AN ABSTRACT OF THE THESIS OF

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Title: Assembly Tolerance Analysis in Geometric Dimensioning and Tolerancing

Abstract approved: Redacted for Privacy

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Tolerance analysis is a major link between design and manufacturing. An assembly or a part should be designed based on its functions, manufacturing processes, desired product quality, and manufacturing cost. Assembly tolerance analysis performed at the design stage can reduce potential manufacturing and assembly problems. Several commonly used assembly tolerance analysis models and their limitations are reviewed in this research. Also, a new assembly tolerance analysis model is developed to improve the limitations of the existing assembly tolerance analysis models. The new model elucidates the impact of the flatness symbol (one of the Geometric Dimensioning and Tolerancing (GD&T) specification symbols) and reduces design variables into simple mathematical equations. The new model is based on beta distribution of part dimensions. In addition, a group of

manufacturing variables, including quality factor, process tolerance, and mean shift, is integrated in the new assembly tolerance analysis model.

A computer integrated system has been developed to handle four support systems for the performance of tolerance analysis in a single computer application. These support systems are: 1) the CAD drawing system, 2) the Geometric Dimensioning and Tolerancing (GD&T) specification system, 3) the assembly tolerance analysis model, and 4) the tolerance database operating under the Windows environment. Dynamic Data Exchange (DDE) is applied to exchange the data between two different window applications, resulting in improvement of information transfer between the support systems. In this way, the user is able to use this integrated system to select a GD&T specification, determine a critical assembly dimension and tolerance, and access the tolerance database during the design stage simultaneously. Examples are presented to illustrate the application of the integrated tolerance analysis system.

Assembly Tolerance Analysis
in
Geometric Dimensioning and Tolerancing

by

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A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

Completed August 25, 1994

Commencement June 1995

APPROVED:

Redacted for Privacy

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Date thesis is presented August 25, 1994

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CHAPTER 1
INTRODUCTION

One of the most difficult research challenges today in manufacturing is the integration of design and manufacturing functions for the promotion of cost effective production. When an engineer designs an assembly, the overall shape and dimensions of each part are specified. In contrast, manufacturing of exactly similar parts is impossible and unnecessary because manufacturing processes and material behavior involve variability by their nature. Thus, manufactured features will always deviate from their dimension design data. For these reasons, tolerances are usually specified at the design stage based on the allowable variability in part dimensions. The assignment of tolerances to part dimensions has a critical repercussion on the overall cost and quality of the assembly.

Both the designer and the manufacturing engineer recognize tolerance allocations in different ways. For example, the design engineer is concerned that the tolerance accumulation will directly affect the assembly tolerance and function. The design engineer is aware that tighter tolerances are preferable to retain the functional behavior of the assembly. On the other hand, the manufacturing engineer knows that tight tolerances will increase the cost and the difficulty of production. They are aware that improper tolerance design results in several problems, not only in manufacturing, but also in assembly functions. Therefore, tolerance analysis and design is the major link between design and manufacturing. Proper assignment of part tolerances without violating the assembly design requirements is realized by development of an assembly tolerance analysis model. Using an assembly tolerance analysis model, both design and manufacturing engineers may determine where the tolerance specifications must be made tighter and where the specifications may be relaxed. Finally, manufacturers and designers can work with the same tolerance specification system to improve the assembly quality and reduce production costs.

1.1 Problem Statement

Tolerance assignment to system components is important in assembly design and production. An assignment that is too "loose" may cause a malfunction of the assembly, whereas an assignment that is too "tight" will definitely raise the cost of production. As a result, tolerance analysis is indispensable. At present, a number of efforts have been made in the area of tolerance analysis. Conducting concurrent tolerance analysis during the design process makes it possible to discover these problems at an early stage.

Usually, tolerance analysis of an assembly requires statistical analysis to determine the effects of the inherent variability in size and shape based on some functional or cost aspect of the design. For instance, referring to conventional tolerance, some statistical distributions, such as normal distribution, are assigned to represent the distribution of actual part dimensions.

Traditionally, tolerance is expressed in a drawing in two different ways, as shown in Figure 101.

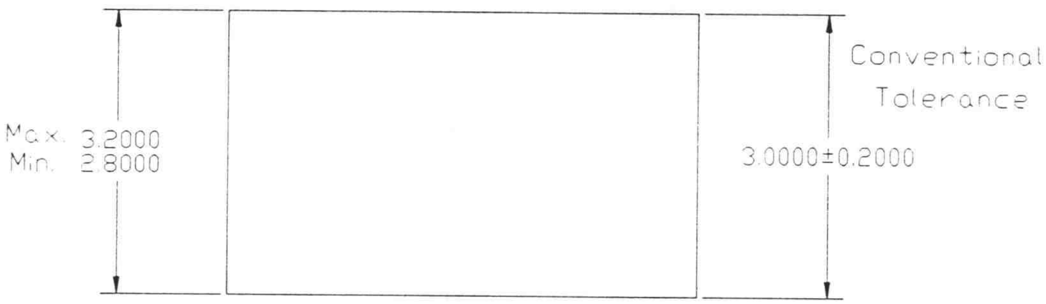


Figure 101. A Part Drawing with Two Different Conventional Tolerance Specifications

The first is shown by the maximum and minimum dimensions. The second, called the conventional tolerancing (+/- system), is specified by giving the basic dimension plus the upper and lower deviation. The maximum and minimum dimensions are displayed on the left side, whereas the conventional tolerance is noted on the right side.

Geometric dimensioning and tolerancing (GD&T) is a new tool used to specify engineering design and drawing requirements [36], supported by the American National Standard (ANSI Y14.5M-1982). The use of GD&T offers tremendous potential for U.S. companies to improve their productivity and profitability by providing a clear way to describe the geometry and size of the part. GD&T has been developed internationally over the past 40 years by committee members representing a broad spectrum of industrial and academic disciplines. By utilizing GD&T, documented in ANSI Y14.5 1982[32], many cost-saving advantages can be gained. It is an engineering language which serves as a common communication medium among designers, manufacturers and inspectors.

Using geometric characteristics and symbols to represent the actual function or relationship of part features, the designer and the customer can visualize the functions or relationship of part features. As a result, the failure of products due to unclear definitions can be reduced, thereby increasing product quality. Actually, GD&T is composed of texts, numbers and symbols which are contained in a feature box, as shown in Figure 102. At present, drawing specifications of GD&T are not available in AutoCAD software. AutoLISP is the computer language which can connect a tolerance analysis system to AutoCAD directly. It is possible to develop an AutoLISP program which allows GD&T specifications during the drawing process in AutoCAD. In addition, the tolerance information could be stored in a CAD databases for further manipulation and analysis.

