orthogonal to each other and to the line direction, and two corresponding RDOFs along the same directions as those for the two TDOFs. A plane has one TDOF along the direction parallel to the plane's normal, and two RDOFs along the two directions, which are orthogonal to each other and are perpendicular to the plane's normal. In addition to appropriate kinematic DOF, a feature of size has a size DOF.

Table 1 shows all metric relations applied to different combinations of point, line, plane, and feature of size. It also shows the number and type of DOFs constrained for each metric relation. Since a feature of size can be represented by its axisymmetric elements with a size dimension, a distance or angular relation specified to a feature of size can be treated as being specified to its axisymmetric element. In Table 1, a feature of size is indicated by the symbol FOS. ''Point (FOS)'' represents a spherical face. ''Plane (FOS)'' represents a slot. ''Line (FOS)'' represents a cylindrical face. The kinematic DOFs of a geometric entity are able to control the same types of DOFs of other entities. Metric relations can be distinguished based on types of controlled kinematic DOF. A shape DOF is defined by a shape/form relation, which could be one or several linear, and orientation relations. Metric relations can be classified into four groups according to the type of DOF they control: size, location, orientation, and shape as shown in Column 1 of Table 1. A size dimension controls size DOF and shape DOF, a location relation controls TDOF, RDOF and shape DOF of an entity, an orientation relation controls RDOF and shape DOF of an entity, a shape relation controls the shape DOF of an entity.

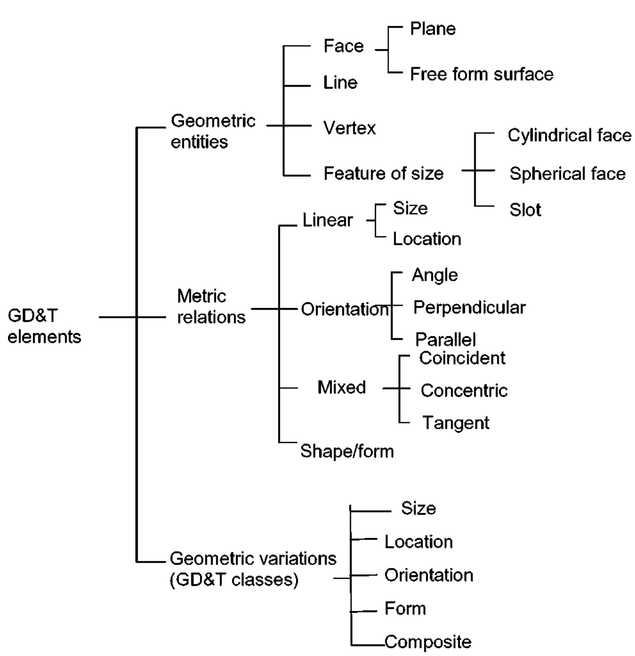


Fig. 1 Elements of GD&T

3.3 Classification of Variations. In order to be consistent with the four metric relation classes discussed in Section 3.2, geometric variations (tolerances) can be classified into the same four classes, i.e., size, location, orientation, and shape (form). Later we will demonstrate that this facilitates tolerance validation and automated tolerance specification. The size class controls the size DOF directly and the shape DOF of the target indirectly. The location variation directly controls the location of a target from a reference. It also indirectly controls the orientation, the shape of the target. Thus, it controls the TDOF, and RDOF and the shape DOF, of the target. The orientation class controls the angular relations (RDOF) of a target with respect to its datum directly and also the shape of the target, indirectly. The shape variation controls the form/shape of a target without any datum (shape DOF).

Instead of classifying tolerances into six classes, as in the Y14.5 standard, one can map these six classes into the four logical classes shown in Table 2. A Y14.5 tolerance may appear in different logical tolerance categories in Table 2 depending on which metric relation is applicable. Although the tolerances that were classified into one class control metric relations of a common type, they still have some differences in that the elements of the target entity and the level of control are different. The characterization of Y14.5 classes includes the type of the target, metric relations constrained by the tolerance, and the qualified datums. This is a parametric CAD oriented characterization that is consistent with Y14.5 and that is needed to satisfy the computability and validity requirements. Establishing the relation between the tolerances and metric relations, tolerance model, CAD model, and constraint model can be integrated. For example, a tolerance can only exist when the corresponding metric relation exists. Tolerance validation based on the logical tolerance class presented here is

Table 1 Metric relations and the controlled DOFs

Metric relation groups	Case #	Entities involved		Metric relations	Implied relation	Constrained TDOF & RDOF (type, number, and direction of control)
Size	1	Cylinder sphere		Radius/ Diameter		1 Size DOF; Shape DOF;
	2	Slot		Width		
Location (Distance)	3	Point	Line	Distance, Coincident		1TDOF: direction⊥ to line direction; Shape DOF if any;
	4	Point-FOS	Line-FOS	Concentric		
	5	Point	Plane	Distance, Coincident		1TDOF: along plane normal; Shape DOF if any;
	6	Line	Plane	Distance, Coincident	Line// plane	1TDOF: along plane normal; 1RDOF: direction = plane normal× line direction; Shape DOF;
	7	Plane	Plane	Distance, coincident, symmetric	Plane// plane	1TDOF: along plane normal; 2RDOF: direction = ⊥ plane normal; Shape DOF;
	8	Plane-FOS	Plane-FOS	Coincident		Same as above, plus Size DOF;
	9	Line	Line	Distance	Not co-planar	(not //) 1TDOF: direction=line1×line2; Shape DOF;
				Distance		(//) 1TDOF: direction = line direction × normal of the plane of two lines or other specified direction ⊥ to both line; Shape DOF;
	10	Line	Line	Coincident		2TDOF: direction⊥ to line direction; 2RDOF: direction⊥ to line direction; Shape DOF;
	11	Line-FOS	Line- FOS	Concentric		Same as above; plus Size DOF
	12	Line-FOS	Line —FOS	Coincident		Same as above, plus 1 Size DOF
	13	Point	Point	Distance		1TDOF: direction along connecting line or other specified direction.
	14	Point	Point	Coincident		3TDOF: along any direction
	15	Point-FOS	Point-FOS	Concentric		3TDOF: along any direction
	16	Point (FOS)	Point (FOS)	Coincident		Same as above, plus 1Size DOF
Orientation (Angular)	17	Line	Line	Angle, Parallel, Perpendicular		Angle≠0(including ⊥) 1RDOF: direction=line1×line2; Shape DOF;
						Angle=0(//) 2RDOF: direction ⊥ line direction Shape DOF;
	18	Line	Plane			(not ⊥)1RDOF: direction = plane normal × line direction; Shape DOF;
						(⊥)2RDOF: direction = ⊥ plane normal; Shape DOF;
	19	Plane	Plane			(angle≠0) 1RDOF: direction =plane normal × plane direction; Shape DOF;
						(angle=0) 2RDOF: direction = ⊥ plane normal; Shape DOF
Shape	20	Line				Shape DOF
	21	Circular Line				
	22	Plane				
	23	Cylindrical face				
	24	Spherical face				
	25	Slot				
	26	Other entity				

more viable, since the validation rules are clearer. Yet the logical definitions can be hidden from the user, who can work entirely with Y14.5.

