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# Comparing and combining off-line feedrate rescheduling strategies in free-form surface machining with feedrate acceleration and deceleration

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#### ABSTRACT

Off-line rescheduling feedrates based on changing surface geometry in free-form surface machining could reduce machining time, increase tool life, and improve surface finish quality. Various feedrate rescheduling strategies have different feedrate rescheduling control parameters. These parameters could be chip thickness, metal removal rate (MRR), or resultant forces. The paper compares these feedrate rescheduling strategies in machining time and feedrate changes. Machining time could be reduced significantly if appropriate values are set for those control parameters. Further, various strategies are combined for better results in two ways: the minimum and the maximum feedrate combination. Machining time could be reduced with both feedrate combinations. The minimum feedrate combination could protect machining against excessive chip thickness, MRR, resultant force, or other conditions that may occur using only one of these rescheduling strategies. The maximum feedrate combination could further reduce machining time with a loose control on those conditions. The paper also points out that advantages of feedrate rescheduling may not been in real if distances between neighboring cutting locations and feedrates. The method to calculate real feedrates and machining time is provided in consideration with feedrate ac/deceleration.

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### 1. Introduction

Multi-axis computerized numerical control (CNC) machines have become the application of choice for free-form surface machining. In feedrate setting of free-form surface machining, the standard practice is to set a constant feedrate based on the worst cut geometry. This conservative method may result in long machining time, tool deflection with wide cutting force fluctuations, and premature tool wear due to light chip loads [1]. Therefore, it is necessary to automatically modify feedrates based on changing cutting conditions. Off-line feedrate rescheduling is to divide existing tool paths into more intervals with different feedrates.

Off-line feedrate rescheduling could use various strategies such as keeping constant chip thickness, keeping constant metal removal rate (MRR), and keeping constant cutting forces. Bailey et al. [1] used maximum force constraint in rough machining operations for feedrate rescheduling. Klopayan and Lee [2] determined feedrates by keeping the feedrate at the centroid of cutting cross area to be constant. Chen et al. [3] optimized feedrate at the maximum MRR under the constraint of allowable surface roughness. Some researchers worked on feedrate rescheduling to keep cutting forces constant in complex surface machining. Lim and Hsiang [4] used a maximum feedrate map to select an optimum cutting direction and feedrate. Their maximum feedrate boundary was determined by applying a force or dimensional constraint in different directions. Those feedrates were selected based on machining tests and data analysis. Yazar et al. [5] used maximum resultant cutting force to schedule feedrates with a proposed cutting force model. Ip [6] proposed a MRR optimization approach to compensate the variation of cutting speed and maintain a constant cutting force by adjusting the cutting feedrate considering tool life and surface gradient. Fussel et al. [7] used tool deflection, surface finish, tool failure, and machine power to set constraints on the cutting force and feedper-tooth based on one discrete mechanistic end milling model. Lee and Cho [8] assumed that the feedrate has a linear relationship with the cutting force in flat end milling for feedrate scheduling. Lazoglu [9] provided a generalized cutting force model of ball end milling and kept the resultant force constant along tool paths. Erdim et al. [10] stated that MRR-based feedrate rescheduling methods output higher feedrate values in comparison with cutting force-based feedrate rescheduling methods. Yan et al. [11] pointed out that an ideal feedrate might not be obtained

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with a short moving distance during high-speed machining. They estimated machining time for productivity evaluation based on linear and exponential feedrate profiles generated by most CNC controllers. In almost all work related to feedrate rescheduling, just like Lee and Cho's assumption [8], new feedrates were inserted into NC programs to allow enough time for feedrate ac/deceleration.

The paper compares several off-line feedrate rescheduling strategies. Experimental results are used to illustrate relationships between the resultant cutting force and cutting process parameters in ball end milling. These relationships are employed in establishing feed-depth of cut (doc) tables to keep maximum or average resultant force close to a given value. The paper also extends previous research by combining rescheduling strategies. For example, feedrates could be rescheduled or optimized based on keeping chip thickness less than one value, keeping MRR less than one value, and keeping the resultant force less than one value. The paper points out that feedrate rescheduling may incur negative effects on tool life, machining time, and resultant forces without considering feedrate ac/deceleration. The reason is in that distances between neighboring cutting locations might be too short for the machine to arrive at rescheduled feedrates. One second round feedrate rescheduling method is provided in the paper to avoid those negative effects.