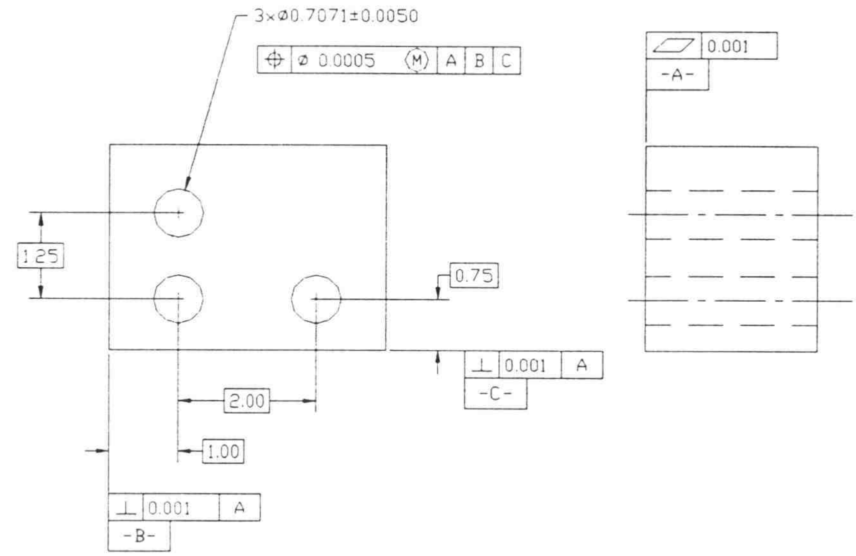


Figure 102. A Drawing with GD&T Specifications

Mathematical models are used to determine unknown critical dimensions of an assembly. Unfortunately, these models use only the conventional representation of tolerances. Also, conventional dimensions have been assumed to follow a normal distribution, which has been shown by Fortini[29] and Spotts [30] not always to be the case. Currently, GD&T is included in 2D drawings but is used only for inspecting the function and the dimension quality of the product. Tolerance analysis is not concerned with GD&T. A new assembly tolerance analysis model is developed in this research to transform GD&T specifications into conventional dimensions and tolerances, for integration with original conventional dimensions.

1.2 Research Objectives

There are three main objectives of this research: 1) creating GD&T specifications in the drawing automatically, 2) developing an assembly tolerance analysis model, and 3) developing an integrated system. These are briefly discussed below.

1) Creating GD&T Specifications in the Drawing Automatically:

An application of CAD software (AutoCAD) is developed to automatically insert both conventional tolerance and GD&T in a 2D drawing during the drawing process. To develop this application, AutoLISP programming language is chosen. The benefit of this language is its ability to access the AutoCAD system. In addition, a new screen menu is established to integrate all AutoLISP programs into AutoCAD. As a result, new users can easily familiarize themselves within a short amount of time if they know AutoCAD.

2) Developing an Assembly Tolerance Analysis Model:

An assembly tolerance analysis model is developed to estimate the nominal and tolerance size of a critical assembly dimension. A knowledge of statistics is necessary to use this model because the distribution of each part dimension in the assembly has been assumed to be a beta distribution. GD&T is considered and included in the assembly tolerance analysis model, but is different from existing assembly tolerance analysis models. There are a lot of symbols in GD&T and each symbol has its own definition and form. This research includes only the flatness feature. The advantage of this research model is its flexibility and efficiency because it involves the evaluation of the influence of GD&T on the assembly tolerance. In addition, the beta distribution is more flexible than the normal distribution. It allows the shape of part dimensions to vary from normal to skewed, indicated by alpha (α) and beta (β) parameters.

3) Developing an Integrated System:

An integrated system of assembly tolerance analysis and GD&T specifications is not available. For this reason, an integrated system has been developed to combine the applications of (1) and (2) concurrently. Using the integrated system, the user can determine the assembly tolerance automatically after finishing an assembly drawing, and dynamically insert the assembly tolerance into the assembly drawing.

The system works within an environment created by Microsoft Windows. It combines both applications with a new linking method, called Dynamic Data Exchange (DDE). Using DDE, the data between both applications (1) and (2) can be transferred dynamically.

1.3 Research Methodology

Research for the development of tolerance analysis was based upon 1) the development of AutoLISP software to create GD&T specifications in AutoCAD automatically, 2) the development of an assembly tolerance analysis model, and 3) the development of an integrated system, as discussed below.

1) Development of GD&T Specifications in AutoCAD:

The AutoCAD software for Windows is chosen and a new application is developed to specify GD&T symbols in part drawings. Tolerance chain generation is also included to create a simple tolerance chain matching the assembly drawing. The development of this application is discussed in Chapter 3.

2) Development of an Assembly Tolerance Analysis Model:

An assembly tolerance analysis model is developed in a simple mathematical form. Design parameters, such as quality factor, mean shift factor, and nominal dimension, are assigned to determine the tolerance on critical assembly dimensions. A knowledge of statistics is required because principle rules of statistics, such as sum of variance, are used in the model. In addition, the beta distribution is chosen to represent the shape of part dimensions.

3) Development of an Integrated System:

An integrated system is developed to make the application of tolerance analysis more powerful. The system is structured around Microsoft Windows. The part drawing is created by AutoCAD for Windows and the assembly tolerance analysis model is operated by Microsoft Visual Basic Version 3.0 for Windows. Both applications are integrated with DDE and work under the Windows environment simultaneously.

1.4 Thesis Organization

The organization of the research, as outlined above, is graphically represented in Figure 103.