3.4 Relations Between Tolerance Classes. The relation among tolerances specified on the same target is called tolerance refinement. Two important aspects for building a correct GD&T model are the validation of the tolerance value and the prevention of redundant tolerancing. The refinement relation derived in this work is based on the logical tolerance classification. Before discussing tolerance refinement relations, different target elements controlled by each tolerance in the same tolerance class are studied in Table 3. Tolerances that are able to control 3D or 2D elements are labeled higher-dimensional tolerances here, while tolerances that only control 2D or 1D elements are labeled lower-dimensional tolerances. In each tolerance class, higher-dimensional tolerances are defined relative to lower-dimensional tolerances. The basic idea behind the tolerance refinement is that the tolerance zone of the tolerance that controls fewer DOF or controls lower dimensional elements can float in the tolerance zone of the tolerance that controls more DOF of the target or controls higher dimensional elements. This general rule can be further interpreted into more operation rules for different types of tolerance. In the following, these operations rules will be derived for three generic cases.

Refinement rule 1: Between two tolerances belonging to the same tolerance class but controlling different target elements, if they control the same set of metric relations, the tolerance value of the tolerance controlling higher-dimensional elements (higher-dimensional tolerance) should be equal or greater than the tolerance value of the tolerance controlling lower-dimensional elements (lower-dimensional tolerance).

Refinement rule 2 Between two tolerances belonging to the same tolerance class and controlling the same target elements, if the metric relations controlled by one tolerance is the sub-set of the metric relations controlled by the other tolerance, the tolerance value of the latter tolerance should be equal or greater than the tolerance value of the former.

Table 2 Classification of tolerance classes and the requirements of their tolerance specifications

Class	Tol.	DOF controlled	Target Type	Metric relation constrained	Datum: Primary	Datum: Secondary /tertiary
Size	±x	Size DOF & Shape	Feature of size	radius/ width	None	
Location	±x	TDOF RDOF & Shape DOF	All	Distance, concentric, or coincidence	HGR♣	None
	⌖		Feature of size		HGR♣ or HPP♣♣ or CCND♠	
	◎		Feature of size of revolution	Distance=0, concentric	Must be of same type as target, HGR♣	None
	⌓⌒		Free form surface	Distance, coincident	HGR♣ or HPP♣♣ or CCND♠	
	⌰↗		Feature of size of revolution	Distance=0, Concentric	1 to 2 datums. One must be an axis, which has HGR♣, the other should be a plane perpendicular to the datum axis	
	⌯		Any symmetric Feature of size	Distance=0, Coincidence	Either be a feature of axis or plane, which has HGR♣	None
Orientation	±x	RDOF & Shape	All, except point	Angle, Perpendicular, parallel	HGR♣	None
	⌖		Feature of size	Angle, Perpendicular, parallel	HGR♣	HPP♣♣
	⊥		All, except point	Perpendicular	HGR♣	HPP♣♣
	//		All, except point	Parallel, refine distance	HGR♣	HPP♣♣
	∠		All, except point	Angle	HGR♣	HPP♣♣
	⌓⌒		Free form surface	Angle, Perpendicular, parallel	HGR♣	HPP♣♣
	⌰↗		Planar face	Perpendicular	should be a feature of size of axis, HGR♣	None
Form	▱	Shape	Planar face	Shape	None	
	—		Axis, planar face, ruled surface	Shape or cylindrical radius	None	
	○		Revolved surface	Radius	None	
	⌭		Cylindrical surface	Cylindrical radius	None	
	⌓⌒		Free form surface	Shape or radius	None	

♣ HGR --- the datum Has the Metric Relation to be toleranced w.r.t. the target. For a tolerance in the location class, the datum needs to have a distance-type constraint with the target; for a tolerance in the orientation class, the datum needs to have an angle- ype constraint with the target.
♣♣ HPP --- the datum has Perpendicular /Parallel/Angle Relation w.r.t. the target to constrain RDOF of the tolerance zone.
♠CCND --- the datum Can Control a New DOF of the target that is not controlled by the previous datums
Datum entity type--- Except there are extra conditions, a datum can have any entity type. For an orientation tolerance, a datum cannot be a point.

Table 3 Tolerances and their controlling elements

Tolerance classes	Higher-dimensional tolerances	Lower-dimensional tolerances
	Control 2D or 3D elements	Control 2D or 1D elements
Location	⌖ ◎ ⌓ ⌰ ⌯	⌒ ↗
Orientation	± ⌖ ⊥ // ∠ ⌓ ⌰	⌒ ↗
Form	▱ ⌓ ⌭	○ ⌒ —

Refinement rule 3 Between two tolerances belonging to different tolerance classes, if there is a datum involved in either tolerance, the tolerance value becomes tighter in the sequence of location tolerance class, orientation tolerance class, and form tolerance class; if there is no datum involved in any tolerance, tolerance value of a size tolerance should be bigger than that of a form tolerance.

When there is a bonus tolerance, operation rule 3 needs to be modified. Consider a hole with a radius tolerance of $\pm\Delta r$, a location tolerance of Δp, and a straightness tolerance of Δs applied to its axis. For RFS condition, the values should satisfy the condition $\Delta p \geqslant \Delta s$. However, for MMC or LMC condition on positional tolerance (assume no material condition on datums), the tolerance values should satisfy: $4\Delta r + \Delta p \geqslant \Delta s$. If both straightness tolerance and positional tolerance apply at MMC, the tolerance values should satisfy $4\Delta r + \Delta p \geqslant 4\Delta r + \Delta s$, which is the same as $\Delta p \geqslant \Delta s$. It is assumed that the value of a size tolerance fully contributes to the bonus tolerance.