The paper is organized as follows. Section 2 provides feedrate and time estimation with linear and exponential feedrate ac/ deceleration functions. Feedrate rescheduling control parameters are defined based on one time optimization model for feedrate rescheduling. Section 3 compares several feedrate rescheduling strategies in machining time and feedrate changes. Those strategies include keeping constant chip thickness, keeping constant MRR, and keep resultant forces close to be constant by using one feed-doc table established from machining tests. Section 4 combines those feedrate rescheduling strategies in two ways, namely, the minimum and the maximum combination. Section 5 discusses real feedrate changes considering feedrate ac/deceleration. Section 6 summarizes results and limitations of current research.

## 2. Time estimation and feedrate optimization considering feedrate acceleration and deceleration

Without considering feedrate ac/deceleration, the total machining time  $t_{\text{total}}$  can be given by

$$t_{\text{total}} = \sum_{i=1}^{k} t_i = \sum_{i=1}^{k} \frac{l_i}{F_i} = \sum_{i=1}^{m} \frac{l_i}{F_i} + \sum_{i=m+1}^{k} \frac{l_i}{F_a},$$
(1)

where  $t_i$  is the machining time of the *i*th machining segment at feedrate  $F_i$ , k is the total number of machining segments, and  $F_a$  is the maximum feedrate without cutting any material,  $l_i$  is the length of one machining segment, namely, the distance between two neighboring cutting locations ( $x_{i-1}$ ,  $y_{i-1}$ ,  $z_{i-1}$ ) and ( $x_i$ ,  $y_i$ ,  $z_i$ ), and it can be given by

$$l_{i} = \sqrt{(x_{i} - x_{i-1})^{2} + (y_{i} - y_{i-1})^{2} + (z_{i} - z_{i-1})^{2}}.$$
 (2)

The optimization model to minimize the machining time can be expressed as follows:

$$\min t_{\text{total}} = \left(\sum_{i=1}^{K} t_i\right)$$
  
Subject to :

 $R \leq R_{\text{max}}$ , resultant cutting force constraint

$$rpm \leqslant rpm_{max}$$
, spindle speed constraint  
 $f \leqslant f_{max}$ , chip thickness constraint  
 $F \leqslant F_{max}$ , feedrate constraint  
 $MRR \leqslant MRR_{max}$ , MRR constraint  
 $W \leqslant W_{max}$ , cutting temperature constraint  
 $P \leqslant P_{max}$ , machine power constraint  
 $T_c \geqslant T_{min}$ , cutting tool life constraint  
 $R_a \leqslant R_{amax}$ , surface roughness constraint  
... and other constraints.

One optimization model to minimize the metal removing cost *C* can be given by

$$\min C = \left(\sum_{i=1}^{K} t_i\right) \left(c_{\text{machine}} + \frac{c_{\text{tool}}}{T} + c_{\text{toolchange}}\right)$$
subject to all constraints in Eq. (3),
(4)

where  $c_{\text{toolchange}}$  is the tool change cost and  $c_{\text{tool}}$  is the tool cost with tool life *T*, and  $c_{\text{machine}}$  is the machining cost rate. Here, tool life is assumed to be constant without considering changes of cutting process parameters such as depth of cut, feedrate, and cutting speed in free-form surface machining.

Commonly speaking, feedrate rescheduling in free-form surface machining is to consider only one constraint in Eq. (3) at all machining segments such as keeping constant chip thickness f, keeping constant *MRR*, keeping constant surface roughness  $R_a$ , or keeping constant resultant cutting force R. Chip thickness, MRR, surface roughness, and resultant force can be defined as *feedrate rescheduling control parameters*. Various feedrate rescheduling strategies have different control parameters and should be combined for better results based on time and cost optimization models in Eqs. (3) and (4).

