Tolerance Control and Tolerance Propagation in rigid assemblies: handicaps to overcome

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Abstract. Geometric Dimensioning and Tolerancing (GD&T) is a crucial step in the life cycle of industrial products and assemblies. Even though, powerful quality tools and product improvement methodologies have been largely addressed through many works, actual assembly analysis methods have considered -at large extent- only 2D-dimensional and geometric variation sources. Besides, major C.A.T. systems developed, to date, have failed to meet, rigorously, the semantic integrity of the GD&T standards (ANSI Y14.5M-82, ASME Y14.5M-94). Since tolerance analysis and tolerance synthesis in assemblies are highly required for assembly optimization, among other tasks, we shall assess some relevant works both in *deterministic* and *statistical* approaches. Limitations and critics regarding these methods are emphasized in the conclusion.

Key words: Tolerance representation, tolerance analysis and synthesis, assembly sequence analysis, ANSIY14.5M-82, ASMEY14.5M-94, Monte-Carlo simulation.

1. Introduction

Tolerances are considered as the production limits that can be held within economic boundaries based on the unit manufacturing cost of the product. The tolerance *representation* and *propagation* in assemblies are, then, required to, among other tasks, ensure functional design, answer Assembly Sequence Analysis (A.S.A.) queries, and optimize the product cost. The aim is to ascertain *optimized* assemblies which are *functionally* acceptable.

In nowadays C.A.D./C.A.M. systems, extensive use of hybrid C.S.G., B-rep and/or Half-space modelers has been performed by methods which have tackled syntax and semantics of tolerances as it is described in the GD&T standards [Dim82] [Dim94]. However, at some extend, only a few works have achieved noteworthy success in addressing the problem, as such [Mat98]. This is because the description provided in of the Y14.5M-82 and other ISO standards do not help enhance reasoning to integrate meaningful tolerance information into solid models. Not only definitions given in the tolerancing standards and the data structure templates implemented in actual C.A.D./C.A.M. systems do not match rigorously with each other, but also, tolerance development and tolerance communication activities have grown-up independently from research in C.A.D./C.A.M. systems. As a result, most C.A.D./C.A.M. systems [Req80] [Req83] [Ros86] [Che87] have achieved a fair success to cope with the tolerance specifications, comprehensively. In fact, tolerancing parts and assemblies is one step more complicated than it is likely seems to be. This is because, till recently, the standard definitions given to tolerances are concerned by geometrical (i.e., form and fit) and functional sufficiency, distinctively. Yet, nowadays, tolerances interrelate with empirical analysis methods (D.O.F. including stressful noise factors, etc.), computer analysis methods(various M.C. and geometric simulation methods, etc.) as well as economic analysis tools (quality loss function, life cycle cost models, etc.)[Cre97].

Section II and III reviewed methods for assembly tolerance analysis and assembly tolerance synthesis, respectively. Along with, limitations inherent to each method are emphasized. The concluding remarks section enumerates problems still under study in tolerance analysis/synthesis areas and questions still waiting for answers.

2. Assembly tolerance analysis

A typical tolerance analysis/synthesis system is shown in Figure 1. Simply speaking, in tolerance analysis (also referred to as tolerance control), the tolerance set assigned to components is checked and interactively redesigned so that the specified functional requirements are satisfied with a given level of confidence. In tolerance propagation (also referred to as tolerance allocation), tolerances are distributed among components and interactively redesigned on the basis of economical criteria.

Tolerance analysis falls under: *worst case* (sometimes called limit stacking, a sure-fit model, or the arithmetic law model) and *statistical* (sometimes called statistical fit model or the variance law model). *Simulated* vs *analytical*, *one*, *two* or *three-dimensional space*, and, *combination* of these approaches are also another variant of classification.

2.1 Worst case approach

The worst case paradigm consists in taking maximum or minimum material condition for each individual component. This is performed regarding the process in use and features functionalities. For male-female assemblies, M.M.C. is considered for functional gage and L.M.C. when the preservation of material for wall thickness is required. The philosophy behind worst case method is to ensure a *complete* interchangeability of the assembly components, i.e., parts are checked, *individually*, rather than a *population* of parts as this is the case with statistical tolerancing.
Since it is unlikely that all dimensions would take their extreme values, at once, the worst case methodology may lead to unjustified tight tolerances which means an unjustified increase of the cost-effective of the assembly.

Given an assembly dimension chain, the design equation can be expressed as:

$$St = f(Ct_1, Ct_2, ..., Ct_n) \qquad (1)$$

where, St is the worst case/random sum dimension tolerance and Ct_i the worst case/random variable relevant to the ith assembly component tolerance.