Table 4 Symbols used in the characteristic sub-graphs

Symbol	Meaning	Symbol	Meaning
□	An entity (a topological entity)	r	A radius
—◯—	A relation (a metric relation).	DI	A distance relation
■	An attribute (a tolerance)	C	A concentric relation
---------	A connection between a relation (metric relation) and an attribute (a tolerance)	P	A parallel relation
x', y', z'	Directions of locational relations	PP	A perpendicular relation
F_k	A topological face	SP	A shape constraint

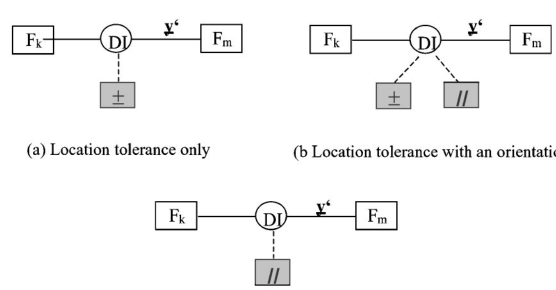
(a) Size constraint and tolerance
(b) Size constraint with size & form tolerance

Fig. 2 Characteristic sub-graphs for size and its tolerances

3.5 Combining Entities and Their DOFs in a DRF. Validation of DRF involves determining if the combination of 1, 2, or 3 datums can fully control the desired DOFs of another entity. One might also want to know what DOF of the target are constrained and if each datum can control new DOF other than these controlled by the previous datums. One can track controlled DOF of the target by identifying the controlled kinematic DOF for each metric relation between the target and the datum. The target is fully constrained by its datums if all of its DOFs are constrained. For example, to fully define a circular pattern with respect to the geometric entities outside the pattern, its axis, its radius/diameter, and its rotational orientation need to be constrained. In order to constrain the axis/line, two TDOFs and two RDOFs of the axis/line should be controlled by the datums. The rotation of the pattern causes the changes of the location of the entities inside the pattern with respect to the entities outside the pattern. This means that the rotation of the plane passing through the axis of any hole in the circular pattern and the axis of the circular pattern should be constrained.

4 Computer Representation of GD&T

This paper reports on part of a larger project that is developing a comprehensive set of tools for tolerance allocation, verification and analysis. This paper focuses only on the global model which is needed to (1) interrelate all D&T controls applied to all features of all parts and assemblies, (2) validate conformance of tolerance specification to Y14.5, (3) extract tolerance stacks for worst case and statistical analyses. The global model uses the idea of DOFs to provide a way to understand the relations between entities, what is controlled and how, and to track over and under constrained conditions. This idea also aids in computing the effects of datum precedence. DOF of a target feature should be fully constrained by the DOF of its datums. The geometric relations en-



(a) Location tolerance only
(b Location tolerance with an orientation
(C) Orientation tolerance only

Fig. 3 Sub-graphs for distance dimension and its tolerances

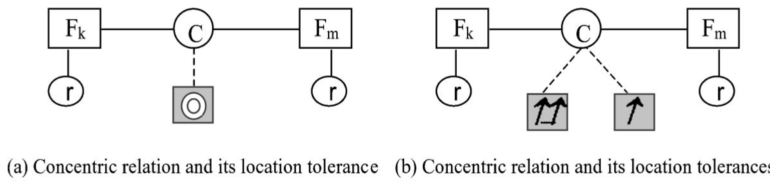


Fig. 4 Sub-graph for concentricity variation

force constraints between the constraints of a target and its datum. Tolerances are the specifications of the variation range of the DOF of a target feature with or without respect to a datum or a DRF.

The GD&T global model is a directed, attributed constraint graph, which combines features of the attribute and DOF approaches [20]. The nodes are topological entities; the arcs are metric relations between the node entities; the attributes of the model are tolerance specifications. Entities in the model (geometric entities) are related by metric relations. A tolerance is encoded as an attribute attached to the corresponding metric relations it constrains. The direction of an arc represents datum-target relation. Tolerances and metric relations are classified into four classes: size, location, orientation, and form (as shown earlier in Table 2).

Compatibility between metric relations (arcs) and tolerances (attributes) is enforced according to Table 2. The existence of a tolerance in the GD&T model requires the existence of the corresponding metric relations. Metric relations can connect to the tolerances belonging to the same type, or connect to the tolerance belonging to the class that controls fewer DOF. For instance, a form tolerance is connected to the size (radius) relation of a feature of size if the form of the face needs to be controlled or to the shape constraint of a feature of size if the form of the axisymmetric element of a feature of size needs to be controlled. Similarly, a parallelism tolerance could be connected to the location relation as a refinement.

A set of generic *characteristic sub-graphs* has been defined to relate tolerance classes to the corresponding metric relations and entities. To illustrate these, the symbology shown in Table 4 and Y14.5 standard [1] will be used. One to three examples for each class of metric relations and their corresponding tolerances will be given. Some special examples will be shown when a tolerance refinement relation is involved. Topological entities are represented by nodes in the graph. In a valid manifold object, each topological entity is associated with a geometric entity. This association is manipulated by a solid modeler and is not represented in the following constraint graphs demonstrating the GD&T representation model.

Figure 2(*a*) illustrates the sub-graphs for size D&T on a cylindrical face. A size tolerance (plus/minus tolerance) is connected to the radius implying the variation limits of the radius. A variation of this is shown in Figure 2(*b*) where the radius is toleranced simultaneously by the +/− size tolerance and the form tolerance (circularity). Thus, the circularity tolerance is also connected to the radius as it refines the size tolerance.

Figure 3 shows the characteristic sub-graph for distance dimension (D1) with applicable tolerances between two parallel faces, (F_k, F_m). The direction of the distance dimension is saved as a unit vector $\mathbf{v}'$. Figure 3(*a*) shows a plus/minus tolerance, applied to location to control the distance dimension, so it is directly connected to the distance arc. Figure 3(*b*) shows the addition of a parallelism tolerance between the same two faces. Since the parallelism tolerance refines the control of the plus/minus tolerance, it is attached to the same arc. Figure 3(*c*) is the sub-graph for a parallelism tolerance specified between two faces without explicit size tolerance; so the parallelism tolerance is connected to the distance relation.

Figure 4 shows two different sub-graphs for concentric (C) cylindrical faces. Both faces have a radius constraint r. When a concentricity tolerance is used to control the concentric relation, it is directly connected to it (Fig. 4(*a*)). When there are two tolerances specified, they are both connected to the concentric relation (Fig. 4(*b*)). The circular runout tolerance refines the total runout. Figure 5(*a*) and (*b*) show sub-graphs for parallel (P) and perpendicular (PP) relations, respectively.