With the conservative feedrate setting method, feedrate  $F_i$  is set to be constant for all machining segments for the worst cutting condition. With feedrate rescheduling or optimization,  $F_i$  should be different at machining segments with changing cutting conditions. Feedrates at three axes are synchronized to obtain the final feedrate  $F_i$  in the moving direction:

$$F_{x,i} = \frac{|x_i - x_{i-1}|}{l_i} F_i, \quad F_{y,i} = \frac{|y_i - y_{i-1}|}{l_i} F_i,$$
  

$$F_{z,i} = \frac{|z_i - z_{i-1}|}{l_i} F_i.$$
(5)

Most CNC controllers generate linear and/or exponential feedrate ac/deceleration profiles. Calculations of real machining time and feedrate at one certain axis considering ac/deceleration are discussed in the following.

#### 2.1. Linear feedrate ac/deceleration

A linear feedrate ac/deceleration function is employed when the feedrate is set to adapt to machining high precision contours. For FANUC control, this high precision contour control mode is set by G05. As shown in Fig. 1, the time to arrive at a given feedrate  $F_{i+1}$  from previous real feedrate  $F'_i$  at one certain axis is

$$t_{i+1} = \frac{|F_{i+1} - F'_i|}{a},\tag{6}$$

where *a* is the linear ac/deceleration rate. The distance  $d_{i+1}$  during the ac/deceleration process is given by

$$d_{i+1} = F'_i \frac{|F_{i+1} - F'_i|}{a} + \frac{1}{2} \frac{(F_{i+1} - F'_i)^2}{a}.$$
(7)

If one machining segment is too short to arrive at the given feedrate  $F_{i+1}$  ( $l_{i+1} < d_{i+1}$ ), the real feedrate  $F'_{i+1}$  and the machining time  $t'_{i+1}$  at the end of the machining segment can be given as



Fig. 1. Time and feedrate considering feedrate ac/deceleration.

follows:

а

$$F'_{i+1} = \sqrt{2al_{i+1} + (F'_i)^2}, \text{ while } F_0 \ge F_{i+1} \ge F'_{i+1},$$

$$F'_{i+1} = 2F'_i - \sqrt{2al_{i+1} + (F'_i)^2}, \text{ while } F_{i+1} < F'_i,$$

$$t'_{i+1} = \frac{F'_{i+1} - F'_i}{a},$$
(8)

where  $F_0$  is the maximum machining feedrate. The maximum acceleration rate of one conventional machine is ranging between 0.2 and 0.3 g and the value can be achieved at 1 g easily with a high-speed machine tool [2]. For instance, with *a* at  $9800 \text{ mm/s}^2$ (1g), the acceleration time is about 3.8 ms and the distance is 0.07 mm from 0 to 2268 mm/min. If the rate changes to 0.25 g, the acceleration time is 15.2 ms and the distance is 0.29 mm with the same feedrate change.

If  $l_{i+1} > d_{i+1}$ , the machine moves at the given feedrate  $F_{i+1}$  after it is accelerated or decelerated to this given feedrate, the machining time can be given as follows:

$$t'_{i+1} = t_{i+1} + \frac{l_{i+1} - d_{i+1}}{F_{i+1}}$$

$$= \frac{|F_{i+1} - F'_i|}{a} + \frac{l_{i+1} - F'_i \frac{|F_{i+1} - F'_i|}{a} - \frac{1}{2} \frac{(F_{i+1} - F'_i)^2}{a}}{F_{i+1}}$$
while  $F'_{i+1} = F_{i+1}$ . (9)

#### 2.2. Exponential feedrate ac/deceleration

Exponential feedrate ac/deceleration functions accurately model instantaneous feedrates in consideration with transit process and time constants of one machine tool during feedrate changes. With the exponential feedrate ac/deceleration, from initial feedrate  $F'_i$  to commanded  $F_{i+1}$ , the real federate  $F'_{i+1}$  at time t can be expressed as [12]:

$$F'_{i+1} = \frac{F'_i - F_{i+1}}{-T_1 + T_2} (-T_1 e^{(-t/T_1)} + T_2 e^{(-t/T_2)}) + F_{i+1},$$
(10)

where  $T_1$  and  $T_2$  are time constants specified by different CNC machine tools.

