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WHITHER 'SIZE' IN GEOMETRIC TOLERANCING ?

by

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Summary

Size is defined indirectly in contemporary Geometric Dimensioning and Tolerancing (GD&T) through Features of Size (FOS), which are subsets of spherical surfaces, cylindrical surfaces, and pairs of opposed parallel planar surfaces. Features of Size have distinctive characteristics – extremal states, locatability – that have set much of the character of GD&T over the past forty years. If GD&T is to live for another forty years, some deep issues associated with size will have to be addressed. One – an ostensibly narrow conformance problem – must be solved soon. The broader issues include such questions as: what is 'size'? Is size fundamental, or can it be defined in terms of other concepts? Can a rational GD&T system be constructed without using size? The paper opens with a short summary of the conformance problem. The focus then shifts to an exploration of size *per se,* and a discussion of properties and issues that emerge from the exploration.[1]

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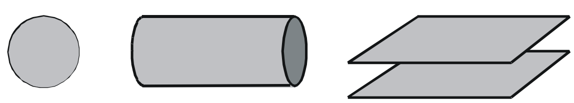
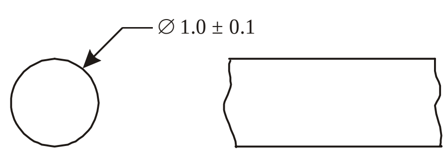
1. INTRODUCTION

'Size' is a convenient concept that everyone uses, but its precise meaning – when it has one – is almost always context dependent and sometimes subjective. The notion of geometric (or spatial) size pervades the mechanical industries: it is used to organize cutters in toolrooms, to catalog standard components, and so forth. Curiously, the American Standard Y14.5 [ASME 94a] for Geometric Dimensioning and Tolerancing (GD&T) provides no definition for size, but instead defines size indirectly through the three Features of Size (FOS) shown in Figure 1.



(a): A Feature of Size is a subset of a spherical or cylindrical surface, or of a pair of parallel planar surfaces with opposing material-side normals.



(b): Limits of Size are defined by limits on nominal parameter values.

Figure 1: Contemporary Features of Size (FOS).

Features of Size have distinctive properties that set much of the character of contemporary GD&T – notably *extremal states* (Maximum and Least Material Conditions determined by the Limits of Size, plus Virtual and Resultant Conditions), and *locatability* (only FOSs can carry position tolerances). However, the variational-limit

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[1] Popular versions of portions of this material have been published in the trade press; see [Voel 95] and [Voel 01].

semantics of FOSs have defied precise definition, and this has led to the conformance problems summarized in Section 2. On a deeper level, there is no agreement on rules for extending the current set of FOSs, or on handling size in some alternative manner. This suggests that the notion of 'size' as a free-standing concept should be examined, and the remainder of the paper is devoted to that exercise.

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2. Assessing Conformance to Size Specifications

GD&T distinguishes between nominal features, which are ideals specified by a designer, and actual or physical features. A nominal FOS carries the limits of size shown in Fig. 1b. Clearly these should control variations in actual size, but the exact nature of the control, and how to assess it (conformance assessment), depend on how size is defined. Examples based on the solid cylindrical FOS shown in Fig. 1b will illustrate the issues.

Actual (measured) size has been determined traditionally by two-point caliper measurements. Such measurements can assess conformance to the nominal specification on a section-by-section basis if the sections are circular. If they are not, the results can be misleading: see Figure 2.

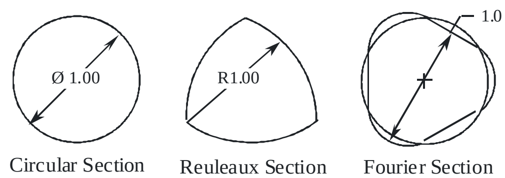

Figure 2: Can two-point measurements distinguish these sections?

The issue is even murkier when size is mixed with form. Rule 1 of Y14.5 requires that an actual cylinder lie on or within a perfect cylinder of MMC (maximum allowed) diameter, as in Figure 3, and that the LMC (least allowed) limit be met on all cross-sections. But how can one check *all* cross-sections, and how is 'cross-section' defined for imperfect cylinders with ambiguous axes? Thus definitions of size based on two-point measurements, while intuitively appealing, carry ambiguities that are difficult to resolve.