A shape constraint can only be toleranced by a form tolerance. On the other hand, the existence of a form tolerance requires the existence of a corresponding metric relation, which is usually a shape constraint. In Fig. 6(*a*), the planar face is controlled by a

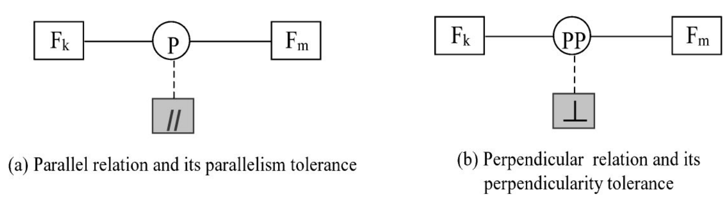


Fig. 5 Sub-graphs for orientation variations

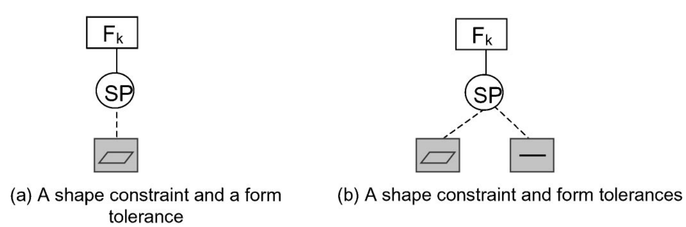


Fig. 6 Sub-graphs for shape variations

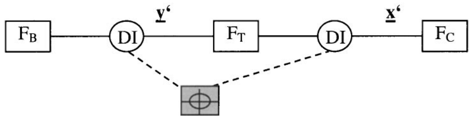


Fig. 7 Sub-graph for axis location

flatness tolerance. The flatness tolerance is directly connected to the shape constraint of the face. A flatness tolerance is a 2D control, which can be refined by a 1D control. A straightness tolerance is a 1D tolerance. In Fig. 6(*b*), the shape constraint of the plane is connected to a flatness and a straightness tolerance. The former tolerance is refined by the latter.

A tolerance needs to be connected to all the metric relations it controls. A hole (face F_T) can be located with two distance relations with two planes (say, plane F_B, F_C) along different directions. The hole also can be perpendicular to a planar face it sits on. One can specify a positional tolerance in the location tolerance class on the hole with respect to this latter plane and F_B, F_C. Since a location tolerance only controls the distance relation, and not the perpendicular relation (the perpendicular relation just helps to orient the tolerance zone), the positional tolerance is connected to the two distance relations between the hole and F_B, between the hole and F_C. The direction of each distance relation is saved as a part of the data of the distance relation (Fig. 7).

To illustrate how the characteristic sub-graphs are combined to

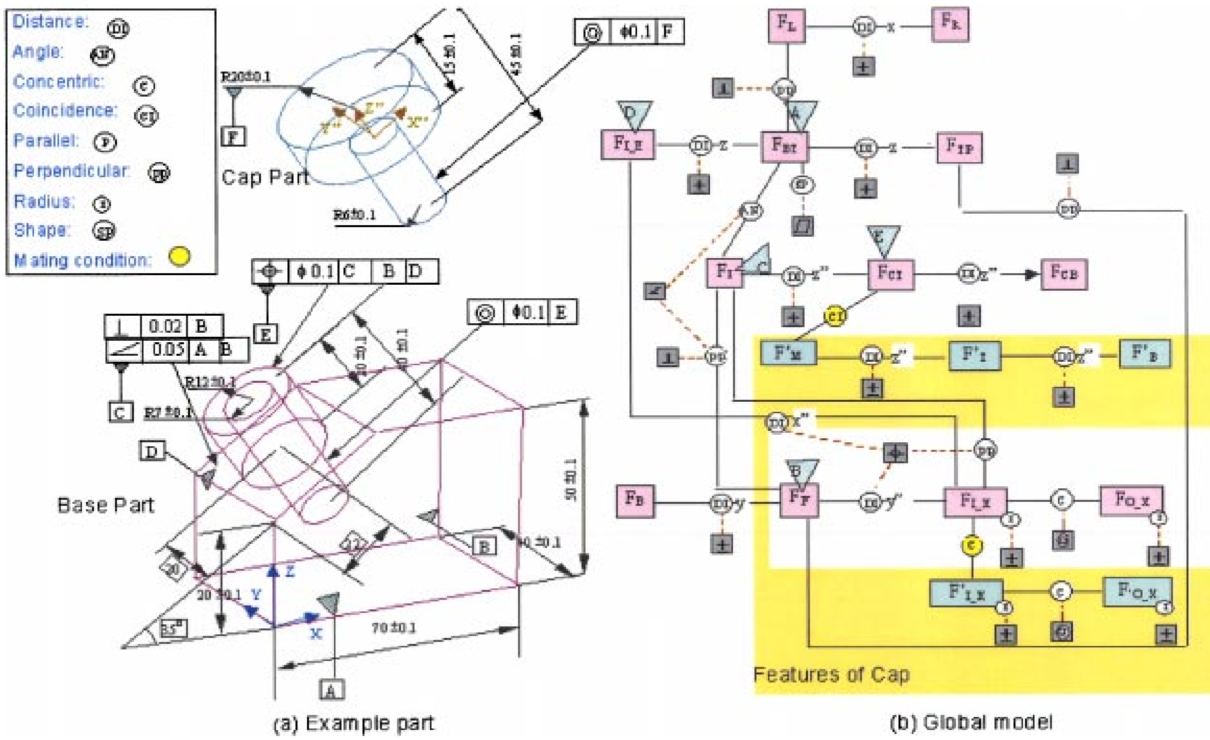


Fig. 8 GD&T global model for a simple assembly (partial)