Integrating Eq. (10) yields the following moving distance:

$$l_{i+1} = \frac{F_{i+1} - F'_i}{T_1 - T_2} (T_1^2 e^{(-t_{i+1}/T_1)} - T_2^2 e^{(-t_{i+1}/T_2)}) + F'_i (T_1 + T_2) + F_{i+1} (t_{i+1} - T_1 - T_2).$$
(11)

For example, given  $T_1 = 32 \text{ ms}$ ,  $T_2 = 33 \text{ ms}$ , machining time  $t_{i+1} = 50 \text{ ms}$ ,  $F_i = 0$ , and  $F_{i+1} = 2286 \text{ mm/min}$ , the moving distance is 0.37 mm and the real feedrate is 1045 mm/min. The time and distance to arrive at 90% of the commanded feedrate is about 128 ms and 2.47 mm. This distance is much longer than the distance with linear feedrate ac/deceleration functions. That implies that the machine might not arrive at or be close to the commanded feedrate in one tiny machining segment with exponential feedrate functions. Real feedrate changes with ac/deceleration are discussed in Section 4.

### 3. Comparing three feedrate rescheduling strategies in freeform surface machining

Fig. 2 shows a tabulated cylinder surface resulting from translating a half ellipse curve along Y-axis. The worst cutting location is at X0 position with the biggest depth-of-cut (doc) at 2.5 mm. The part length is 508 mm. The cutting tool is one 4-flute 19.05 mm high-speed steel ball end mill. The work-piece material is Al6061-T6. The width of cut in Y direction is 3.1 mm. The fixed feedrate *F* used in the conservative method is 736.6 mm/min and the spindle speed is 2500 rpm. The feed or chip thickness is about 0.074 mm/rev. There are 28 cutter locations at each tool path. Machining times of tiny machining segments are from 8 to 30 ms. The initial and maximum machining feedrate  $F_0$  is 2286 mm/min. The original machining time is 2.69 min with the constant feedrate along the curved tool path. Commercial software, VeriCut OPTI, is used for off-line feedrate rescheduling. This section compares feedrate rescheduling strategies with different feedrate rescheduling control parameters. Those rescheduling control parameters are chip thickness, MRR, and maximum or average resultant force. Experiments tests are used to establish a feed-doc table to keep peak or average resultant forces close to or less than a constant value. With feedrate rescheduling, the lowest feedrate happens at the worst cutting position, namely, the X0 position.

#### 3.1. Feedrate rescheduling with constant chip thickness

First, maintaining a constant chip thickness at all cutting locations is selected for feedrate rescheduling. Fig. 3 shows feedrate changes in one tool path by keeping constant chip thickness or feed from 0.05 to 0.15 mm/rev. Feedrates are rescheduled at cutting locations close to X0 position. Bigger chip thickness incurs higher feedrate at X0 position and narrows the feedrate rescheduling range. Feedrates are less than the maximum and initial machining feedrate  $F_0$  in the feedrate rescheduling range. The figure shows that a higher  $F_0$  does not change feedrates at or close to the worst cutting position X0, but widens the feedrate rescheduling range. For example, for chip thickness at 0.1 mm with  $F_0$  at 2286 mm/min, C0.1 in the figure, the minimum feedrate is 1600 mm/min and the feedrate rescheduling range is from X-0.58 mm to X0.58 mm. For chip thickness at 0.1 mm with  $F_0$  at 3810 mm/min (150 in/min), C0.1f150 in the figure, the minimum feedrate is still 1600 mm/min and the feedrate rescheduling range is from X-1.27 to X1.27 mm. If the chip thickness is 0.15 mm, C0.15 in the figure, feedrates at most cutting locations are kept at the allowed maximum feedrate  $F_0$ (2286 mm/min). That implies that chip thickness at 0.15 mm might be too high for rescheduling feedrates in this machining example.

#### 3.2. Feedrate rescheduling with constant MRR

Fig. 4 shows feedrate changes by keeping constant MRR. Similar to rescheduling with constant chip thickness, a larger MRR incurs a higher feedrate at the worst cutting position and narrows the feedrate rescheduling range. For example, for MRR at



Fig. 2. Tool paths and NC codes for machining a tabulated cylinder.



Fig. 3. Feedrate changes with constant chip thickness.



Fig. 4. Feedrate changes with constant MRR.

16,387 mm<sup>3</sup>/min, MRR 16387 in the figure, the minimum feedrate is 584 mm/min and the feedrate rescheduling range is from X-1.98 to X1.98 mm. If the constant MRR is set at 48,161 mm<sup>3</sup>/min, MRR 48161 in the figure, feedrates at most cutting locations are kept at the initial feedrate  $F_0$  except three positions at and close to the X0 position and the lowest feedrate is 1727 mm/min. Therefore, MRR at 48161 mm<sup>3</sup>/min may not be an appropriate value for feedrate rescheduling in this example.