In the worst case approach, the mathematical model for the tolerance stack up is:

$$St = \sum_{i=1}^{n} a_i Ct_i \qquad \text{for 1D assemblies}$$

$$St = \sum_{i=1}^{n} (\partial f(Ct_1, Ct_2, .., Ct_n) / \partial Ct_i) Ct_i \qquad \text{for multi-dimensional assemblies}$$

Where, Ct_i represents the Component tolerance of component i, St is the sum tolerance of the assembly dimension chain, f(Ct_1, Ct_2, …, Ct_n) is the assembly function describing the resulting dimension tolerance of the assembly and a_i is the sensitivity of the i^{th} component tolerance. Note, in most cases, a_i equals +1 or –1 depending on the direction of variation.

Figure 2 gives an example for one-dimension worst case tolerance analysis.

2.2 Statistical tolerance approach

Statistical tolerance (S.T.) analysis implicitly supposes that a given fraction of the assemblies would be rejected in exchange of wider tolerances and better cost control [Sri97] [Bra97]. Statistical tolerancing seeks to shift the engineer's focus from unit-to-unit inspection to inspection of randomly selected samples of a product population. The S.T. is attractive because, among others, i) the ST analysis is less expensive in comparison with worst case tolerance analysis, ii) it is better adapted to answer design queries in some industrial fields, where the design and the manufacturing precision far surpass the manufacturing process capability, iii) and finally, because quality based methods such as the S.P.C. (Statistical Process Control) or the S.Q.C. (Statistical Quality Control) are nowadays totally embraced by statistical design practitioners. Therefore, many statistical methods have been developed to tackle statistical tolerance analysis/synthesis. In the following sections, some statistical based methods are described

2.2.1 *The moment based methods*

Early, the linear stack up method, also termed the Root Sum Squares method (RSS), was proposed by Mansoor [Man63]. Consider equation (1), Mansoor [Man63] stipules that the worst case combination would occur when St is linear, the Ct_i are assumed independent and normally distributed.

$$St = \left[\sum_{i=1}^{n} a_i Ct_i^2 \right]^{1/2} \qquad \text{for 1D assemblies}$$

$$St = \left[\sum_{i=1}^{n} (\partial f(Ct_1, Ct_2, .., Ct_n) / \partial Ct_i)^2 Ct_i^2 \right]^{1/2} \quad \text{for multi-dimensional assemblies}$$

For one dimensional assemblies, let consider:

- component dimensions combine linearly
  i.e.,, $St = f(Ct_1, Ct_2, ..., Ct_n)$ linear,
- the Ct_i are independent random variables and are assumed normally distributed from the nominal,
- the mean and standard deviation of each component dimension are known *a priori*,

- a_i is the sensitivity of the i^{th} component tolerance which equals +1 or –1 depending on the direction of variation,
- we made use of the 'three sigmas' rule, hereinafter, presented,

The first two moments are:

$$\mu_{st} = \sum_{i=1}^{n} \mu_{Cti} \qquad \text{and} \qquad \sigma_{St}^2 = \sum_{i=1}^{n} \sigma_{Cti}^2$$

where, μ_{St}, $\mu_{Cti,}$ σ_{st} and σ_{Cti} are the means and standard deviations of St and Ct_i, respectively
and, the sum tolerance is:

$$St = (St_{max} - St_{min}) = \mu_{St} + 3\sqrt{\sum_{i=1}^{n}\sigma_{Cti}^2} - \mu_{St} + 3\sqrt{\sum_{i=1}^{n}\sigma_{Cti}^2} = 6\sqrt{\sum_{i=1}^{n}\sigma_{Cti}^2}$$

Based on the central limit theorem assumption, St is normally distributed, regardless of the distributions of the assembly component dimensions. The convergence to the normal distribution can be more rapid if the tolerances, Ct_i, are inversely proportional [Geo86] to the sensitivities, a_i. Another point; major mechanical assemblies have been assumed to combine in a linear way. This has made the linear stack up method very popular. Yet, in some applications, the linear stack up cannot be applied. Evans [Eva75] proposed an alternative using Taylor series approximation.
Parkinson [Par85] generalized the RSS method. St is formally expressed as:

$$St = \frac{Z}{3}\left[\sum_{i=1}^{n} a_i Ct_i^2\right]^{1/2}$$

Where, Z is the number of standard deviations desired for St and the Ct_i which are supposed normally distributed.
The rule of '± three sigmas' (or the rule of six sigmas) is a special case (Z equals 3) of the Parkinson model.

$$P\,\{\mu_{St} - 3\sigma_{St} < St < \mu_{St} + 3\sigma_{St}\} = \phi\,(3) = 0.9973 = 99{,}73\% \cong 1$$

$$\Phi(x) = \frac{2}{\sqrt{2\pi}}\int_0^x e^{\frac{-t^2}{2}}\,dt$$

Where, ϕ (x) is the error function.