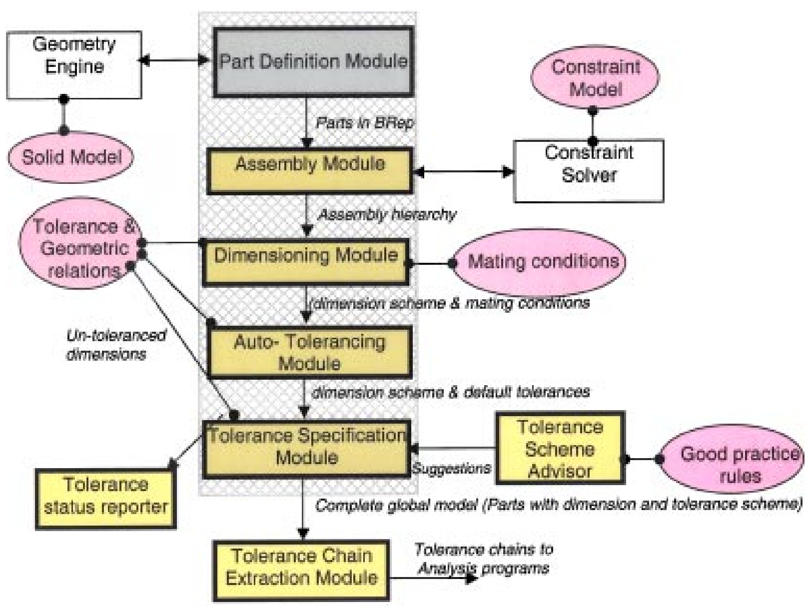


Fig. 9 Architecture of the Integrated GD&T System

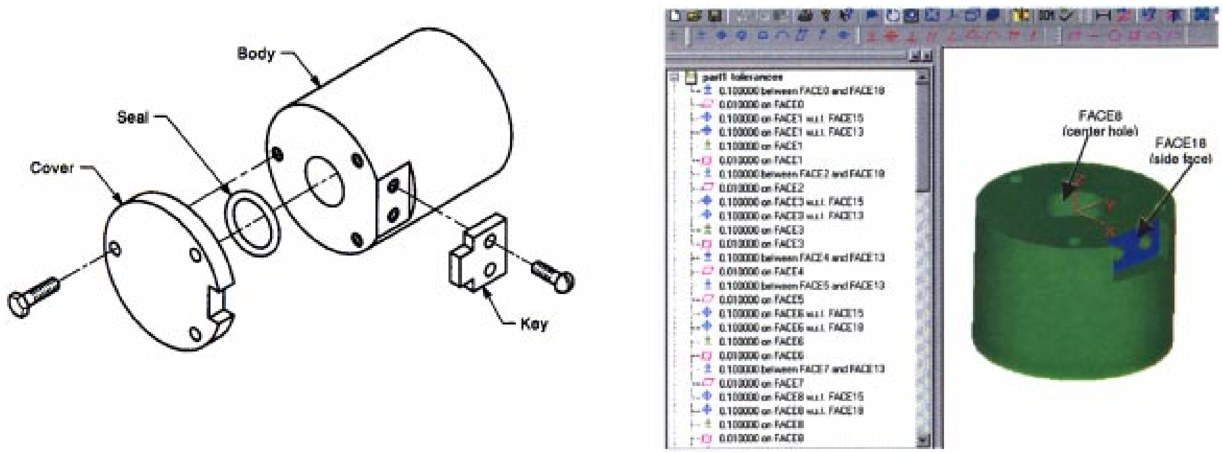


Fig. 10 Auto-tolerancing applied to part "Body"

get the global model, consider an assembly consisting of two parts (base+cap) as given in Fig. 8. The face labels are shown only in the graph and not the geometry to simplify the illustration. For the base, the face labels are: left face F_L, right face F_R, bottom face F_{BT}, inclined planar face F_I, top face F_{TP}, back face F_B; front face F_F; F_{I_E} is an edge on the inclined face F_I; F_{CT} is the top face of the inclined boss; F_{I_X} is the hole face of the inclined boss; F_{O_X} is the outside face of the inclined boss; F_{CB} is the bottom face of the hole. For the cap, F'_{I_X} is the smaller and F'_{O_X} is the larger cylindrical face; F'_M is the planar face on the cap that mates with F_{CT} of the base; F'_T is the larger planar face of the cap; F'_B is the bottom face of the cap.

It can be seen from the graph, that the same type of metric relation can appear between the same pair of geometric entities only once. In most cases, each metric relation is connected to only one tolerance. If there are more on the same metric relation (e.g., the perpendicular relation between face F_I and face F_F in Fig. 8), tolerance refinement relation should be satisfied between these tolerance specifications (the angularity tolerance and the perpendicularity tolerance in this case). One tolerance may constrain one or more metric relations. For example, the perpendicularity tolerance specified on the inclined face (F_I) controls only the perpendicular relation with respect to datum B (the front face F_F); however, the angularity tolerance specified to the same face (F_I) controls two metric relations (perpendicular relation with respect to datum B or face F_F and the angle relation with respect to datum A or F_{BT}). The metric relations are consistent in this model with respect to the relations within the same part and between different parts.

The process of constructing a global GD&T model for a part or an assembly involves two steps: (1) identify the metric relations of the dimension scheme, (2) connect the tolerance to the corresponding metric relations it controls. Since the prerequisite of the tolerance analysis is an assembly including geometrically and topologically valid parts, no geometry and topology change is conducted during GD&T model construction, but the relative positions of the parts can be changed. For the metric relations on the same part, only those consistent with the part shape can be added to the GD&T model. Compatibility among the geometric entities, metric relations, and tolerances can be maintained according to the relations listed in Tables 1 and 2. Each tolerance specification is attached as an attribute of the metric relations the tolerance controls. Inside each tolerance specification, the related tolerance data is saved in the global model.

5 Implementation

5.1 Integrated System Architecture. An integrated GD&T specification and analysis system based on the proposed GD&T global model has been implemented in C++ with a modular architecture, as shown in Fig. 9. Some of the modules are: Part Definition (Parametric CAD), Assembly Definition, Dimensioning, Default Tolerancing, and Tolerance Specification. When the user identifies the dimension of interest, the Tolerance Chain Extraction Module can find the corresponding tolerance chain for analysis. Tolerance Analysis modules will not be discussed here.

The Part Definition Module, based on ACIS,[3] is used for the creation of the parameterized solid models of parts. The Assembly Module supports the building of the assembly structure. When positioning a part, the mating conditions are solved through a commercial constraint solver (DCM[4]). The Dimensioning Module is used for specifying a dimensioning scheme or mating conditions, i.e. to specify metric relations among geometric entities on the same part or on different parts. Auto-dimensioning can generate default dimensions based on the functionality of the part. The default dimensions can be over-ridden. The values of dimensions (metric relations) are automatically extracted from the solid model. If the metric relation selected by the user is not consistent with the geometry of the model itself, it cannot be added to the part. The user is required to give the value of the metric relation when it is a mating condition, which is applied to the geometric entities on the different parts. At any stage during dimensioning, scheme validation can be conducted for each entity and metric relation on a part, if fully constrained, under or over constrained, and to check if a metric relation can be solved or if it is redundant. The output of the Dimensioning Module is a partial GD&T model that contains the dimension scheme and mating conditions. The Auto-Tolerancing Module provides the user default tolerances to all the existing metric relations of a part. The Tolerance Specification Module allows the designer to interactively specify and concurrently validate tolerances. Modules relevant to this paper are discussed below.

5.2 Auto-Tolerancing. Tolerances on mating features are dependent on part function and design intent; much of this knowledge is experiential and domain specific. This raises the question: is it even feasible to create an intelligent auto-tolerancing tool across all mechanical functions? Whereas this goal is not realizable today without a repository or model relating functions, ge-

[3]ACIS is a registered trademark of Spatial Technologies Corporation
[4]DCM 2D, DCM 3D are trademarks of D-Cubed Ltd

