NURBS SURFACE RECONSTRUCTION FROM CROSS SECTION IMAGES

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Abstract. Digital images, like those according to the DICOM medical image standard, are one of the most common types obtained nowadays in computer data acquisition processes like tomography and magnetic resonance, basically consisting of cross-sectional gray level digital images. Some non-invasive techniques produce a great number of images, creating a non adequate visualization in terms of volume, being more adequately the construction of a surface from these data (surface fitting). Using smooth curves and surfaces, this technique is called surface skinning, and NURBS curves and surfaces are the best choice for biological entities models. For this sake, this work shows a method for surface reconstruction from a given set of cross sections, performing the NURBS surface skinning.

Keywords: NURBS, Skinning, Medical imaging, Surface reconstruction, Interpolation.

1. INTRODUCTION

In many research and application areas such as medical science, reverse engineering and CAD/CAM, the construction of a surface is an important problem. The points obtained from data acquisition techniques, like ultrasound imaging, have made possible the construction of computer models that presents many advantages such as diminish the cost and the time for the creation cycle of a variety of products (Ashley, 1993).

The proposed method described in this paper was developed with the aim of fitting a surface to a set of cross sections images to obtain a computer model of real human organs, to be used in CAD/CAM systems, using surface skinning or lofting, that consists in pass a smooth surface through a set of so called cross sectional curves. For smoothness and continuity in the curves and surfaces, NURBS – Non Uniform Rational Basis Spline surfaces are a suitable choice to represent the surface and the cross sections curves, being included in standards like IGES (1986), PHIGS (van Dam, 1988) and STEP (1994). Some reasons to use NURBS are given in Piegl and Tiller (1997). For automatic point extraction from the image, some data constraints must be defined before the process, like the final degree of the curves, being the choice here the degree 3. In this work, the cross sections are obtained segmenting the digital images of each set extracting some contour points in each plane, to be interpolated by NURBS curves and surface, respectively.

2. NURBS

A NURBS curve of degree p is a piecewise polynomial curve defined as:

\[ C(u) = \sum_{i=0}^{N} w_i P_i N_{i,p}(u) \]  

Where \( u \) is a parameter value, \( P_i \) form the so called curve control polygon points weighted by \( w_i \) and \( N_{i,p}(u) \), \( i = 0,...,n \), are the B-spline basis functions defined over a knot vector

\[ U = \{u_0,...,u_m\}, u_i \leq u_{i+1}, i = 0,...,m - 1 \]  

\[ N_{i,p}(u) = \begin{cases} 1 & \text{if } u_i \leq u < u_{i+1} \ 
0 & \text{otherwise} \end{cases} \]  

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\[ N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p}(u) \]  

We assume throughout this paper that the knot vector has the following form:

\[ U = \{ a, a, \ldots, a, a, a, b, b, \ldots, b \} \]

where, in most practical applications, \( a = 0 \) and \( b = 1 \). A NURBS surface of degree \((p, q)\) is defined similarly as:

\[ S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} w_{i,j} P_{i,j} N_{i,p}(u) N_{j,q}(v) \]

where \( u \) and \( v \) are the parameter values in the longitudinal and isoparametric directions of surface construction, \( P_{i,j} \), \( i = 0, \ldots, n; \ j = 0, \ldots, m \), form the so-called control net defined by a set of points weighted by \( w_{i,j} \) and the basis functions \( N_{i,p}(u), i = 0, \ldots, n \), and \( N_{j,q}(v), j = 0, \ldots, m \), are defined as above (the construction of \( N_{j,q}(v) \) is similar), over the knot vectors:

\[ U = [u_0, \ldots, u_r] \quad 0 \leq u_i \leq u_{i+1}, i = 0, \ldots, r - 1 \]

\[ V = [v_0, \ldots, v_s] \quad 0 \leq v_j \leq v_{j+1}, j = 0, \ldots, s - 1 \]

2.1. NURBS interpolation:

In the interpolation by a NURBS curve or surface, the control points are obtained by the inversion in the matrix form:

\[ [D] = [N][B] \]  \hspace{1cm} (7)
\[ [B] = [N]^+ [D] \]  \hspace{1cm} (8)

where, \( D \) – interpolating points matrix, \( N \) – rational basis function matrix, \( B \) – control point matrix (control polygon). The system of equations can be solved using the pseudo-inverse method if the matrix \( N \) is not square. Thus:

\[ [N]^+[D] = [N]^+[N][B] \]  \hspace{1cm} (9)
\[ [B] = [I][N]^+[N]^+[D] \]  \hspace{1cm} (10)

Although, this solution can lead to numerical instability and other methods, such as LRU decomposition, are preferable. For a given set of data points, the initial parameters values are calculated by the chord length method.

2.2. Cross Section extraction

In the skinning process, a surface pass through a set of so called cross-sectional curves, being necessary make the image segmentation: extraction of the pixels that form the image contour.

Figure 1. (a) Original Dicom image, (b) Binary image, (c) Segmented image.

For the Dicom image features, the background has a lower gray level, the first step is a image binarization to two levels (one or zero) using a threshold given by the Otsu's method. The image is segmented generating regions, that could be used to reconstruct inner surfaces. According to the image acquisition process, is required a filter step with a
Figure 2. Series of contours from the segmentation of the cross section images.

As each first point of a contour are not necessarily aligned, the generated NURBS curves of each cross section could be twisted regarded the next. To avoid this problem, before to start the interpolation, it is necessary to determine the alignment of the contours by using a baseline reference.

2.3. Control Net

In this step it is computed a NURBS interpolation obtaining the control points of each cross section contour curve. These points will form the control net in the isoparametric direction (perpendicular to the longitudinal direction) and if the number of knots is different in a contour, is necessary apply the curve compatibility: there must be the knot. The surface control points can be obtained by interpolating the control points of each cross section disposed in the longitudinal direction. The NURBS surface could be represented by:

\[
S(u,v) = \sum_{i=0}^{n} N_{i,p}(u) \left( \sum_{j=0}^{m} w_{ij} P_{ij} N_{j,q}(v) \right) = \sum_{i=0}^{n} N_{i,p}(u) R_{i,l}
\]

\[
R_{i,l} = \sum_{j=0}^{m} w_{ij} P_{ij} N_{j,q}(v)
\]

The \( R_{i,l} \) are the control points of one cross section curve. Interpolating through the curves points \( S(u,v) \) yields the \( R_{i,l} \) and interpolating through the \( R_{i,l} \) in the longitudinal direction yields the control points net.

Figure 3. Surface control net.

3. TESTS AND RESULTS

The proposed method was tested using a set of 20 DICOM images from a knee, each of 512x512 pixels dimension, resulting in two contours of open curves and surfaces. Each image was segmented extracting two contours, being sampled resulting in 479 points for each contour (958 points for image). These points are scaled, because it’s position could not satisfy the degree condition of the final curves, being the choice the degree 3.

Figure 4. Final surface with 4 times the original number of points.
4. CONCLUSION

Differently of most skinning methods this work is a approach from a set a cross sections, resulting in a surface with dependency mainly to the contour extraction. The whole process can be divided in three phases: cross section extraction or segmentation, contour extraction and NURBS skinning. Some issues in the features of the images, in the first step, could produce erroneous points in the contours.

Using NURBS to approximate the surfaces has advantages over other surface models because these modeling curves are commonly found in CAD systems. This is useful in applications such as reverse engineering where a computer model must be produced and processed from real objects.

5. REFERENCES


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