Because $\pm 3\sigma$ range of the St or any component dimension distribution, $Ct_{i,}$, would lay within 99.73% (so near to 100%) of its tolerance range, the rule of 'three sigmas' is more than satisfactory for statistical tolerance analysis.

Bjorke [Bjo89] used a similar method based on the Beta distribution of the component tolerances and a Gaussian distribution for the sum function. However, the method has made use of the central limit theorem, which means, inaccurate calculations when a few assembly components are being considered(<30).

Chase and Greenwood [Cha88] used a modified statistical model with a correction factor, C (C ≅ 1.5), to adjust statistical models which do not represent rigorously the actual assembly distribution. For the $\pm 3\sigma$ normal distribution:

$$St = C\sqrt{\sum_{i=1}^{n} Ct_i^{\,2}}$$

Chase and Greenwood [Cha88] have also developed a mean shift model ($f_i \in [0, 1]$) arguing that the mid-point of the tolerance zone of each assembly component can be defined as a fraction of its tolerance range. The sum dimension tolerance is, then,

$$St = \sum_{i=1}^{n} f_i Ct_i + \sqrt{\sum_{i=1}^{n} (1 - f_i)^2 Ct_i^{\,2}}$$

Note that the method [Cha88] do not help assign the shift factors and supposes that all component dimensions are normally distributed.

In [Nig95], a rectangular distribution to approximate linear tolerance chains has been used. The method yields an important concentration at the extremes limits and ignores any skewness of the real distribution.

$$St = \sum_{i=1}^{n} Ct_i - \left[\frac{n!\,p \prod_{i=1}^{n} Ct_i}{2} \right]^{1/n}$$

p is the allowable percentage of the assemblies which are out of the specified limits.

In the case where the number of the assembly components is rather low (≤ 30), which is not rare in mechanical assemblies, the assume that all component dimensions are *independent* and the sum being *normally* distributed is misleading (central theorem). Furthermore, normal distributions assume that the location of the tolerance zones are symmetric which is inappropriate to describe major manufacturing processes because of the unpredictable nature of the systematic variations (tool wear, material cutting, etc.) which are inherent to the manufacturing processes.

Non-linearity is another problem encountered in statistical tolerance analysis, especially, when geometric tolerances of form, location, runout or/and profile are considered. Inaccuracies in non-linear stack up analysis can lead to false yield of assemblies. With an exception made for one dimensional chains, this is true regardless of the type of the chain link [Ash98]. For one-dimensional chains, one can reasonably build on linear combination of populations of parts by linearizing non-linear dimensions using Taylor series, or a Monte-Carlo simulation.

The composition rules (i.e., methods for combining variabilities and composing criteria when tolerances interact) which are currently considered for worst case and statistical approaches differ from one criterion to another. In the worst case parametric tolerancing we use the worst case stack up and minimum chain rule. In the worst case geometric tolerancing there exist many rules controlling the interaction of dimension chains [Voe98], though, there is no rule for dimensional chaining. An example of interacting dimension chains is given in Figure 3. For the statistically chained parametric dimensions [Voe98], the only widely adopted rule is the summed mean and summed variances rule.

2.2.2 The Taguchi method

The Taguchi method is a statistical Design Of Experiment (DOE) technique which has been broadly implemented in both statistical and deterministic tolerance analysis/ synthesis [Kus95][Sko97]. The Taguchi quality loss function is used as a metric to evaluate the impact of the design parameters on the functional variability.

$$L(y) = \frac{1}{N}\sum_{i=1}^{N}\int_{0}^{T} L_i(t, y)dt$$

This integral equation shows how, on average, money L(y) is lost over time T (product life time) as a measurable design parameter (y) deviates from its intended value [Cre97]. Tolerances are designed so that the functional variability for optimized control factors (set of points) is established with limit customer loss. Typically, the limits are based on the $\pm 3\sigma$ rule. The method developed by Taguchi is capable of designing an efficient cost effective experiment in an easy way. The factors (independent variables) are identified using an appropriate D.O.E. technique so that the main effects, as well as, their interactions can be ascertained. The factors control the performance function which is made up of the dependant variables. Many D.O.F. techniques are available [Kus95]. In the case the number of factors is small, the Latin Square (L.S.) technique, a variant of the Latin Hypercube Sampling (L.H.S.) technique [Rub81] which is used to generate a random vector X = (X_1,.., X_k) with a fixed k, is recommended. However, when the number of factors is, rather, large, we can use, either, the fractional factorial method (developed by Dr. Taguchi) or other orthogonal arrays to determine the effect of factors and their interactions.
In tolerance analysis, component dimensions are considered as factors to which a limited set of discrete values (levels) are given. This differs with the Monte-Carlo simulation method where a significant number or trials are being considered. In tolerance synthesis, the component dimensions correspond to the factors and the manufacturing processes, which are used to obtain each component, correspond to the levels. The total cost (function of response) is the objective function to be minimized. Further details on the use of the Taguchi method in tolerance analysis/synthesis are provided in references [Kus95][Cre97].