# 3.3. Feedrate rescheduling with one feed-doc table for constant resultant forces

Feedrate could be rescheduled based on feedrates or feeds specified with various docs. In this paper, feedrates between two docs are linearly interpolated with one feed-doc table to keep maximum or average resultant forces close to a given value. The table is established based on experimental tests. Machining tests of straight cuts with the ball end mill are performed on a HAAS VF-1 vertical machining center. Cutting forces in X, Y, Z directions are measured using a 3-component dynamometer KISTLER 9257B. Docs are set at 1, 1.875, and 3.75 mm. Feeds are set at 0.05, 0.1, and 0.15 mm/rev. Fig. 5 shows partial cutting forces with doc at 3.75 mm and feed at 0.05 mm/rev. Three components of cutting forces ( $F_X$ ,  $F_Y$ ,  $F_Z$ ), resultant forces in XOY plane ( $R_{XY}$ ), and resultant forces  $(R_{XYZ})$  are shown in the figure. In the figure, resultant forces  $R_{XY}$  and  $R_{XYZ}$  are very close to each other although peak values of  $F_Z$ are larger than peak values of  $F_X$  or  $F_Y$ . Fig. 6 illustrates maximum and average resultant forces R as functions of feed f under different docs. Using approximated force-feed relationships in the figure, feeds f (mm/rev) and docs (mm) for maximum resultant forces at 1000, 750, and 500 N and average resultant forces at 375, 250, and 125 N are listed in Table 1 for feedrate rescheduling.

Fig. 7 shows feedrate changes when feedrates are scheduled with Table 1. In the figure, a higher resultant cutting force incurs a narrower feedrate rescheduling range and a higher feedrate at X0 position. Using average resultant force for feedrate rescheduling, the feedrate rescheduling range is wider than using maximum resultant forces, that is to say, feedrates change more smoothly by using average resultant force than using maximum resultant force even with similar feedrate at X0 position. For instance, with maximum  $R_{XYZ}$  at 500 N, Force 500 N in the figure, the lowest feedrate is about 737 mm/min and the feedrate rescheduling range is between X-1.27 and X1.27 mm. For average  $R_{XYZ}$  at 125 N, AForce 125 N in the figure, the lowest feedrate is about 635 mm/min and the feedrate rescheduling range is from X-2.72 to X2.72 mm. If the average  $R_{XYZ}$  is set at 375 N, AForce 375 N in the figure, feedrates at all cutting locations are kept at the initial feedrate. If the constant maximum force  $R_{XYZ}$  is set at 1000 N, Force 1000 N in the figure, feedrates at most cutting locations are

kept at the initial feedrate  $F_0$  except three positions at or close to X0 and the lowest feedrate is as high as 2108 mm/min. This implies that average force at 375 N and maximum force at 1000 N might be too high for feedrate rescheduling in the example.

# 3.4. Comparing feedrate rescheduling strategies in machining time and feedrate changes

Fig. 8 shows the machining time as a function of the lowest feedrate at the worst cutting position for various feedrate rescheduling strategies. With the conservative *constant feedrate method*, the machining time changes about 62% when feed *f* increases from 0.05 to 0.15 mm/rev. If feedrates are rescheduled using *constant chip thickness* at 0.05 mm/rev, machining time reduces 54% compared to the machining time with constant feedrate at 500 mm/min (f = 0.05 mm/rev). With this reschedul-



**Fig. 5.** Cutting forces under doc = 3.75 mm and f = 0.05 mm/rev.

Resultant force Rxyz vs Feed Max. Force doc=1.875 Max. Force doc=1 --- Max. Force doc=3.75 --- Avg. force doc=1 --- Avg. force doc=1.875 --- Avg. force doc=3.75 1750 Resultant Forces Rxyz (N) 1500 1250 1000 750 500 250 0 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.21 0.23 0.25 Feed (mm/rev)

Fig. 6. Relationships between feed and resultant force R<sub>XYZ</sub>.