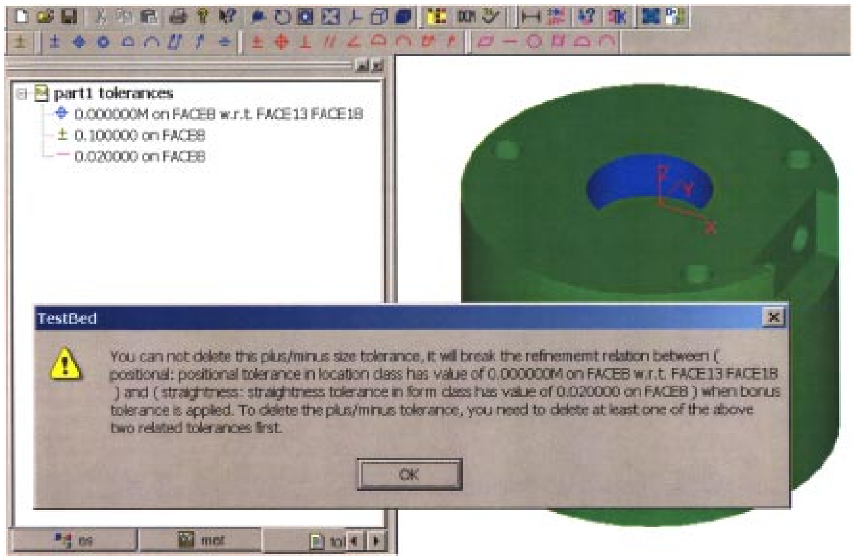


Fig. 11 Example of tolerance validation

ometry and performance level, it is possible to generate plausible tolerance schemes based on feature types, metric relations and/or mating constraints, on the basis of Table 2. Auto-tolerancing is not just an effort saver, but it shows the designer *what* needs to be controlled and *how* to control it. In the Tolerance Spec module, one can explore and validate alternative ways of achieving the same result.

In CAD, a metric relation is bi-directional—no distinction between target and datum. But most geometric tolerances have a control direction. Before auto-tolerancing, it is necessary to ''directionalize'' a metric relation for distance or angular dimension between 2 entities, to determine the default datum. The following datum selection rules have been formulated.

1. if one face is non-planar and the other planar, the planar face is chosen as the datum;
2. the geometric entity that has more metric relations should be chosen as the datum;
3. the entity that has a bigger area should be chosen as the datum.

These rules are based on good practice rules that consider manufacturing and inspection feasibility.

Auto-tolerancing proceeds as follows. Shape constraints generate a form tolerance of the corresponding type (Table 2): flatness for planar faces, cylindricity for cylindrical faces, and profile tolerance on other types of faces. A size dimension is used to generate a plus/minus tolerance by default. A location dimension (distance, coincident or concentric relation) generates a location tolerance associated with the metric relation (Table 2). For concentric relation, a concentricity tolerance will be associated with it. For other types of location, if the target entity is a feature of size, its default tolerance is a positional tolerance; otherwise, a plus/minus tolerance is specified to the dimension. An angular dimension will generate an associated orientation tolerance (Row 9–15, Table 2); parallelism, perpendicularity, angularity for the corresponding type of orientational metric relation. For location and orientation tolerances the datum is selected based on datum rules given above.

Figure 10 shows an example assembly taken from the ASME standard [1]. On the right side, it shows the default tolerances generated by Auto-Tolerancing for the main body. There is a distance relation of 30 between the center hole FACE8 and the side planar face FACE18. Since FACE18 is a planar face, it is chosen as the datum for the tolerance applied on the distance between FACE18 and FACE8. FACE8 is a feature of size, so a positional tolerance is specified as the default tolerance to control the distance relation between FACE8 and FACE18. The shape of FACE18 is controlled by a flatness tolerance. The shape of FACE8 is constrained by a cylindricity tolerance. The radius of FACE18 is specified with a plus/minus tolerance.

5.3 Tolerance Validation. Validation of tolerance specifications consists of two parts: individual tolerances and tolerance specifications on the same target with respect to different datums. Individual tolerance checks include:

1. Feature type validation: The tolerance should be applicable to the target entity type (Column 4, Table 2).
2. Tolerance class validation: The tolerance class should be consistent with the type of metric relations that it controls (Column 5, Table 2).
3. Datum validation 1: Each datum should have the required entity type (Column 6, Table 2).
4. Datum validation 2: Each datum must be related to the target with metric relations corresponding to the geometric tolerance class. (Column 5, Table 2)
5. Datum validation 3: Each datum in a DRF is valid if it contributes a new DOF that cannot be controlled by others in the DRF.
6. Tolerance value validation 1: A tolerance value should be appropriate for the tolerance type. A tolerance value of zero is permitted for a feature of size under MMC or LMC condition when a positional tolerance, an angularity tolerance, a parallelism tolerance, or a perpendicularity tolerance is applied.
7. Tolerance value validation 2: When validating the tolerances specified on the same target entity, the tolerance refinement relation (Refinement rule 1 to 3) discussed in Section 3.4 can be applied. One also has to check that the dimensions controlled by the geometric tolerances in the location and orientation tolerance classes are basic dimensions. This means that no tolerance refinement relation can be applied to +/− tolerances within the location and the orientation tolerance classes.
8. Tolerance zone validation: The tolerance zone depends on the tolerance and the target entity type. If the target is a cylindrical face, when the axis is the element to be controlled, the zone can be cylindrical for a straightness tolerance, an angularity tolerance, a perpendicularity tolerance, a parallelism tolerance, a positional tolerance, or a concentricity tolerance.