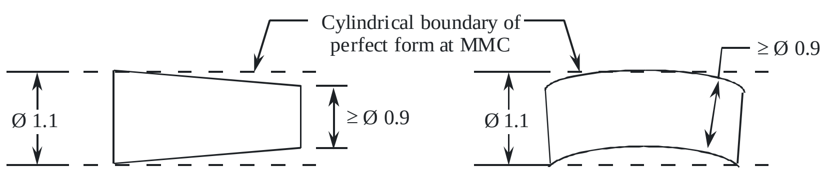

Figure 3: The Y14.5 interpretation of size under Rule 1.

Requicha [Requ 83] proposed a rigorous definition for size based on a containment zone constructed by offsetting the perfect-form boundary inward and outward. For a solid cylinder, the outer (MMC) boundary would honor Y14.5's Rule 1 (perfect form at MMC), but the perfect form inner (LMC) boundary would be more stringent than Y14.5 requires; it could not be met, for example, by the Y14.5-acceptable 'bent cylinder' in Fig. 3. Thus Requicha's definition was rejected, but its primary mechanism – a containment zone – is used in the current 'official' definition of size.

The current definition in Y14.5.1 [ASME 94b][2] was provided by Srinivasan and Requicha [Srin 93]. Figure 5 illustrates the concepts for a solid cylinder. Succinctly:

- sweep a solid ball of MMC diameter on the 'spine' (a space curve) labeled S_{MMC} to generate the solid R_{MMC};
- sweep a solid ball of LMC diameter on the spine S_{LMC} to genereate R_{LMC};
- construct zone Z as the set difference of the two R-solids;

[2] Y14.5.1 is the new (in 1994) 'mathematical companion' to Y14.5; it provides definitions in algebraic geometry for the tolerances defined in Y14.5 via prose and special-case examples.

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- actual feature G (a subset of an imperfect cylindrical surface) conforms to the size specification if G lies within Z.

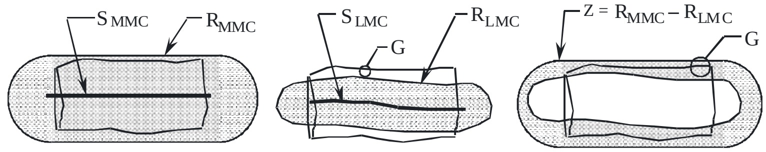

Figure 4: The Y14.5.1 definition of size conformance.

The spines are the key to this definition. S_{MMC} is a line segment under Rule 1, but S_{LMC} (and S_{MMC} if Rule 1 does not apply) can be any curve segments that satisfy some mathematical conditions.[3] The definition is existential and almost certainly impractical, because one must either exhibit spines that establish zone containment, or prove (somehow) that no such spines exist. See [Sure 94] for more information and early results on finessing existentiality, and Section 5 below for a different solution to the current impasse.

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3. FIVE CONCEPTIONS OF SIZE

This section summarizes five conceptions of size, with the aim of finding at least one that will lead to a formal characterization of size in GD&T. We begin with natural language semantics, since every natural language provides expressions for the notion of size.

3.1 VERNACULAR SIZE

In English dictionaries we find, for example:

size (noun) : the physical dimensions, proportions, magnitude, or extent of an object; any of a series of graduated categories of dimension whereby manufactured articles, such as shoes and clothing, are classified; … and others less relevant. These definitions capture everyday human use, but they are not sharp enough for our purposes. A dictionary of mathematics contains no entries for size.

3.2 PARAMETRIC SIZE

The parameter 'A' of the disk in Figure 5a can fairly be called a size parameter because (for example)

- the diameter, perimeter, and area vary monotonically with A;
- a containment property holds, *viz.*

$$Solid(A) \subset Solid(A^+), \ where\ A < A^+ ; \qquad (1)$$

- the solid retains its shape if it is indeed a disk.

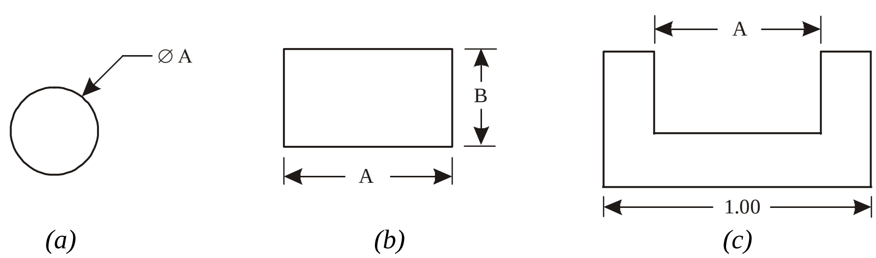

Figure 5: Parametric solids.