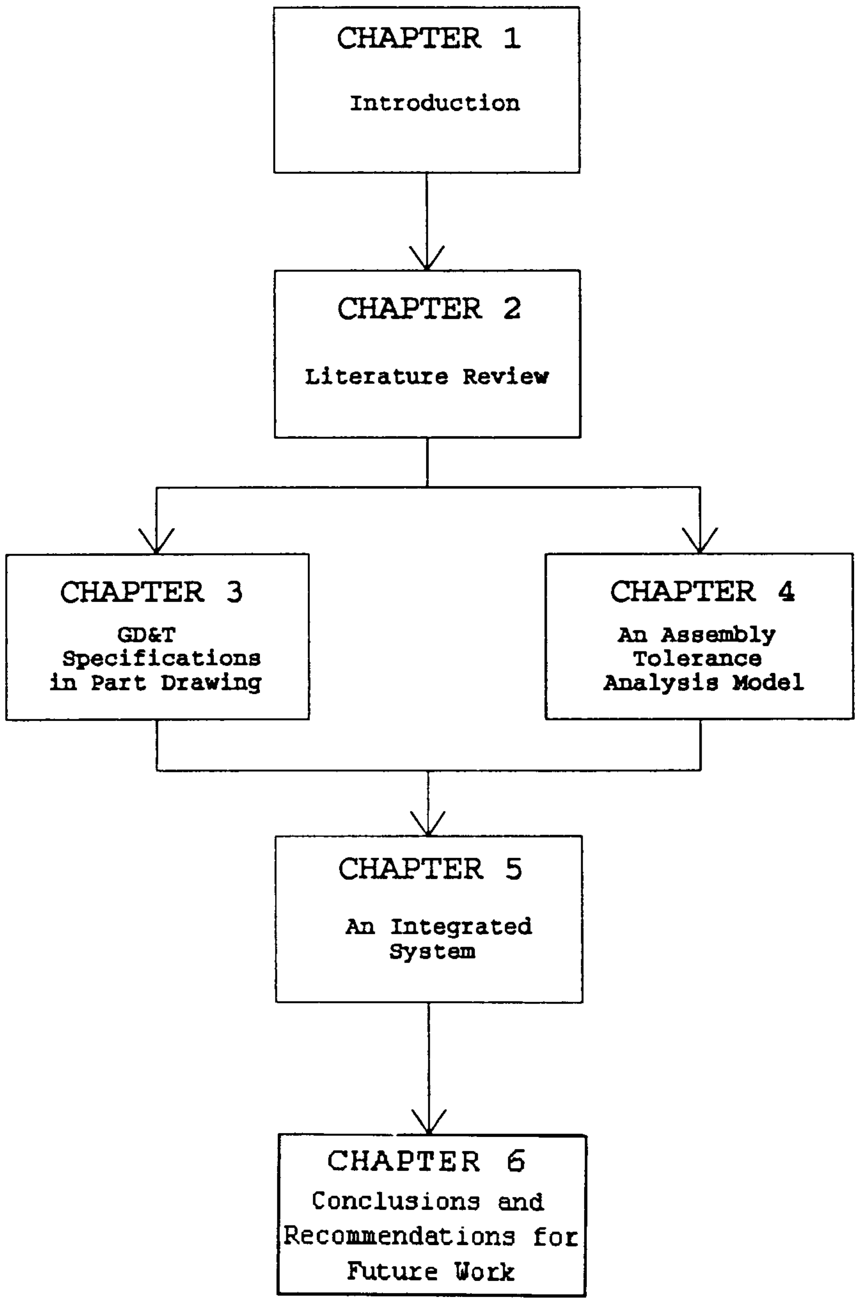


Figure 103. Research Organization Chart

CHAPTER 2
LITERATURE REVIEW

2.1 Existing Tolerance Analysis Models

Many studies have been made in the area of tolerance analysis. Mathematical models are used to determine the assembly, or unknown dimension. The most commonly used models are the Worst Case (WC) and the Root Sum Squares (RSS) models, described below.

Worst Case :

$$Tol_{asm} = tol_1 + tol_2 + tol_3 + \ldots + tol_n,$$

Root Sum Squares :

$$Tol_{asm} = (Z/3)[tol_1^2 + tol_2^2 + tol_3^2 + \ldots + tol_n^2]^{1/2},$$

where

Tol_{asm} represents the tolerance of assembly dimension,

tol_i represents the tolerance of part i (i = 1,2,..n),

and Z represents the confidence level factor.

The Worst Case model is not concerned with the distribution of part dimensions. If it is assumed that no part exceeds the tolerance zone, then the parts are distributed on either side of the nominal dimension as the worst case condition, as shown in Figure 201. In contrast, the Root Sum Squares model assumes that the distribution of part dimensions is normal with a mean at the tolerance midpoint. Moreover, the standard deviation (σ) is usually assumed to be 1/3 of the equal bilateral tolerance, as shown in Figure 202. When Z is equal to 3, the acceptance rate includes 99.73%, which means that there are only 2.7 components per one thousand exceeding tolerance. The factor Z can directly influence the acceptance rate depending on the cost of rejection relative to the cost of component tolerances. The larger the value of Z, the fewer rejects are expected. Conditionally, the Root Sum Squares model requires a random procedure so that all part dimensions are randomly selected and assembled. In fact, non-random factors occur during the manufacturing process. This behavior impacts not only the part dimension distribution, but also the mean dimension. The distribution is not normal and the mean dimension is not located on the midpoint. To solve this problem, the Modified Root Sum Squares model was developed.

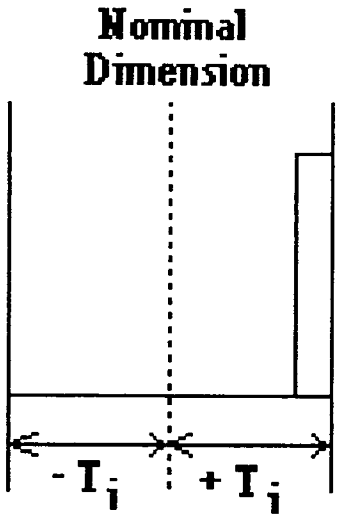


Figure 201. WC Model: Part dimensions are stacked at both extreme sides.

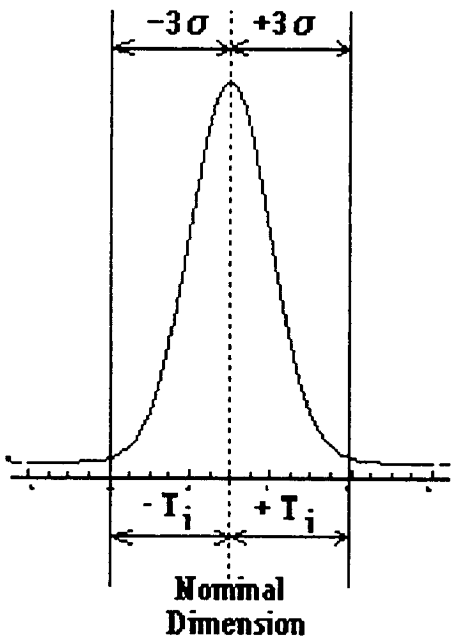


Figure 202. RSS Model: Part dimensions are normally distributed with the mean at the nominal dimension.

Modified Root Sum Squares:

$$Tol_{asm} = C_f (Z/3) [tol_1^2 + tol_2^2 + tol_3^2 + \ldots + tol_n^2]^{1/2},$$

where C_f represents the correction factor.

This model improves the accuracy of the Root Sum Squares model by introducing a correction factor (C_f). A correction factor of 1.5, suggested by Bendor[1] and Levy[2], is commonly used. Gladman[35], however, has recommended the range between 1.4 to 1.8 for the correction factor.

During the manufacturing process, it is impossible to keep the mean dimension exactly centered between the tolerance limits because of the mean shift. The mean shift results from effects such as tool wear when the machine produces a large number of parts. As a result, the mean shift certainly moves the mean dimension away from the designed mean (Figure 203). To incorporate this effect, the process variability should be less than the tolerance specification. Otherwise, significant component rejects will occur, resulting in increased costs due to reworking or scrap waste. The mean shift should not be ignored. GreenWood and Chase[3] have developed a new mathematical model, called the Estimated Mean Shift Model, which includes the mean shift as described below.