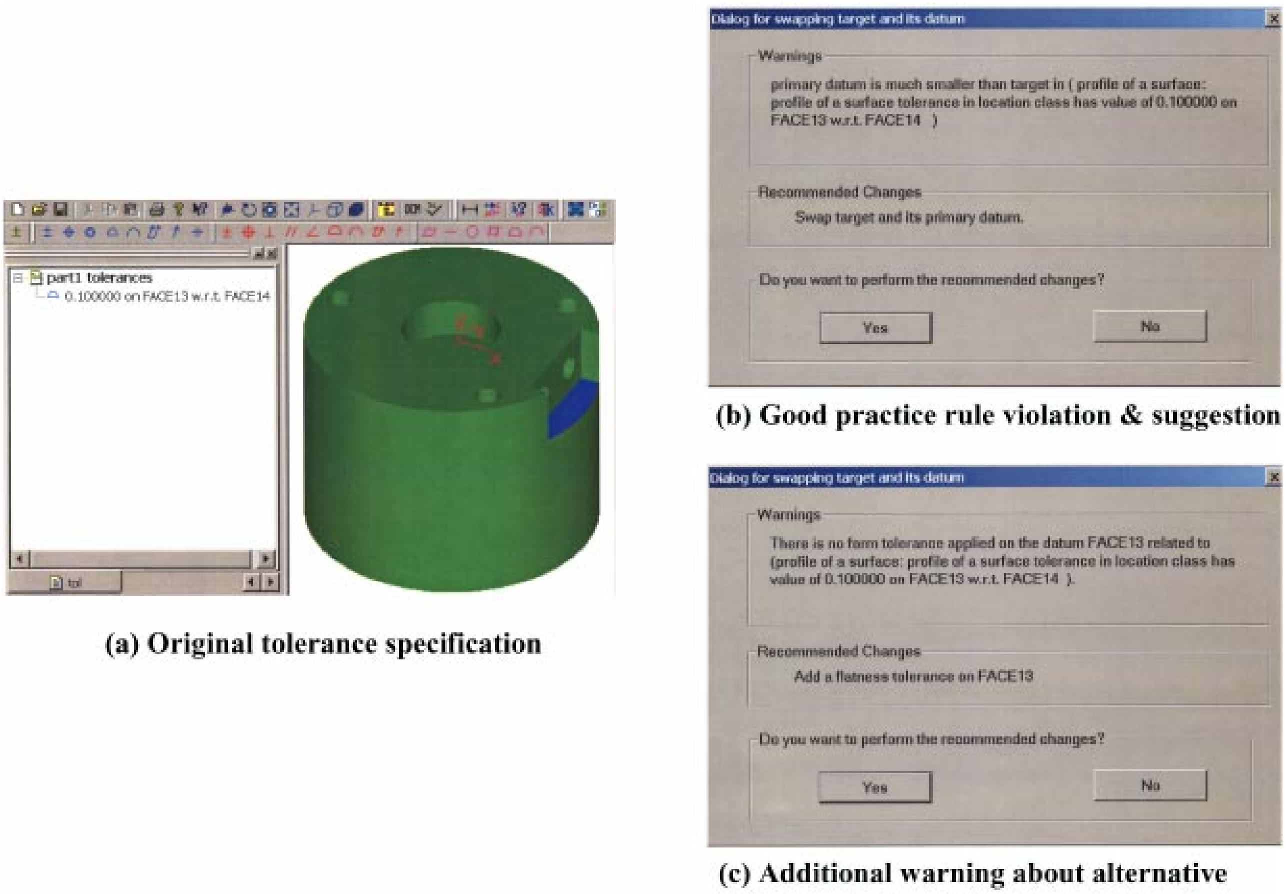


Fig. 12 GD&T Advisor

9. Material modifier validation: A material modifier can only be specified to a feature of size for straightness, position, angularity, parallelism, or a perpendicularity tolerance.

Validation of individual tolerance frames is necessary but not sufficient for GD&T validation. The Dimension Scheme and the Tolerancing Scheme must also be validated. The validation of a dimension scheme should precede the validation of a tolerance scheme. Dimensioning is complete when all of DOFs of all geometric entities are fully constrained with respect to each other through valid metric relations. For Tolerance scheme validation, each metric relation on the part should be controlled by the tolerance in the class corresponding to the metric relation. For example, a distance relation is fully toleranced only after it has been constrained by a tolerance in the location tolerance class; if a distance relation is only controlled by a parallelism tolerance, it is not fully toleranced. Since in the GD&T model presented here, a tolerance specification is directly connected to a metric relation, a tolerance specification cannot be added into the model if there is no relevant metric relation.

Implementation of Dimension Scheme validation is based on the use of a commercial geometric constraint solver, DCM-3D® which supports points, curves and surfaces in three dimension and distance, radius and angle dimensions between these entities. This includes logical constraints such as parallel, perpendicular, tangent and coincident. DCM-3D® manages a collection of geometric entities that are constrained to be rigid with respect to each other by sets, within which all geometric entities are independent of any constraints between them. Because commercial solvers are designed for parametric CAD and positioning rigid parts in assembly modeling, not for the GD&T, several enhancements are required. Topological information (interrelations among vertices, edges, and faces) is not retained by geometric solvers. This information needs to be added through coincident constraints. Since we are only interested in relative ‘‘motion,’’ rigid body motions need to be removed by fixing some entities. A 3D model has 6 DOF due to rigid body motion; the basic combination of entities that has 6 DOF in a body is a plane plus a straight line lying on it and a vertex lying on the line.

The integrated GD&T system can verify if a part is fully constrained or not. Also, when the user deletes an existing dimension or tolerance, the system checks if the deletion causes any invalidation of tolerance refinement relation. The part shown in Fig. 11 has a positional tolerance specified on the center hole (FACE8) with respect to the top face (FACE13) and the side face (FACE18), a size tolerance specified on the radius of the center hole (FACE8), and a straightness tolerance specified on the axis of the center hole (FACE8). If the user wants to delete the size tolerance on the center hole (FACE 8), it is not allowed by the system, since the size tolerance acts as a bonus tolerance in the tolerance refinement relation (Section 3.4). The system suggests that in order to remove the plus/minus tolerance on the part, the user needs to remove either the positional tolerance or the straightness tolerance first. The user can also choose to modify the tolerance value to make plus/minus tolerance deletable.

5.4 Support for Tolerance Analysis. The GD&T global representation model developed in this work supports 3D tolerance analysis. The input required is a tolerance chain and the geometric information of the geometric entities involved. A tolerance chain consists of a list of tolerance specifications. Each tolerance specification includes the target, tolerance type, tolerance class, tolerance value, DRF, material modifiers. The geometric information of geometric entities involved in the tolerance chain can be retrieved from the solid model since the GD&T global model is closely related to the solid model. The first step of the tolerance analysis is the extraction of tolerance chain for a pair of entities between which the geometric or dimensional variation is to be analyzed. Tolerance chains can be directly retrieved from the GD&T graph. The distance relations are directed relations in the GD&T model, and the controlling tolerances are directly associated with the metric relations. This helps identify the dimension chain for tolerance analysis. After the dimension chain is detected, recognizing the tolerance chain is easy since all the corresponding tolerances are directly saved as a part of the dimension data. The analysis procedures are outside the scope of this paper.

5.5 GD&T Advisor. Guaranteeing the correctness of the GD&T scheme is not enough. A good GD&T scheme should be able to assure the manufacturability and inspectability. Towards this goal, the integrated GD&T system includes a Tolerance Scheme Advisor to help the designer balance the tradeoffs between the cost and functionality. Good practice rules accumulated from previous works are reported in [25]. The rules collected are classified into two categories, validation rules and recommendation rules. Validation rules are enforced through the GD&T representation model, discussed in Section 5.3. Recommendation rules are used for the designer to achieve a better tolerance scheme.

When there is a rule violation detected, the system provides an alternative that is consistent with the good practice rules. In Fig. 12(*a*), the top face (FACE13) has a profile tolerance with respect to the small horizontal face (FACE14). When the GD&T scheme advisor checks the tolerance against good practice rules, the advisor finds that the datum is much smaller than the target, it provides an alternative solution of swapping the target and the datum. If the user accepts this option, the advisor then finds that there is no form tolerance controlled on the new datum (FACE13), the system asks the user if a form control can be added. If the user agrees, a flatness tolerance is added on FACE13. Finally, in Fig. 12, the target of the profile tolerance is the small face (FACE14). The datum of the profile tolerance now is the bigger face (the top face, FACE13), which has a form tolerance on it.