2.3 Simulated methods

Simulated methods have also been developed to support propagation/control of tolerances in assemblies [Ash98] [Sko97] [Tur87] [San99]. Tolerance analysis using the Monte-Carlo simulation consists in simulating each instance of the assembly component dimensions, then, accumulate them to deduce the sum dimension (functional requirement). Repeating the process many times for sufficient number of samples, the distribution function of the sum dimension can be obtained. Among all, the Monte-Carlo (M.C) simulation technique is the most popular. Even though, the M.C technique is simple to implement in computers, however, it requires a large number of samples, sometimes, its is slow in converging and it is usually said that the M.C method is useful for solving problems with moderate accuracy (5-10%). For accurate computation of simulated data with a reasonable number of samples, numerous variance reduction techniques have been devised; we quote, the antithetic variables [Rub81], the Latin hypercube sampling [Rub81], the importance sampling [Sko97] (i.e., a technique which samples from another distribution -the importance function- rather than the distribution being evaluated), and, the correlation method [Sko97] (i.e., a technique which substitutes the function being evaluated by an approximation function).
In the Product Assembly Modeler (P.A.M.) system [Ash98], tolerance analysis and synthesis has been performed using the M.C simulation method and the assumption of a beta distribution of the sum dimension.

3. Assembly tolerance synthesis

Tolerance synthesis, also referred to as tolerance propagation, tolerance allocation or tolerance assignment, is the 'opposite' of tolerance analysis procedure.
The tolerance synthesis procedure is controlled by two opposing requirements, namely, maintaining optimal quality and minimizing the production cost. The former objective would lead to tight tolerances, however, the latter would release them. In Varghese *et al* [Var96], the tolerance synthesis process was defined as an optimization problem whose objective function is the manufacturing cost and the constraints include the assembly yield, the confidence level, and the tolerance limits.

Since, it is impossible to work out a cost tolerance model for each type of manufacturing process and machine, several production cost models have been devised [Don93]. Table 1 gives some among others. The choice of the allocation method depends on the cost tolerance model being chosen. Even though, it is admitted that the Lagrange multipliers method yields the best results in tolerance allocation, it cannot be applied steadily to most tolerance functions. In Kumar *et al* [Kum92], a summary of commonly known allocation methods is given (see Table 2).
Deterministic tolerance synthesis methods use process tolerances data handbooks and empirical best practice rules. However, statistical approaches made use of statistical design techniques such as the Taguchi method [And93].

For statistical tolerance synthesis, numerous methods are available. The Finite Difference (F.D.) methods are used in the design of the objective function from samples of data. The Monte-Carlo simulation is, then, run for each point to obtain the design function data. Because of the accuracy of the Monte-Carlo technique (proportional to the square root of the number of samples used), the conventional finite difference methods prove prohibitive. Gadallah and Elmaraghy [Gad94] proposed a regression analysis technique which is similar to finite differences but the points are more spaced.
Recently, to remedy the finite difference limitations, global optimization algorithms such as the genetic algorithms [Sko97] [Lee93] [Ian95] and the simulated annealing [Kir83] have been employed for statistical tolerance synthesis. Both methods are heuristic search methods. Genetic algorithms are suitable to model the tolerance synthesis problem and they have shown a high capability to tackle assemblies with interacting dimension chains (an example is given in Figure 3). The Simulated Annealing algorithm (S.A.) (i.e., a simulation of the thermodynamic process of cooling a liquid *very* slowly) is appropriate to solve non-linear optimization problems. Originally, the S.A. algorithm has been used by Zang and Wang [Zha93], for discrete tolerance optimization. Then, it has been extended to cope with continuous distributions and statistical propagation. An implementation is found in the PAM system [Ash98].

4. Concluding remarks

The paper has surveyed some research issues relevant to tolerance analysis and tolerance synthesis in assemblies. Despite, much attractive theoretical work has been performed, on one hand, and numerous implementations have been experimented, on the other hand, still numerous drawbacks and limitations need be overcome [Tra00].

- In many works, component tolerances are assumed distributed on a range of six sigma range ($\pm 3\sigma$) with the process nominal value *symmetrically centered* at the design nominal value. This does not meet the Y14.5M-82 specifications of classes and grads of fit which are not obligatory centered between the bilateral tolerance limits. Besides numerous geometric tolerances of form and localization are far representative by normal distribution laws.