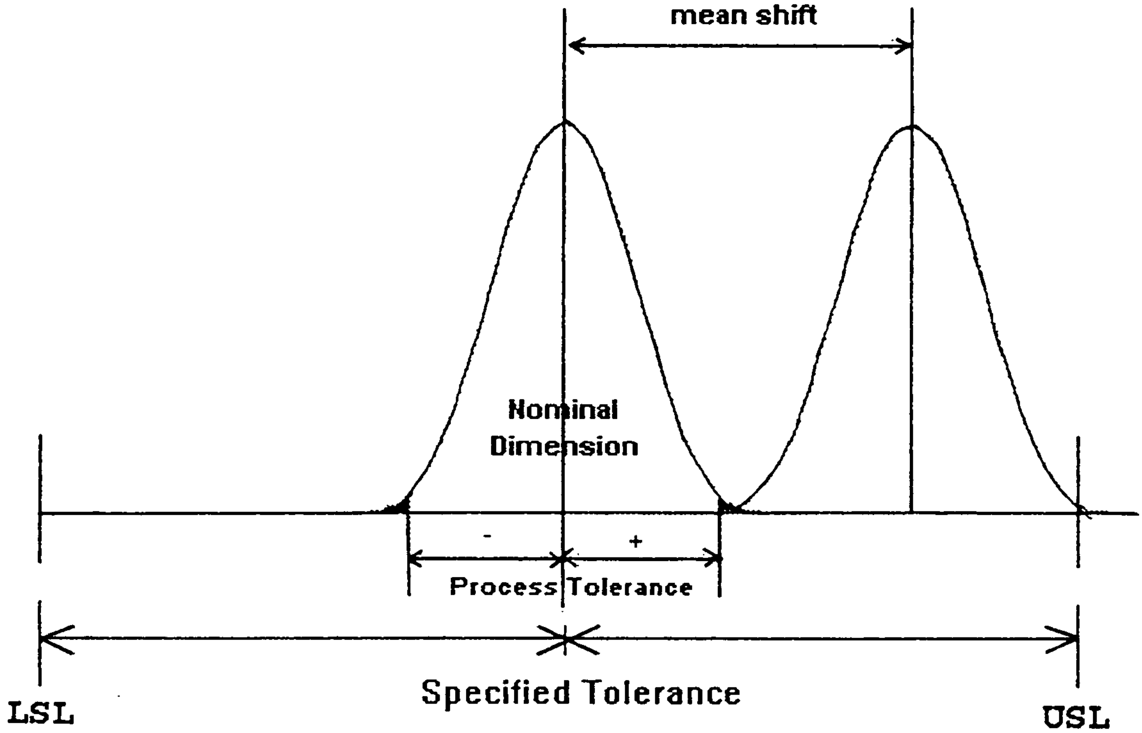


Figure 203. Mean Shift Occurs During the Manufacturing Process (LSL: Lower Specified Limit, USL: Upper Specified Limit)

Estimated Mean Shift Model:

$$Tol_{asm} = f_1tol_1 + f_2tol_2 + f_3tol_3 +...+ f_ntol_n + (Z/3)[(1-f_1)^2tol_1^2 + (1-f_2)^2tol_2^2 + (1-f_3)^2tol_3^2 + ...+ (1-f_n)^2tol_n^2]^{1/2} ,$$

where f_i is the mean shift factor of part i , $0 \le f_i \le 1$.

This model combines both Worst Case and Root Sum Squares together. The first summation $[f_1tol_1 + f_2tol_2 + f_3tol_3 +...+ f_ntol_n]$ will end up as the Worst Case if all parts have the same mean shift factor, 1. On the other hand, if all parts have the same mean shift factor 0 then the result will be the same as the Root Sum Squares. The component variability is still assumed to be three sigma (σ) from the mean(not the midpoint) to the nearest tolerance limit. When the mean shift increases, the standard deviation (σ) must decrease; therefore, the shape of part distribution varies between the Worst Case and the Root Sum Squares depending on the mean shift factor, as illustrated in Figure 204.

Mansoor[4] presented the use of probability methods and analyzed the assembly tolerance using different distributions, such as normal, triangle, sine, moving normal and rectangular (Figure 205).

A new algorithm by Wong and Ozsoy[5] can automatically generate the tolerance chain. This method is concerned with the mating relationship between various components of an

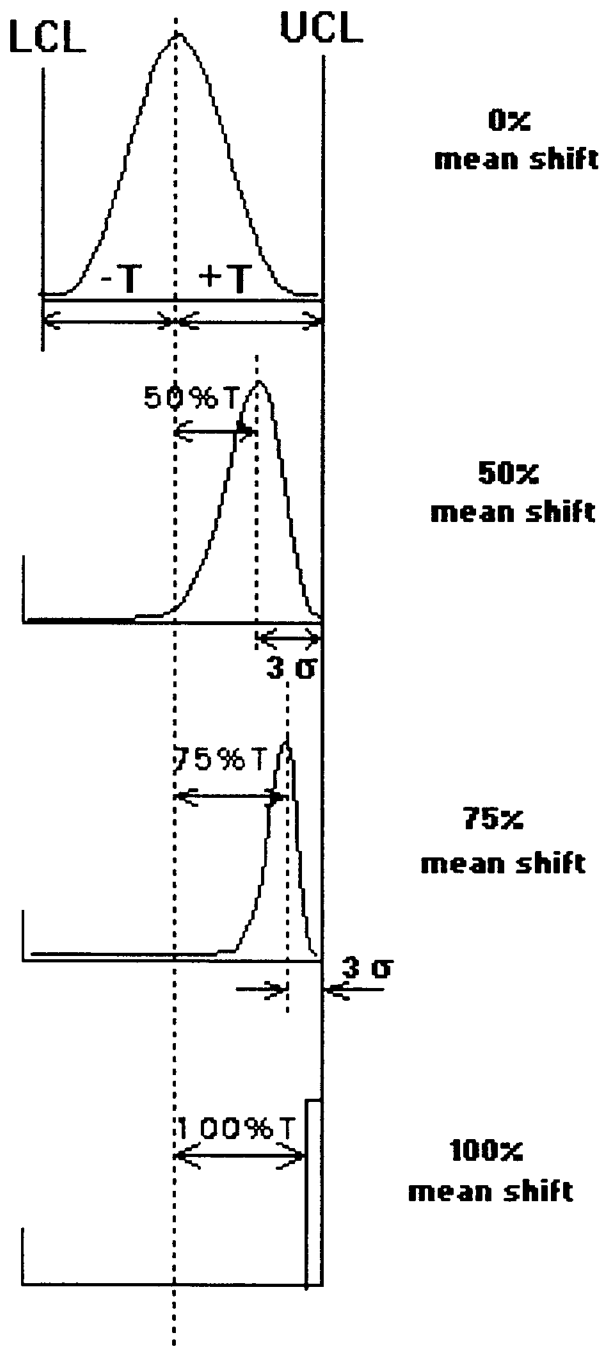


Figure 204. The Resulting Distributions Following The Assumption of Estimated Mean Shift Model (Greenwood and Chase [3])

assembly. For example, for the assembly shown in Figure 206, each part can link to other parts with mating links, as shown in Figure 207, and finally result in the tolerance chain as shown in Figure 208. However, the user must do a lot of work in terms of providing mating relationships if the assembly is more complicated.

Bjorke[6] has developed the concept of the fundamental equation based on the relationship between components in an assembly.

Fundamental equation:

$$X_{\Sigma} = f(X_i), \qquad (2.1)$$

where

X_{Σ} is the sum or critical assembly dimension, and
X_i is the individual variable or dimension in the assembly.