6 Closure

The GD&T global model presented achieves the four basic requirements for a computer model for GD&T representation proposed in Section 1-completeness and richness, compatibility, computability, and validity, but the goal of self-validating structures is not fully achieved. The proposed GD&T global model satisfies the completeness criterion, since all of the data of GD&T of a part or an assembly is fully stored and all Y14.5 classes can be represented. To meet the compatibility requirements, a tolerance is treated as an attribute of a metric relation in the global model. The tolerance data is saved as a part of the data of its corresponding metric relation. The allowed geometric variation (tolerance) is defined as a tolerance zone. Computability is satisfied for supporting GD&T reasoning based on a GD&T global model. This requires the close relation between the GD&T representation model and the solid model and the ability to manipulate the data to answer questions of interest. Tolerance chains containing all the tolerances in the stack can be extracted for use by computer aided analysis packages. The direction of each dimensional relation is stored in the global model for the ease of extracting the dimension chain. A direct relation between the tolerance and the metric relations the tolerance controls helps to obtain the tolerance chain if a dimension chain is given. To accomplish the validity requirement, the relations among the elements in GD&T have been fully built into the structure of the GD&T model. The existence of a corresponding metric relation is the pre-requisite for the existence of a tolerance. When a dimension gets deleted, the corresponding tolerance gets deleted, too. A tolerance of a certain class is only allowed to connect to certain types of metric relations. This helps avoid conflicting tolerances. Some tolerances might control different elements on the target, although specified to the same target. In the GD&T model, tolerance attributes are connected to the corresponding metric relation in order to differentiate the tolerance control specified on the different geometric elements.

Acknowledgments

The authors gratefully acknowledge support of the National Science Foundation (grant DMI-9821008). Views expressed in this paper are those of the authors and do not imply any endorsement by NSF. An earlier version was presented at ASME CIE 2002.

References

[1] ASME, 1994, Dimensioning and Tolerancing, ASME Y14.5M-1994a, American Society of Mechanical Engineers, New York.
[2] ASME, 1994, *Mathematical Definition of Dimensioning and Tolerancing Principles*. ASME Y14.5.1M-1994b, American Society of Mechanical Engineers, New York.
[3] ISO 1101-Dimensioning & Tolerancing standard.
[4] Johnson, R. H., 1985, "Dimensioning and Tolerancing-Final Report," *R84-GM-02-2, CAM-I*. Arlington, Texas.
[5] Ranyak, P. S., and Fridshal, R., 1988, "Features for Tolerancing a Solid Model," *ASME Computers in Engineering Conference*, **1**, pp. 262–274.
[6] Shah, J. J., and Miller, D., 1990, "A Structure for Supporting Geometric Tolerances in Product Definition Systems for CIM," Manuf. Rev., **3**(1).
[7] Roy, U., and Liu, C. R., 1993, "Integrated CAD Frameworks: Tolerance Representation Scheme in a Solid Model," Computers & Industrial Engineering, **24**(3), pp. 495–509.
[8] Roy, U., and Fang, Y. C., 1996, "Tolerance Representation Scheme for a Three-dimensional Product in an Object-oriented Programming Environment," IIE Trans., **28**, pp. 809–819.
[9] Maeda, T., and Tokuoka, N., 1995, "Toleranced Feature Modeling by Constraint of Degree of Freedom for Assignment of Tolerance," *Proceedings of 4th CIRP Design Seminar*, Tokyo, Japan, April 5–6 1995, pp. 89–103.
[10] Tsai, J. C., and Cutkosky, M. R., 1997, "Representation and Reasoning of Geometric Tolerances in Design," Artificial Intelligence for Engineering Design, Analysis and Manufacturing, **11**, pp. 325–341.
[11] Requicha, A. A. G., 1983, "Toward a Theory of Geometric Tolerancing," Int. J. Robot. Res., **2**(4), pp. 45–60.
[12] Jayaraman, R., and Srinivasan, V., 1989, "Geometric Tolerancing: I. Virtual Boundary Requirements," IBM J. Res. Dev., **33**(2), pp. 90–104.
[13] Hillyard, R. C., and Braid, I. C., 1978, "Analysis of Dimensions and Tolerances in Computer Aided Mechanical Design," Computer Aided Design, **10**(3), pp. 161–166.
[14] Krishna, K., Krishnan, Osama, Eyada, K., and Ong, Jin B., 1997, "Modeling of Manufacturing Processes Characteristics for Automated Tolerance Analysis," International Journal of Industrial Engineering, **4**(3), pp. 187–196.
[15] Turner, J. U., 1993, "A Feasibility Space Approach for Automated Tolerancing," J. Eng. Ind., **115**(3), pp. 341–346.
[16] Rivest, L., Fortin, C., and Desrochers, A., 1993, "Tolerance Modeling for 3D Analysis-Presenting a Kinamatic Formulation," *Proceedings of 3rd CIRP Seminars on Computer Aided Tolerancing*, France, April 27–28, pp. 51–74.
[17] Desrochers, A., and Riviere, A., 1997, "A Matrix Approach to the Representation of Tolerance Zone and Clearances. Source," The International Journal of Advanced Manufacturing Technology, **13**(9), pp. 630–636.
[18] Chase K. W., Magleby S. P., and Gao J. S., 2000, "Tolerance Analysis of 2-D and 3-D Mechanical Assemblies with Small Kinematic Adjustments," http://adcats.et.byu.edu/WWW/Publication/index.html.
[19] Zhang, B. C., 1992, "Geometric Modeling of Dimensioning and Tolerancing," *Ph.D. Thesis*, Arizona State University, Tempe, Arizona.
[20] Wu, Y., 2002, "Development of Mathematical Tools for Modeling Geometric Dimensioning and Tolerancing," *Ph.D. Dissertation*, August 2002, Arizona State University, Tempe, AZ 85287.
[21] Kandikjian, T., Shah, J., and Davidson, J., 1999, "A Mechanism for Validating Dimensioning & Tolerancing Schemes in CAD Systems," Computer aided Design, **33**(10), pp. 721–37.
[22] Clément A., Riviére A., and Serre P., 1995, "A Declarative Information Model for Functional Requirement," *Proceedings of the 4th CIRP Design Seminar*, Tokyo, Japan, April 5–6, pp. 1–16.
[23] Desrochers, A., and Maranzana, R., 1995, "Constrained Dimensioning and Tolerancing Assistance for Mechanisms," *Proceedings of the 4th CIRP Design Seminar*, Tokyo, Japan, April 5–6, pp. 17–30.
[24] Davidson, J., Mujezinovic, A., and Shah, J., 2002, "A New Mathematical Model for Geometric Tolerances as Applied to Round Faces," ASME J. Mech. Des., **124** pp. 609–623.
[25] Ramaswamy, S., Shah J., Davidson, J., 2001, "Computer Aided GD&T Advisor Based on Y14.5 Conformance and Good Practice," ASME DFM conference, Sep., Pittsburgh.