ing strategy, the machining time changes about 40% if chip thickness increases from 0.05 to 0.15 mm/rev. If feedrates are rescheduled using *constant MRR* at 16,387 mm<sup>3</sup>/min, time reduces about 45% in comparison with the original machining time. The machining time changes about 29% if MRR increases from 16,387 to 49,161 mm<sup>3</sup>/min. In fact, the machining time changes to 5.81 min with constant MRR at 2622 mm<sup>3</sup>/min and it is longer than original 2.69 min. Rescheduling feedrate with the *feed-doc table for maximum resultant force at 500 N*, time reduces 51% from the original machining time. Time changes about 21% if the maximum cutting force decreases from 1000 N to 500 N. Rescheduling feedrate with the feed-doc table for average resultant force



Fig. 7. Feedrate changes with constant resultant forces based on the feed-doc table.



Fig. 8. Time changes with various feedrate rescheduling strategies.

Table	1
Table	1

Feed-doc table for constant resultant force

doc (mm)	<i>f</i> at 1000 N	<i>f</i> at 750 N	f at 500 N	f at avg. 375 N	f at avg. 250 N	f at avg. 125 N
<0.8	0.25	0.25	0.25	0.25	0.25/0.54	0.25/0.25
1	0.25	0.25	0.11	0.25	0.176	0.077
1.875	0.18	0.095	0.048	0.204	0.124	0.055
3.75	0.117	0.073	0.034	0.178	0.086	0.025



Fig. 9. Feedrate changes with various feedrate rescheduling strategies.

125 N, time reduces 49% from the original time. Time changes about 33% if the average cutting force decreases from 375 N to 125 N. This proves that using constant chip thickness, constant MRR, or constant resultant force in feedrate rescheduling could reduce machining time much if these feedrate rescheduling control parameters can be set at appropriate values. Using a higher chip thickness, MRR, and resultant forces for feedrate rescheduling could be further reduce machining time. But these values are limited by other constraints listed in Eq. (3).

Fig. 9 shows feedrate changes with four rescheduling strategies. These strategies are keeping chip thickness at 0.043 mm, keeping MRR at 20,661 mm<sup>3</sup>/min, keeping maximum resultant force close to 500 N, and keeping average resultant force close to 125 N. The lowest feedrates at X0 position are close to 737 mm/min for three strategies except the strategy of keeping average resultant force. Machining times are 1.60, 1.33, 1.34, and 1.37 min, respectively, and reduce much from original 2.69 min. With the same lowest feedrate at X0 position, feedrate changes with constant MRR and maximum resultant forces are very similar. That implies that resultant force might have linear relationship with MRR. Among those strategies, rescheduling with constant chip thickness has the largest feedrate rescheduling range and the longest machining time.

# 4. Combining feedrate rescheduling strategies in free-form surface machining

Different feedrate rescheduling strategies have different control parameters and could be combined for better results based on Eq. (3). There are two ways to combine various strategies: feedrate at one cutting position could be the minimum or the maximum value of feedrates rescheduled individually based on different rescheduling strategies. In this section, at first, feedrates are rescheduled by combining constant chip thickness and constant MRR strategies with the minimum approach. Then, three strategies compared in Section 3 are combined with the minimum and the maximum approach.

#### 4.1. Rescheduling with constant chip thickness and constant MRR

First, feedrate rescheduling strategies with constant chip thickness and constant MRR are combined with the minimum approach. Optimized feedrates based on each strategy are calculated, respectively, and then smaller values are used. This could protect the machining against excessive chip thickness or MRR conditions that may occur using only one of these two rescheduling strategies [13].

Fig. 10 shows feedrate changes by keeping MRR at 8279, 16,387, and  $32,774 \text{ mm}^3/\text{min}$  while keeping maximum chip thickness f less than 0.05 mm/rev. The machining time reduces about 25-34% in comparison with the original machining time. A smaller MRR results in a lower feedrate at the worst cutting location and does not change feedrate rescheduling range. For example, keeping MRR less than 8279 mm<sup>3</sup>/min, MRR0.7 chip 0.05 in the figure, the lowest feedrate changes to 406 mm/min from 660 mm/min at keeping chip thickness at 0.05 mm/rev, chip 0.05 in the figure. If MRR is set at 16387 mm<sup>3</sup>/min, MRR2 chip 0.05 in the figure, feedrates at all cutting locations are kept same as feedrates rescheduled based on constant chip thickness 0.05 mm/rev. That implies that constant MRR. if larger than 16.387 mm<sup>3</sup>/min. MRR2 in the figure, might be too high for combining with constant chip thickness at 0.05 mm/rev. In this case, feedrates are determined only by one rescheduling strategy and another strategy does not have any effects on feedrates.