For the assembly shown in Figure 209, the fundamental equation is:

$$X_{\Sigma} = X_1 + X_2 - X_3 - X_4 - X_5 .$$

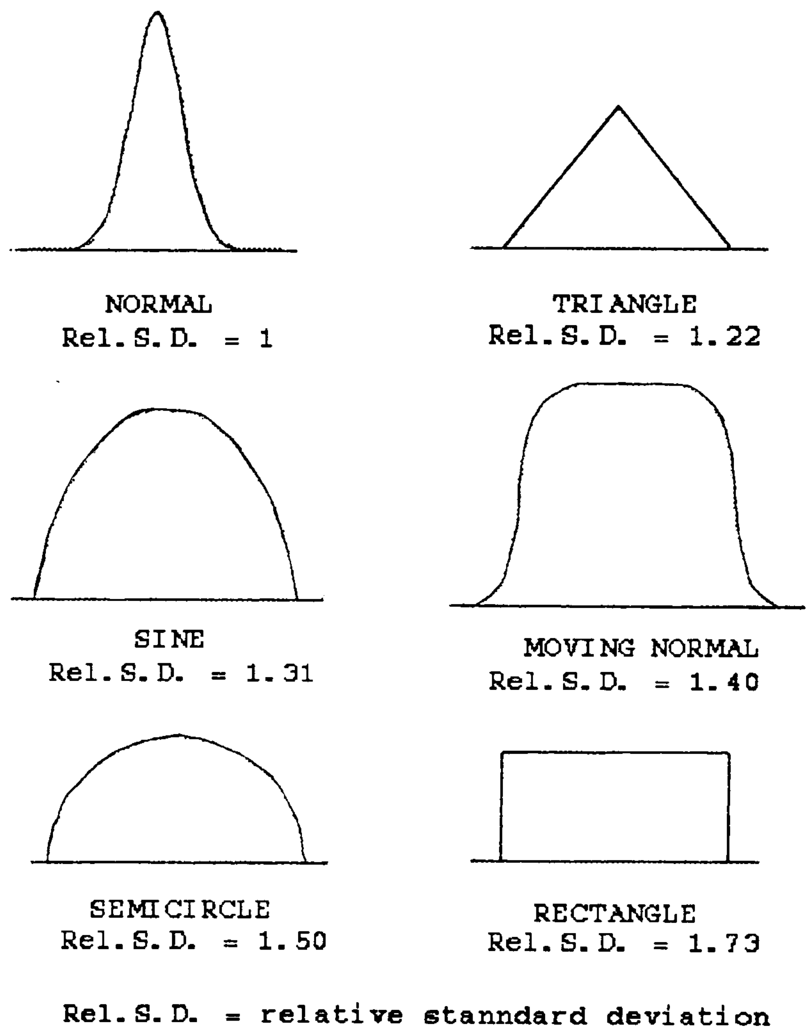


Figure 205. The Variation Shapes of Tolerance Distribution (Mansoor [4])

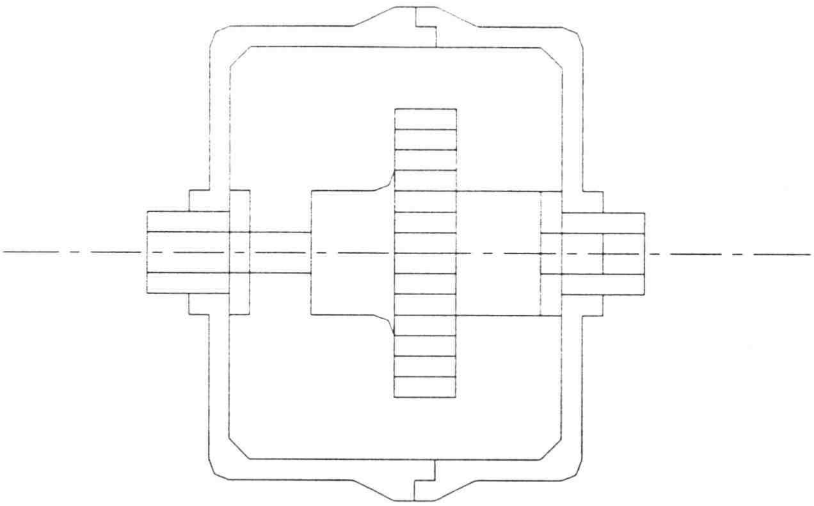


Figure 206. Gear Box Assembly (Bjorke [6])

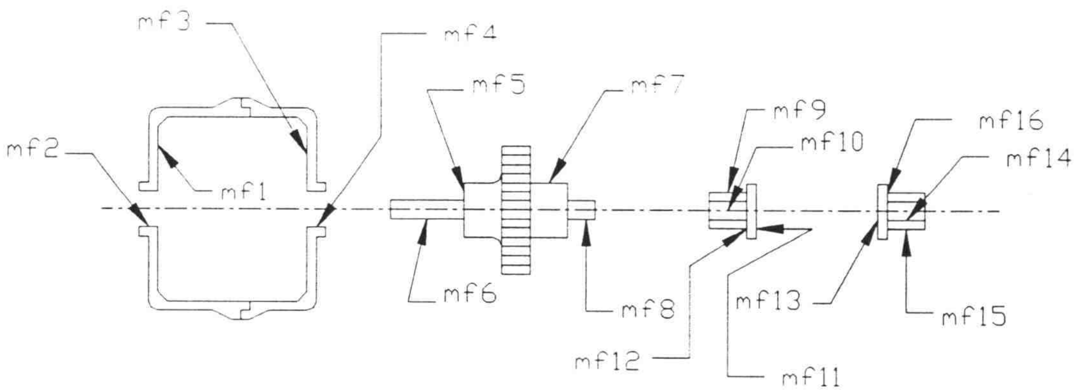


Figure 207. The Mating Features of the Components of the Gear Box (Wong & Ozsoy [5])

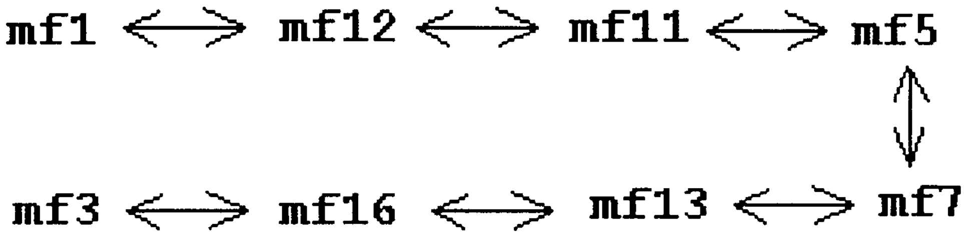


Figure 208. The Tolerance Chain Link of the Gear Box (Wong & Ozsoy[5])