- Major tolerance control and tolerance allocation methods which have been developed to date, implicitly, used the Regardless of Feature Size (R.F.S.) mating condition. Maximum Material Conditions (M.M.C.) and Least Material Conditions (L.M.C.) are less developed to chart size and geometric deviations. Although, in real life practice, M.M.C. and L.M.C. are much more required either in inspection or manufacture.
- Many works have described mathematical models for Virtual Conditions Boundaries, yet, most have failed to stick *scrupulously* to the definitions provided by the ANSI Y14.5M-82 or the ASME Y14.5M-94 standards.
- In many tolerance analysis/synthesis implementations, topological, geometric and manufacturing information inherent to tolerance chains are still defined manually. The development of reasonings for automatic tolerance chains generation are still necessary.
- Because the simulation methods are not capable to generate *exact* solutions and may demand important computational loads, much emphasis should be placed on developing further methods which are based on sensitivity analysis approaches. The contributing tolerances are, therefore, identified and necessary level adjustments are carried out so that converge to the solution with minimum iterations can be achieved,
- Most methods developed in tolerance analysis are 1D or 2D. A very few methods have addressed the 3D chain tolerance analysis models, extensively, yet highly required in actual C.A.E. packages. Moreover, little work have experimented how 1D, 2D or 3D chains can intervene in applications such as process planning, or the inspection processes,
- Many broadly distributed C.A.T. solutions proved prone to errors because of the unreliable and non-repeatable measurements taken with touch-trigger C.M.M. probes. Also, datum planes and axes models have made "abusive" use of the best fit surface methods. Though, some critics emerge whether the best fit surfaces techniques do represent appropriately high points and set up features. The considerations put on the probe sizes, the number of points as well as the scattering of these points have made the best fit surfaces techniques controversial. Furthermore, most C.A.T. solutions have totally ignored fixtures as prohibitive, yet, required to help inspect size, form and location gages.
- In many circumstances, size alone is not worth charting, yet, many C.A.T. solutions have adopted such a concept. A truer functional design mean should, rather, consider the difference between the *virtual condition*, i.e., a constant worst case boundary which is generated by the additive effect of size and geometric controls (this is obtained from either the M.M.C. or the L.M.C.) and the *resultant condition*, i.e., a nonconstant worst case boundary which is generated by controls of the size and the geometry , concurrently. Instead, the true design mean would be the intermediate size between the *virtual condition* (M.M.C. or L.M.C) and the L.M.C (whether in M.M.C. concept) or M.M.C (whether in L.M.C concept). Meadows [Mea 98] went further and proposed graphing the difference between the functional mean (halfway mark between the *virtual condition* and the L.M.C) and the actual mean (halfway mark between the as-produced worst case boundary and the L.M.C).

- For compliant sheet metal assemblies, variations do not stack up as with rigid assembly components. Additional models are still required to better assess tolerance analysis and synthesis for compliant sheet metal assembly.

References

[And93] Andrew Liou, Y.H., Lin, P.P., Lindeke, R.R., and Chiang, H.D., *Tolerance specification of robot kinematic parameters using an experimental design technique- the Taguchi method,* Robotics & Computer Integrated Manufacturing, 3 (10), 199-207, 1993.

[Ash98] Ashiagbor, A., Liu, H.C., and Nnaji, B.O., *Tolerance control and propagation for the product assembly modeler,* Int. J. PROD. RES, 1 (36), 75-93, 1998.

[Bjo89] Bjorke, O., *Computed aided tolerancing,* 2nd edition, ASME Press, New York, 1989.

[Bra97] Braun, P.R, Morse, E.P, and Voelcker, H.B., *Research in statistical tolerancing: Example of intrinsic non normalities and their effects,* Proc. 5th CIRP seminar on Computer Aided Tolerancing, edit Elmaraghy, H. A., University of Toronto, Canada, 1-12, 1997.

[Cha88] Chase, K.W., and Greenwood, W.H., *Design issue in mechanical tolerance analysis,* Mfng Review, 1 (1), 50-59, 1988.

[Che87] Ckeikh, A., *Integrating functional tolerancing in computer aided engineering environments,* JIMEC, Aman Jordan, 960-969,1987.

[Cre97] Creveling, C.M, *Tolerance design, A handbook for developing optimal specifications,* Addison Wesley Longman, 1997

[Dim82] Dimensioning and Tolerancing. *ANSI Y 14.5M,* ASME, USA, 1982.

[Dim94] Dimensioning and Tolerancing. ASME *Y 14.5M, ASME, USA, 1994.*

[Don93] Dong, Z., *Design for automated manufacturing,* Concurrent Engineering: Automation, Tools and Techniques, A. Kusiak (ed.), New York, Wiley, 1993.

[Eva75] Evan, D.H., *Statistical tolerancing: the state of the art. Part II, methods for estimating moments,* J. Quality Tech., 1 (7), 1-12, 1975.

[Gad94] Gadallah, M., and Elmaraghy, H., *The tolerance optimization problem using a system of experimental design,* In Advances in design Automation, ASME Publishers, N.Y., 69, 251-265, 1994.