Fig. 11 shows feedrate changes by keeping chip thickness at 0.025, 0.05, and 0.1 mm/rev while keeping MRR less than 16,387 mm<sup>3</sup>/min, MRR1 in the figure. The machining time reduces about 9–44% from the original machining time. If chip thickness is set at 0.1 mm, MRR1 chip 0.1 in the figure, feedrates at all cutting locations are kept same as feedrates rescheduled based on constant MRR16,387 mm<sup>3</sup>/min. That further proves that the machining time could be reduced if feedrates are rescheduled by combining the rescheduling strategy with constant chip thickness or MRR could further reduce machining time, but those values are constrained by the value of MRR or chip thickness set for the minimum feedrate combination.

#### 4.2. Combining three feedrate rescheduling strategies

Those feedrate rescheduling strategies shown in Fig. 9 could be combined in two ways. One approach is to keep chip thickness less than 0.043 mm, keep MRR less than 20,661 mm<sup>3</sup>/min, and keep maximum resultant force less than 500 N at the same time, that is to say, use the minimum value of rescheduled feedrates at each machining segment. Feedrate changes are shown as min(MRR,CHIP,FORCE) in Fig. 12. The machining time changes to 1.61 min. Another approach is to satisfy only one of three control constraints, i.e. use the maximum value of optimized feedrates at each machining segment. Feedrate changes are shown as Max(MRR,CHIP,FORCE) in the figure. The machining time changes



Fig. 10. Feedrate changes rescheduled with various MRR and f = 0.05 mm/rev.



Fig. 11. Feedrate changes rescheduled with various chip thickness and  $MRR = 16387 \text{ mm}^3/\text{min}$ .



Fig. 12. Feedrate changes with feedrate combination and with exponential ac/ deceleration.

to 1.31 min. The feedrate rescheduling range with the minimum feedrate combination is larger than the range with the maximum feedrate combination.

Feedrate rescheduling by combining three strategies (MRR, Chip thickness, and force) still reduces machining time if three control parameters are set at appropriate values for each strategy. The machining time after a minimum feedrate combination is larger than the maximum value of machining times with each single strategy. The machining time after a maximum feedrate combination is smaller than the minimum value of machining times with each single strategy. Therefore, the machining time reduces more with the maximum feedrate combination than the minimum feedrate combination. The minimum feedrate combination could protect the machining against excessive chip thickness, MRR, force, or other conditions that may occur using only one of these rescheduling strategies while the maximum feedrate combination could further reduce the machining time with a loose control on those conditions.

### 5. Real feedrate changes with feedrate ac/deceleration

As discussed in Section 2, one commanded feedrate  $F_{i+1}$  cannot be achieved if the machining time is less than the required ac/

deceleration time for feedrate changing from  $F_i$  to  $F_{i+1}$ . Fig. 12 shows real feedrates with one exponential feedrate ac/deceleration profile ( $T_1 = 32 \text{ ms}$  and  $T_2 = 33 \text{ ms}$ ). Possible feedrates at the worst cutting position X0 are 1257 and 1493 mm/min from the initial 2268 mm/min. These values are much higher than the lowest feedrate 737 mm/min rescheduled. The maximum resultant cutting force would be larger than 500N at X0 and the estimated value is about 1000N with Fig. 6. The real chip thickness and the real MRR at the worst cutting position would be far more than those values set for feedrate rescheduling. That is to say, the machine cuts too fast and it may break the tool quickly and damage the workpiece at and close to the worst cutting position. Hence, feedrate rescheduling without considering ac/deceleration may have negative effects on tool life and other machining performances in free-form surface machining. A second round feedrate rescheduling is required to solve the problem. In the second round rescheduling, feedrates at all cutting locations can be adjusted to make sure that the feedrate at the worst condition are close to the feedrate rescheduled. The method is to decelerate the feedrate at an earlier cutting location of one tool path to lengthen the time for deceleration. Sometimes the initial feedrate at the beginning of feedrate rescheduling has to been reduced to shorten the ac/deceleration time. The following gives one algorithm to adjust feedrates at one certain axis with known feedrate ac/deceleration functions.