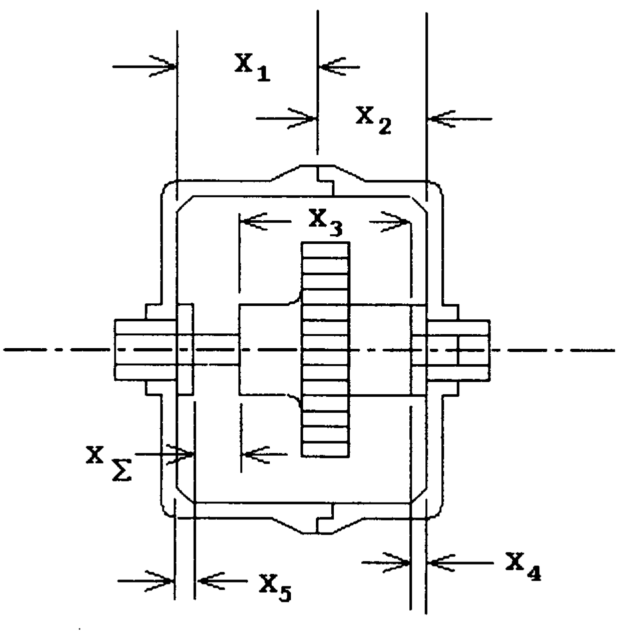


Figure 209. Mechanical Assembly (Bjorke [6])

This is simple and linear. In addition, Bjorke[6] suggested the beta distribution for manufacturing conditions such as, small lot sizes, truncated distributions or skewed distributions. A more recent work by Turner[7] uses the Monte Carlo method in solving the tolerance analysis problems in several simple assemblies. Turner's work is geometric-model-based, and the system derives all its necessary input geometric relationships from the geometric model.

2.2 Computer Applications and Algorithms for Tolerance Analysis

Computer Aided Design (CAD) software is currently the most efficient tool for designers. The CATI system developed by Soderberg[8] can interface the CAD software with an external calculation program which determines unknown assembly tolerances. Unfortunately, it can analyze only normal distribution. Fleming[9] discussed a representation of parts which includes geometric tolerances. He introduced the concept of building networks of tolerance zones and datums for the analysis of parts and assembly tolerances. In the network, each arc represents a relationship implied by the tolerance specification or by contact (in the assembled condition). Porchet and Zhang[10] proposed a way in which geometrical tolerances and process inaccuracies can be considered simultaneously. Kapur[11] developed a rational basis to analyze tolerance using quality loss functions. A general optimization model is developed to allocate tolerances to assembly parts. Different quality loss functions are also discussed. A new approach to tolerance synthesis -- CASCADE-T -- (Lu and Wilhelm)[12] uses a complete representation of the conditional tolerance relations that exist between features of a part under design. In CASCADE-T, three errors in tolerance specification, validity, consistency, and sufficiency, are adjusted during design review. CASCADE-T utilizes several artificial intelligence techniques to deal with emergent problems. A detailed account is maintained of functional requirements, the relationship among implied geometric features, and the checking of computations. Computations that check for consistency, validity, and sufficiency are done only for related geometric features.

Karolin and Ahluwalia[13] have developed a software called CATC (computer aided tolerance control system). Their method of analyzing and controlling tolerances is based on the tolerance-chart technique developed in the early 1950s. The tolerance-chart technique is the simplest method of selecting the tolerance for the resulting dimension. Machined dimensions are combined in closed chains of n dimensions, the resulting one being a combination of the other n-1 independent working dimensions. This method is limited to simple 1D cases; it is almost impractical for 2D and 3D tolerance analysis. The CATC program is iterative in nature, and uses computer graphics for information display. Zang's[14] work, which uses dimensions and tolerancing calculations in CAPP (computer aided process planning) for precision manufacturing, is also based on tolerance chain. Raman and Panchai[15 and 16] used CAD software in tolerance analysis. Both works identify and computerize a procedure for automatic tolerance assignment in CAD drawing. Martino[17] developed a new method for linear programming analysis of complex geometric tolerances. Turner and Guilford[18] presented a representation for the ANSI Y14.5 geometric tolerances in a solid model, but the representation is concise, using only a small number of primitives and some generally applicable concepts. The work by Turner and Sodhi[19] proposed a method for representing tolerance and assembly information in a feature-based environment. Assembly and tolerance analysis are performed using the dependency graph and relative positioning capabilities. Irani, Mittal and Lehtihet[20] introduced a graph theoretic representation of the tolerance chart and used linear programming to optimize tolerance allocation. The graph representation of tolerance chains is suitable for representation within, and extraction from, the CAD database. Truslove[21] created a concept of data re-use to link computer aided design and manufacturing (CAD/CAM) by improving the ability of information flow between design and manufacturing activities. However, the inadequacy of CAD systems to cater for tolerance information also has some serious implications for the information flows because all annotational information is removed from a drawing when this information is passed downstream. Shah and Miller[22] describe a product modeling system which combines a geometric modeler with a form feature modeler and a tolerance modeler. The methodology consists of rules, symbols and their interpretation, definitions, and conditions for inspection. Using this method, the user can mark datums and specify tolerances symbolically by the use of "control frames", so that new tolerances can be added without major revision. Unfortunately, their work does not go into enough detail to determine how each of the Y14.5 tolerances would be expressed or what the limitations might be. Moreover, Wu and Elmaraghy[23] discuss the geometric tolerance by using measurement data captured by a coordinate measuring machine(CMM) in order to detect out-of-tolerance conditions. This work considers only cylindrical features. Turner and Wozny[25] also provide a mathematical theory of tolerance based on the M-Space formulation. The essence of their approach is the association of instances of in-tolerance parts with points in a standard vector space of model variations. Model variables are associated with the boundary features of the parts. Although this approach seems to be powerful, the theory relies on a specific model representation and is quite difficult to express for general 3-D parts. GEOTOL software developed by Turner and Wozny[26] can solve the tolerance problem involving the use of solid modeling technology. Linear programming and Monte Carlo methods are used in this software. It was shown that the worst case tolerance analysis problem may be formulated as an optimization problem and solved using linear programming. Alternatively, Monte Carlo Simulation may be applied. The Monte Carlo method allows for nonlinear design variables and supports statistical as well as worst case analysis, but is generally more expensive than the linear programming method. Srinivasan and Jayaraman[27] have expressed tolerance constraints in terms of virtual boundaries between parts, which are shown to accurately describe the function requirements for certain classes of problems, such as fit. In addition, they propose an idea which is the main objective of this thesis,--to establish a converter. Using the converter, one can convert virtual boundary requirements into condition tolerances. Chase and Parkinson[28] surveyed a number of research papers which recommended the analysis of geometric tolerances statistically, including geometric variation in a vector loop model. Using these features, the effects on complex assemblies along with dimensional variations can be computed.

CHAPTER 3
GEOMETRIC DIMENSIONING AND TOLERANCING SPECIFICATIONS IN PART DRAWING

Part drawing is the language by which design and manufacturing engineers visualize and communicate part features. Drawing a complicated part, however, is time consuming. Computer systems are widely used to increase task potential and reduce drawing time. Computer aided design (CAD) software is one such system of design packages used to enhance the capability of the drawing process. CAD technology has developed greatly during the last decade so that now a number of CAD packages, such as AutoCAD, and DesignView, are commonly used in manufacturing and design engineering.