'A' also seems to be a size parameter for the rectangular plate in Fig. 5b, but perhaps a weaker size parameter. The diameter (as measured by the diameter of a minimal enclosing sphere), perimeter, and area grow with A, the containment property holds, but the solid's aspect ratio (hence its shape?) changes unless $B = kA$.

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[3] The conditions on the spines in the Y14.5.1 definition are not strong enough. More rigorous conditions are proposed in [Sure 95], but these have subtle consequences that may be unacceptable to the Y14.5 community.

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Is 'A' a size parameter for the U-shaped solid in Fig. 5c? The solid's diameter and perimeter are invariant under small changes in A, its area shrinks, the containment property fails, and the solid's shape may change … depending on how we construe 'shape'. So – probably not. But A might be a size parameter for the slot in the solid!

These examples suggest that (some) simple solids defined by one- or two-parameter constitutive equations (e.g. spheres, ellipsoids) may be naturally size-parametric in the constitutive parameters. For more complex, composed solids, parametric size seems to be in the eye of the beholder.

3.3 RELATIVE SIZE: SCALING

Here we seek a characterization of size through a characterization of shape. Figure 6a shows three triangles generated by rigid motions (rotations and translations) which are members of a *congruence class.* They differ in location and orientation, but have the same size and shape by any reasonable interpretation of those terms. The rigid motions that generate the triangles preserve the distances between all pairs of points in a solid, and the signs of angles defined on triples of points.

Fig. 6b shows a *similarity class* generated by uniform scalings of members of the congruence class. Clearly distances between points in the triangles are not preserved, but signed angles and *ratios* of distances are preserved, and these capture the notion of *shape.* Thus

- scaling preserves shape;
- the size of $k_2 A$ *relative to* $k_1 A$ is k_2 / k_1
- greater relative size does not guarantee containment, i.e.
$$(k_2 > k_1) \not\Rightarrow (k_2 B \supset k_1 B)\ , \tag{2}$$
unless the scaled solid is convex: see Figure 6c.[4]

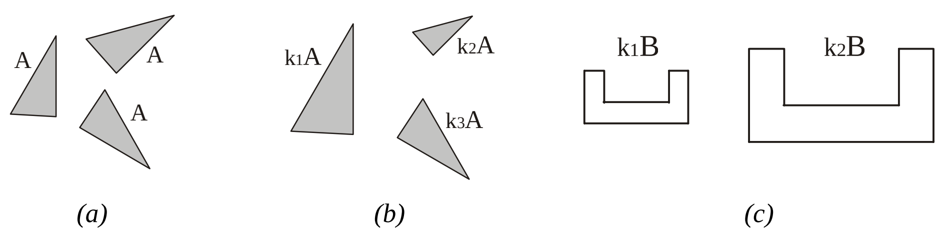

Figure 6: Defining relative size through isotropic scaling.

3.4 INCREMENTAL SIZE: OFFSETTING

Scaling expands or shrinks objects homogeneously. A different, incremental conception of size can be based on changes at an object's boundary. Constant-distance offsetting, defined through Minkowski sums and differences, provides a convenient model.

Figure 7a shows a (two-dimensional) solid A. Fig. 7b illustrates spherical dilation (offsetting) of A, *viz.*

$$\begin{aligned} Offset(A, B(t)) &= A \oplus B(t) @ \underline{o} \\ &= A \cup (\partial A \oplus B(t) @ \underline{o}), \end{aligned} \tag{3}$$

where B(t) @ $\underline{o}$ is a ball (sphere) of radius t located at the origin of a common coordinate system, $\oplus$ denotes Minkowski (exhaustive vector) summation, and ∂A denotes the boundary of A. Fig. 7c illustrates spherical contraction (insetting) of A through the Minkowski difference operation, *viz.*

$$\begin{aligned} Inset(A, B(t)) &= A \ominus B(t) @ \underline{o} \\ &= A - (\partial A \oplus B(t) @ \underline{o}). \end{aligned} \tag{4}$$

[4] Containment conditions are generally stated in this paper modulo rigid motions, i.e. "There exists a rigid motion M such that $M(k_2 B) \supset k_1 B$."

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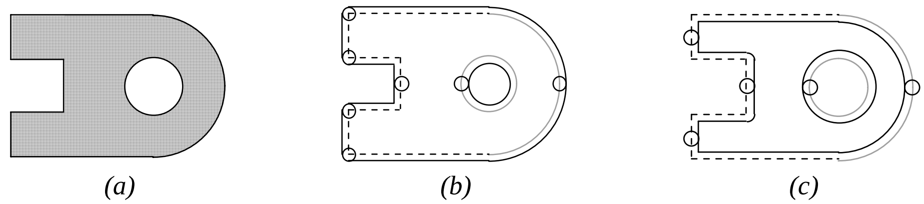

Figure 7: Defining incremental size through offsetting.