[Geo86] Geoff, A., *Some statistical aspects of tolerancing,* SPIE, Optical System Design, Analysis, and Production for Advanced Technology systems, 655, 67-79, 1986.

[Ian95] Iannuzzi, M.P., and Sandgren, E., *Tolerance optimization using genetic algorithms: Benchmarking with manual analysis.* In CIRP conference on Computer Aided Tolerancing, Paris, 1995.

[Kir83] Kirkpatrick, S., Gelatt, C.D., and Vecchi, M.P., *Optimization by simulated annealing,* Science, 220, 671-680, 1983.

[Kum92] Kumar, S., and Raman, D., *Computer aided tolerancing, the past, the present and the future,* In Journal of Design and Manufacturing, 2, 1992.

[Kus95] Kusiak, A., and Feng, C.X., *Deterministic tolerance synthesis, a comparative study,* Computer Aided Design, 10 (27), 759-768, 1995.

[Lee93] Lee, J., and Johnson, G., *Optimal tolerance allotment using a genetic algorithm and truncated Monte-Carlo simulation,* Computer Aided Design, 25 (9), 601-611, 1993.

[Man63] Mansoor, E.M., *The application of probability to tolerances used in engineering designs*, Proc. Inst. of Mech. Eng., 1 (178), 29-51, 1963.

[Mat98] Mathieu, L., Clement, A., and Bourdet, P., *Modeling, representation and processing of tolerances, tolerance inspection: A survey of current hypothesis*, edit Elmaraghy, H. A., Geometric Design Tolerancing: Theories, Standards and Applications, Chapman & Hall, 1-33, 1998.

[Mea98] Meadows, J.D, *Measurement of geometric tolerances in manufacturing*, Marcel Dekker, Inc., 1998

[Nig95] Nigam, S.D, and Turner, J. U., *Review of statistical approaches to tolerance analysis*, Computer Aided Design, 1 (27), 6-15, 1995.

[Par85] Parkinson, D.B., *assessment and optimization of dimensional tolerances,* Computer Aided Design, 17, 191-199, 1985.

[Req80] Requicha, A. A. G., *Representations for rigid Solids: theory, methods and systems*, ACM Comput. Surv., 4 (12), 437-464, 1980.

[Req83] Requicha, A. A. G., Voelcker, H., B., *Solid modeling: current status and research directions*, IEEE Comput. Graph & Applic., 7 (3), 25-37, 1983.

[Ros86] Rossignac, J., R., and Requicha, A. A. G., *Offsetting operations in solid modeling*, Computer Aided Geom. Des. (3), 129-148, 1986.

[Rub81] Rubinstein, Y.R., *simulation and the Monte-Carlo method*, John Wiley & Sons Inc. New York, 1981.

[San99] Sanderson, A.C., *Assemblability base on maximum likelihood configuration of tolerances,* IEEE Transactions on Robotics and Automation, 3 (15), 568-572, 1999.

[Sko97] Skowronski, V.J., and Turner, J.U., *Using Monte-Carlo variance reduction in statistical tolerance synthesis,* Computer Aided Design, 1 (29), 63-69, 1997.

[Sri97] Srinivasan, V., *ISO debates statistical tolerancing*, Proc. 5th CIRP seminar on Computer Aided Tolerancing, edit Elmaraghy, H. A., University of Toronto, Canada, 25-35, 1997.

[Tur87] Turner, J.U., and Wozny, M.J., *Tolerance in computer aided geometric design,* Visual Computer, 4 (3), 214-226, 1987.

[Var96] Varghese, P., Braswell, R.N., Wang, B., and Zhang, C., *Statistical tolerance analysis using FRPDF and numerical convolution.* Computer Aided Design, 9 (28), 723-732, 1996.

[Voe98] Voelcker, H.B., *Remarks on the essential elements of tolerancing schemes,* edit Elmaraghy, H. A., Geometric Design Tolerancing: Theories, Standards and Applications, Chapman & Hall, 119-121, 1998.

[Zha93] Zhang, C., and Wang, H., *The discrete tolerance optimization problem,* Manufacturing Review, 6 (1), 60-71, 1993.

[Tra00] Trabelsi, A. and Delchambre, A., *Assessment on tolerance analysis in assemblies*, International Journal of Concurrent Engineering: Research and Applications, 4(8), 244-263, 2000.