A part drawing represents the image of part features. It consists of linear, circular, and curved lines. Dimension specifications are described by numbers, text, or symbols, and included to represent the size of geometric features. Without tolerance specifications, however, the manufacturer cannot produce parts of exact design dimensions economically. Therefore, tolerances must be included in a part drawing. Tolerance specifications can be displayed in two different formats: 1) the minimum and maximum dimensions, and 2) the conventional tolerancing (+/-) system, as was shown in Figure 101. Tolerance specifications produce inadequate information for designers and manufacturers to accurately visualize images of part features. In addition, significant geometric characteristics related to assembly functions are needed, such as flatness, roundness, or straightness. Even though parts are produced within tolerance specifications, they may not fit well during assembly. A new geometric engineering design and drawing specification standard[32] called, "geometric dimensioning and tolerancing" or "GD&T," has been developed to improve geometric specifications in part drawing.

3.1 Geometric Dimensioning and Tolerancing (GD&T)

Tolerance specifications are used to specify control limits (maximum and minimum dimensions) of a part. In the manufacturing process, a part, if produced within tolerance specifications, is considered acceptable to be put into assembly. However, an acceptable part may not fit or work appropriately in the assembly due to some geometric features, such as flatness, straightness, or roundness. These specifications are often ignored or not recognized to be important in a part drawing. Geometric notes, therefore, are added into the part drawing to describe a geometric feature in a critical section. Unfortunately, geometric notes are stated inconsistently. They require much more time and space, tend to be scattered on the drawing, and often appear as footnotes which separate the note from the feature to which it applies. Also, notes may require translation if the drawing is used in another country. As a result, the possibility of misunderstanding geometric notes is apparently high.

A new geometric specification standard, named geometric dimensioning and tolerancing (GD&T), has been developed to improve the ability in geometric specifications in part drawings. GD&T specification symbols are used to specify, not only the geometric characteristics of a part feature, but also the relationship between part features. These symbols are compact, quickly drawn, and can be quickly placed in the drawing where the control applies.

3.1.1 Definition of Geometric Dimensioning and Tolerancing (GD&T)

GD&T specifies a geometric part's characteristics (flatness, roundness, straightness, etc.), and identifies the control value of these features.

GD&T is defined in the American National Standards (ANSI Y14.5-1982) [36]:

> Geometric dimensioning and tolerancing is a means of specifying engineering design and drawing requirements with respect to actual "function and relationship" of part features. Furthermore, it is a technique which, properly applied, ensures the most economical and effective production of these features. Thus geometric dimensioning and tolerancing can be considered both an engineering design drawing language and a functional production and inspection technique.

This standard has evolved out of a consolidation of earlier standards (SAE Automotive Aerospace Drawing Standards Section A6, 7 - 8 September, 1963; MIL-STD-8C, October 1963; USAASI Y14.5-1966; ANSI Y14.5-1973; and ASA Y14.5-1975). Updated and expanded practices, however, have been initiated in the present ANSI Y14.5-1982 standard.

3.1.2 Classification of Geometric Dimensioning and Tolerancing Specifications

GD&T specifications are classified into five types: form, profile, orientation, runout and location[10]. Each type has its own group of symbols, as shown in Figure 301. First, the form type has four symbols: flatness, straightness, circularity and cylindricity. Second, the profile type has two symbols: profile of line and profile of surface. Third, the orientation type has three symbols:

TYPE OF TOLERANCE	CHARACTERISTIC	SYMBOL
FORM	FLATNESS	⏥
	STRAIGHTNESS	—
	CIRCULARITY	○
	CYLINDRICITY	⌭
ORIENTATION	PERPENDICULARITY	⊥
	ANGULARITY	∠
	PARALLELISM	//
LOCATION	POSITION	⌖
	CONCENTRICITY	◎
RUNOUT	CIRCULAR RUNOUT	↗
	TOTAL RUNOUT	⌰
PROFILE	PROFILE OF A LINE	⌒
	PROFILE OF A SURFACE	⌓

Figure 301. Five Types of GD&T Specifications with Their Own Group of Symbols

perpendicularity, angularity and parallelism. Fourth, the runout type has two symbols: circular runout and total runout. Finally, the location type has two symbols: position and concentricity. Each symbol has its own format depending on its regulation and definition. Some symbols require more parameters than others. For example, the position symbol requires the datum reference, whereas the flatness does not. The position tolerance needs the user to specify not only the material condition, such as maximum material condition(MMC), but also the datum reference and basic dimensions. As a result, there are many variations of different formats within GD&T that the user could create during the drawing process. Figure 302 displays some of these various types of GD&T specifications in a part drawing.

Due to the complexity of part drawings, which include both conventional tolerancing and GD&T specifications, a CAD system should be selected to help the design process more easily and efficiently. AutoCAD is a well known CAD system which has served in several design fields, such as architectural drawing, interior design, civil drawing, mechanical design, and aerospace design. This research focuses on adapting the AutoCAD system to efficiently accommodate both conventional tolerancing and GD&T specifications. A brief explanation of the AutoCAD system follows.

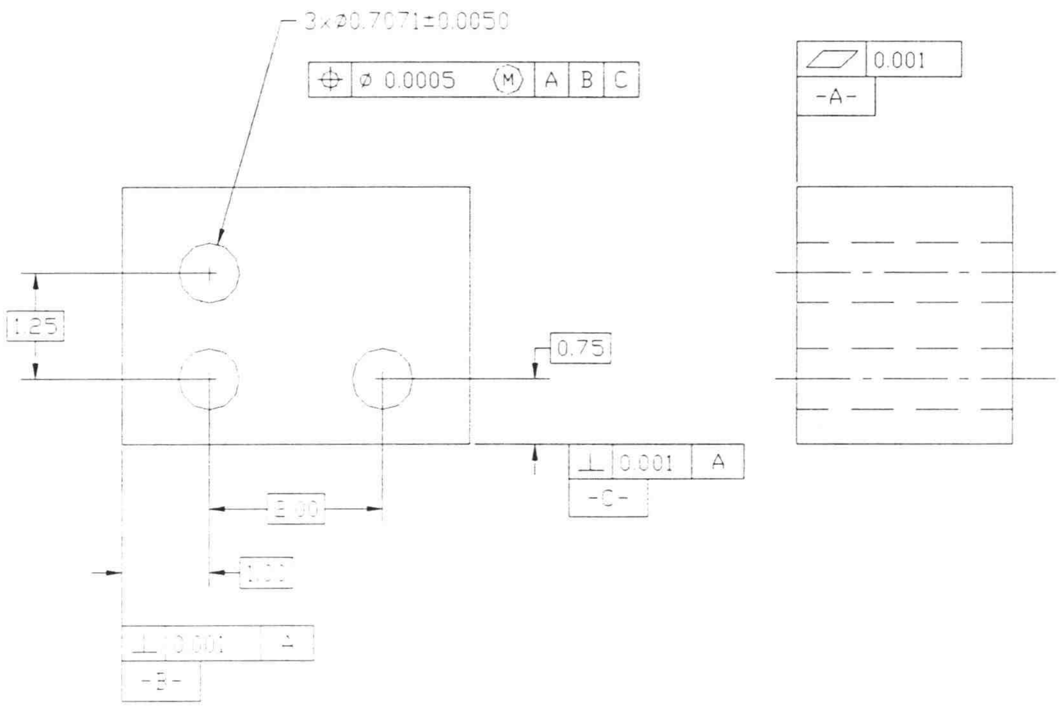


Figure 302. A Part Drawing with GD&T Specifications

3.2 AutoCAD System

The AutoCAD system design package is a powerful drawing tool. It is a general purpose computer-aided design/drafting application for computer systems. Using a computer, which offers a tremendous advantage over hand preparation, a drawing can be prepared and modified proficiently. Previously, AutoCAD was available only as a large and costly system. Due to sophisticated technology and development, it now runs in a personal computer (PC) or a network.