Observe that these definitions admit absolute *larger* and *smaller* size relations based on containment. Specifically,

- $A \subset Offset(A) \Rightarrow$ Offset(A) is *incrementally larger* than A. (5a)
- $Inset(A) \subset A \Rightarrow$ Inset(A) is *incrementally smaller* than A. (5b)

It is evident from Fig. 7, and easy to prove formally, that offsetting/insetting generally does not preserve shape.

3.5 Generative Size: Medial Axis Methods

Generative size entered geometric tolerancing with the Y14.5.1 definition summarized in Section 2. The paradigm: *generate* a solid by sweeping a standard simple solid – typically a sphere – over a spatial trajectory. The size of the generated entity is, by definition, the size of the swept sphere.

Suresh and Voelcker suggested [Sure 94] that this conception of size, which applies to the three generic FOSs of contemporary GD&T, can be generalized through well known medial axis concepts. In essence (see Figure 8): any general solid can be represented unambiguously by a union of variably sized spheres, determined by a Radius Function, arrayed over the solid's medial axis (spine). Note that the Radius Function varies over the spine (only two instances are shown in Fig. 8), and provides a measure of *local size.* Unfortunately, medial axis methods – while conceptually elegant – are computationally prohibitive at present.

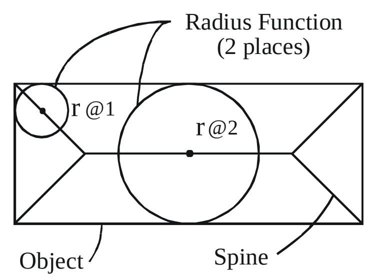

Figure 8: Medial Axis representation

3.6 What can we conclude?

Vernacular size is too vague to be useful, and parametric size does not provide a proper definition of size … but it can be a useful for simple solids. The two strongest candidates for a mathematical semantics of size are

- *relative* size, which is defined through scaling and preserves shape but generally not containment, and
- *incremental* size, which is defined through offsetting and guarantees containment but generally does not preserve shape.

Generative size, which is defined by sweeping, is currently a 'wild card'. It is clearly a useful representation for simple solids (e.g. the classical FOSs), but we don't know enough about generative methods to assess their intrinsic properties and practical significance.

* *

4. Observations

1) *Containment* and *shape invariance* are important. *Containment* is a basic principle used in assembly design, and offsetting (our incremental size mechanism) provides a powerful tool for managing containment. *Shape invariance* is paramount in applications where shape determines function. Constrained motions (e.g. shafts rotating in bearings – the shaft and bearing should remain *round* under small variations) offer obvious examples, but there are many others.

2) The classic three Features of Size play major roles in both nominal (ideal form) and variational (toleranced) mechanical design. How can such a small set of features do so much? The answer seems to lie in simple but powerful geometric properties. For nominal design, the classic three FOSs embody the most important symme-

tries found in the seven symmetry groups that underlie kinematics [Srin 99]. For variational design, the classic three *retain their shapes under offsetting,* and thus offer *containment **and** shape invariance.* In addition, all three are (conveniently) parametrically monotonic.

3) Shape invariance under offsetting is a special and rare property. [Voel 02] describes an initial study of solids that exhibit the property … a reasonably rich family that should provide candidates for extending the set of FOSs.

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5. WHITHER 'SIZE' IN GEOMETRIC TOLERANCING?

The problem posed by the current Y14.5.1 definition should either be fixed or a new definition should be sought. To date the mooted fixes have introduced almost as many problem as they have cured. The root of the definitional problem lies in Rule 1, which mixes *size* with *form*. If those two can be separated, the definitional problem should either vanish or become much easier.

On generalizing GD&T beyond the classic three FOSs: apparently one can either expand the set of FOSs using an intrinsic property (shape invariance under offsetting?) as a discriminant, *or* one can abandon the FOS concept and work wholly through offsetting and/or scaling. At present neither alternative is 'politically viable' in the Y14.5 committee world.

Finally, I am convinced that we are missing one or more facts or properties pertinent to size. Thus I view the material in this paper as pertinent but incomplete.

* *

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7. ACKNOWLEDGEMENTS

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