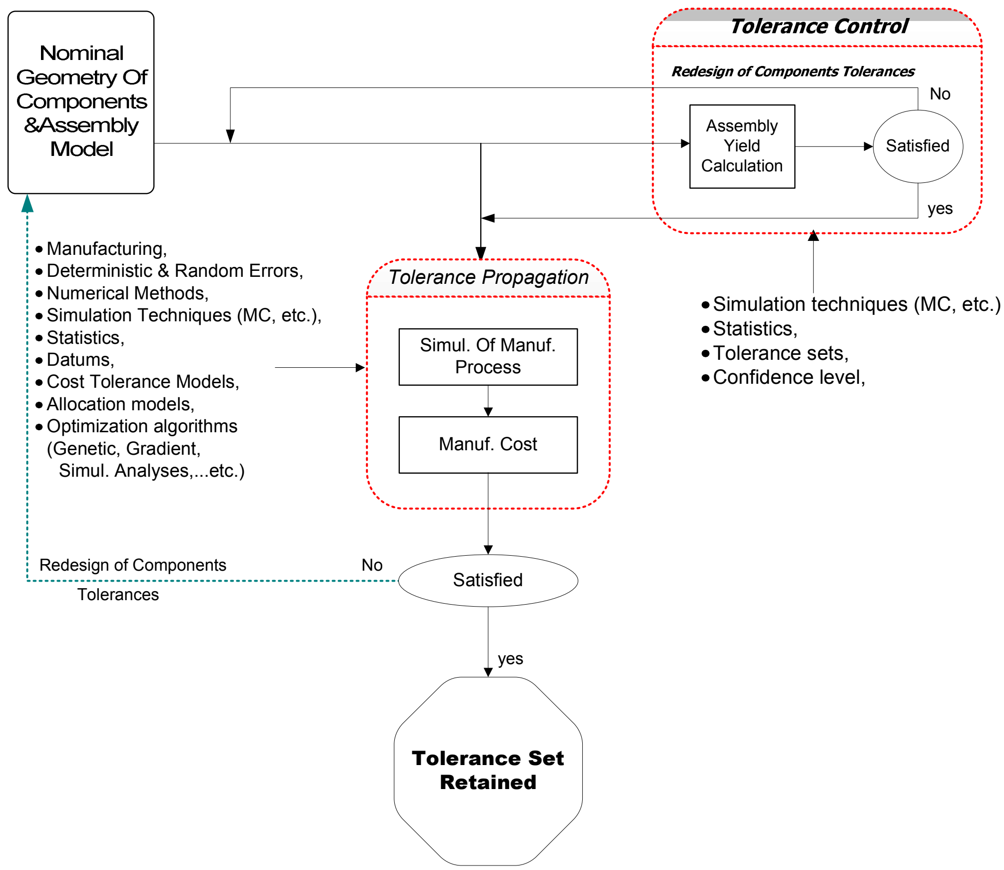

Figure 1. A block diagram of a typical assembly tolerance analysis/synthesis system.

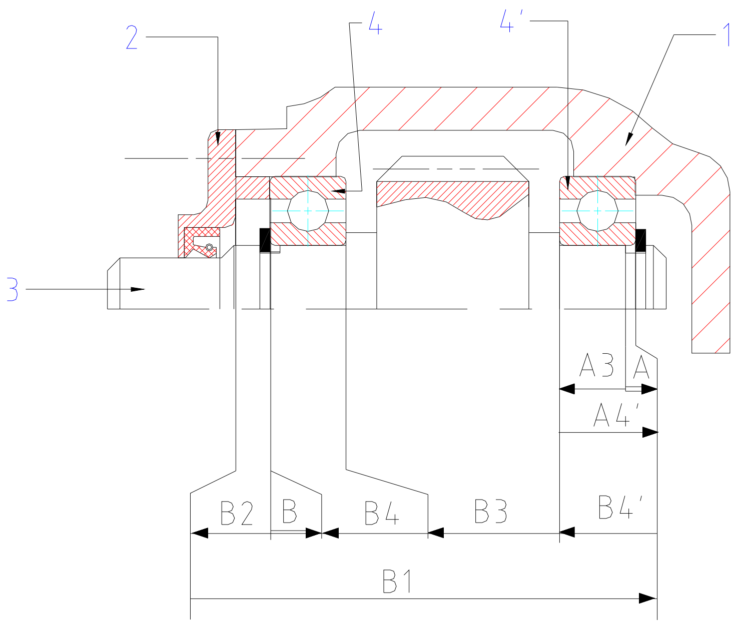

Figure 2. Design function A, assures the circlips not to make contact with both the shaft groove (3) and the roller bearing (4), simultaneously. The associated assembly dimension chain involves the component dimensions ai, with i the pieces number:

B = B1-(B4+B3+B4'+B2)