- (1) Assume the initial feedrate  $F_0$  at the beginning of one tool path to be the maximum machining feedrate and the lowest feedrate at the worst cutting condition to be  $F_m$  at the *m*th tool path segment in the tool path.
- (2) For (m-j)th tool path segment from (m-j)th to (m-j+1)th cutting locations, calculate the distance  $l_{m-j}$  with Eq. (2), where *j* is a positive integer number and its initial value is 1.
- (3) Calculated the possible maximum feedrate  $F_{m-j,max}$  with Eqs. (8) and (10) for this (m-j)th tool path segment. If the commanded feedrate  $F_{m-j}$  in the NC program is larger than  $F_{m-j,max}$ , then adjust the feedrate  $F'_{m-j}$  to be  $F_{m-j,max}$ ; otherwise, keep the recommended feedrate, namely,  $F'_{m-j} = F_{m-j}$ . The machining time for the tool path segment can be calculated using Eqs. (8) and (11). If the federate  $F'_{m-j} \ge F_0$ , then keep  $F'_{m-j} = F_0$ , and stop.
- (4) If  $F'_{m-j} < F_0$ , increase *j* by 1, namely, j = j+1, if  $j \le m-1$ , return to step (2). If  $F'_{m-j} < F_0$  and j = m, that means the lowest feedrate at the worst cutting condition can not be achieved with the initial feedrate  $F_0$ . The initial feedrate has to be reduced to  $F_0$  and stop.

The algorithm makes sure that the feedrate at every tool path segment is less than or equal to the feedrate initially scheduled.

### 6. Summary

This paper presents one time and feedrate estimation method considering feedrate ac/deceleration in free-form surface machining. It compares various feedrate rescheduling strategies with one machining example. Those strategies include keeping constant chip thickness, keeping constant MRR, and using one feed-doc table for constant maximum or average resultant force along the curved tool path. Feedrate rescheduling with constant maximum resultant force could keep peak values of cutting forces to be or close to a constant value. Machining tests are used to illustrate relationships between resultant forces and cutting process parameters such as depth of cut and chip thickness in ball end milling. These relationships are used to establish one feed-doc table for constant maximum or average resultant forces. Feedrate rescheduling could reduce machining time if feedrate rescheduling control parameters such as MRR, chip thickness, and resultant forces could be set at appropriate values. Setting a higher control parameter could be further reduce machining time but those values are limited by other control parameters such as MRR, chip thickness, force, surfaces finish, and tool deflection. Combining various feedrate rescheduling strategies could reflect these constraints. The paper combines various feedrate rescheduling strategies with the machining example in two ways-the minimum and the maximum feedrate combination.

To extent current work, spindle speed or cutting speed could also be rescheduled together with off-line feedrate rescheduling. The criteria on how to choose a feedrate rescheduling strategy and how to choose an appropriate value of one rescheduling parameter need to be analyzed in further work. The time or the cost optimization model in the paper could be used to develop other new off-line feedrate or spindle speed rescheduling strategies in free-form surface machining by changing the rescheduling control parameter. Other variables such as tool size, number of teeth in one cutter, cutting tool geometry, width of cut, tool life or tool wear rate, tool cost, tool-work temperature, workpiece quality including surface finish and integrity, and power consumed in machining could also be considered in further work. Combination of off-line rescheduling strategies focused in this paper and on-line adaptive control could be used to deal with tool wear, workpiece hardness variation or depth of cut variation encountered in actual machining [13]. The work presented in the paper can be also extended to high speed machining hardened steels or other difficult-to-cut materials.

Dozens of machining experiments were conducted for feeddoc tables used in the paper. Using results from numerical experiments with a small number of machining tests for calibration and validation could be a good alternative for this. One machining process modeling software was used to predict resultant forces in previous work [14]. But accurate 3D machining process modeling and simulation are not only time consuming but also not reliable with current technology. Developing a much quicker and more accurate 3D machining process simulation software remains to be an important and difficult task for multiaxis free from surface machining.

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