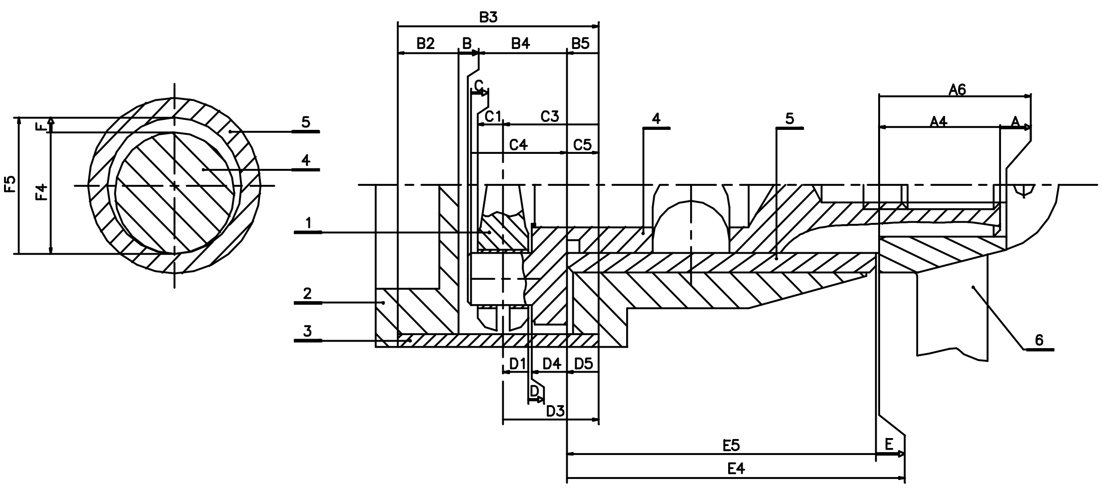

FIGURE 3. An example of an *implicit* interrelated tolerance dimension chains. The tolerance chains associated to the design functions C and D involve the same component dimension C3 and D3. This is also true with C5, B5, and D5.

Math Principle	Cost Model	Model author
Discrete	$Ct_i = t_i$; i= 1, 2,.., n	0-1 programming: Ostwald & Huang Combinatorial: Monte & Datseris Branch and bound: Lee & Woo
Reciprocal	$Ct = a_0 / t$	Lagr. Multipl: Chase & Greenwood
Reciprocal Squared	$Ct = a_0 / t^2$	Lagr. Multipl: Spotts
Reciprocal Power	$Ct = a_0\ t^{-a_1}$	Lagr. Multipl: Sutherland & Roth
Reciprocal Power	$Ct = a_0\ t^{-a_i}$	Non linear Program.: Lee & Woo
Exponential	$Ct = a_0 \exp(-a_1 t)$	Lagr. Multipl: Speckhart
Exponential/reciprocal power	$Ct = a_0\ t^{-a_1} \exp(-a_2 t)$	Non linear Prog.: Michael & Siddal
Piecewise linear	$Ct = a_i - b_i\ t_i$	Linear programming: Bjorke & Patel
Modified Exponential	$Ct = a_0 \exp(-a_1 a_2\ t) + a_3$	-
Combined RP and Exponential	$Ct = a_0 + a_1/t^2 + a_3 \exp(-a_4\ t)$	-
Linear and Exponential	$Ct_t = a_0 + a_1\ t + a_2 \exp(-a_3\ t)$	-
Cubic Polynomial	$Ct = a_0 + a_1\ t + a_2\ t^2 + a_3\ t^3$	-
Forth order Polynomial	$Ct = \sum_{i=0}^{4} a_i\ t^i$	-
Fifth order Polynomial	$Ct = \sum_{i=0}^{5} a_i\ t^i$	-

Table 1*. Cost-tolerance

Math Principle	Math Cost Model	Model Author
Discrete	$St - \sum Ct_i \geq 0$	-
Proportional scaling	$d_1/Ct_1 = d_2/Ct_2 = \ldots = d_n/Ct_n$ $\sum Ct_i \leq St$	-
Constant Precision factor	$Ct_i = P(d_i)^{1/3}$ $P = St \big/ \sum d_i^{1/3}$ where, $\sum Ct_i \leq St$	-
LaGrange Multiplier	$Ct_i = -(1/b_i) \ln(\exp(k) / a_i b_i)$ where, $K = \sum (\ln a_i b_i / b_i) - St / \sum (1/b_i)$	Spotts
Geometric Programming	$Ct_i = St / A + \ln((a_i b_i G / R)) / b_i$ where, $A = \sum (1/b_i)$; $R = \Pi a_i^{(1/A\, bi)}$ $G = \Pi(1/b_i)^{(1/A\, bi)}$	-
Linear Programming	$\text{Min} \sum C_i Ct_i + A'$ $\sum Ct_i \leq St$ $Ct_i \geq 0$	-
Non Linear Programming	Non linear programming function	-

Table 2*. tolerance allocation methods[48]

*: d_i: dimension of component i;
Ct_i: Component tolerance of part i;
S_t: sum tolerance of an tolerance chain in assembly;
C_i: slope of the ith linearized cost tolerance curve;
a_i, b_i: cost tolerance parameters;
and, P: precision factor.