With AutoCAD, an engineer can draw and correct drawing errors easily, producing clean and precise final drawings. AutoCAD provides a set of entities for constructing a part drawing. An entity is a drawing element such as a line, circle, or text string. To draw an entity, the user types a command (for instance, LINE) on the keyboard or selects it from a menu. Then the user responds to prompt on the screen, supplying parameters for the selected entity. These parameters always include the desired location of the entity in the drawing. Sometimes a size or rotation angle is also required. After the user supplies this information, the entity is drawn and displayed on the screen. The user can then enter a new command in order to draw another entity or perform another AutoCAD function. Every change the user makes appears immediately on the screen. Other AutoCAD functions let the user modify the drawing in a variety of ways. Changing a parameter of an existing entity, the user can then erase, move or copy it to form repeated patterns. Drawing aids are provided to help the user position entities accurately. The simple command format of AutoCAD allows the user to accomplish all these functions easily. The user can enter commands by typing on the keyboard, or by using a pointing device to select choices from a menu. The most updated AutoCAD system is AutoCAD Release 12 [33,34]. There are a lot of tools to help the user draw an entity more easily. A standard AutoCAD menu, as shown in Figure 303, is available to let the user enter commands by using a pointing device.

AutoCAD Release 12 provides commands to draw tolerance specifications. Using dimension commands (DIMTOL, DIMTP and DIMTM), two different types of tolerance specifications can be created in a part drawing: the maximum and minimum dimension, and conventional tolerancing (+/-)system. These procedures, however, take a lot of time if the values of + and - tolerance specifications are dissimilar.

A typical GD&T specification is contained in a rectangular box which is partitioned into several sections. These sections consist of symbols, numbers, and letters. Unfortunately, only one of the GD&T specifications (basic dimension) is available in the AutoCAD system. In addition to basic dimensions, it is possible to introduce GD&T specifications into a drawing using the standard AutoCAD

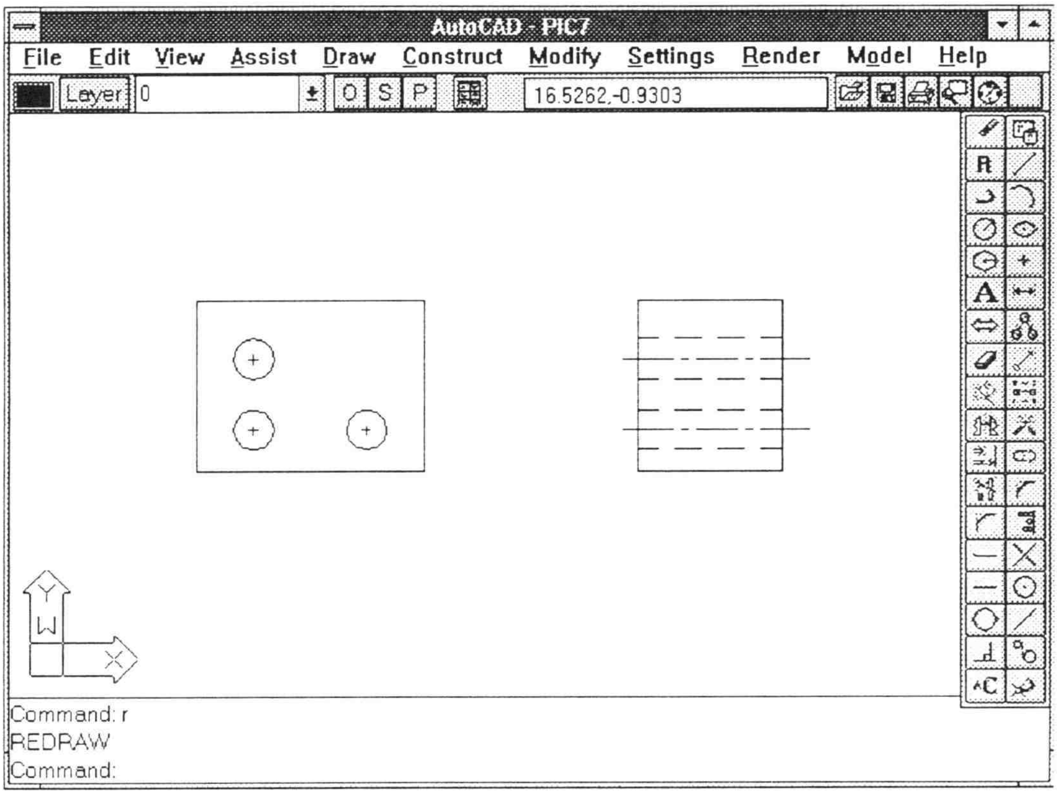


Figure 303. A Part Drawing with the Standard AutoCAD Menu

menu, but this is a very time consuming procedure. Without enhancing the standard AutoCAD menu, the user may spend a lot of time creating GD&T specifications from scratch in the AutoCAD system.

3.3 Development of GD&T Specifications in the AutoCAD system

In the AutoCAD system, a drawing element is represented by an entity. Each entity has its own data list, controlling its appearance on the screen. For example, the data list of a line entity consists of an entity name, a coordinate of two end points, a layer name, and a line type. In addition, AutoCAD environment elements, such as the snap mode, screen size, or gird display, are controlled by a number of system variables. The response of a system variable is either a continuous or a zero or one value, depending on its application. For example, if the system variable, "GRIDMODE," controls the appearance of grid display (ON/OFF), then its response is zero or one. In contrast, the system variable, "GRIDUNIT," controls the distance between grid points, so its response is a continuous value such as 1.2, 3.4, or 4.8. To set the AutoCAD environment, the user types a command on the keyboard or selects choices from a menu with a pointing device, activating a new value for the chosen system variable.

Tolerance specifications in the AutoCAD system can be drawn by setting three system variables: DIMTOL, DIMTP, and DIMTM. Using the different values for + and - tolerance specifications takes more time. Therefore, tolerance specifications within the AutoCAD system need to be developed.

As previously noted, GD&T specifications in the AutoCAD system are not currently available. Drawing a GD&T specification with a standard AutoCAD menu is time consuming because it requires a large number of drawing entities, such as lines, circles, texts, or blocks. The automatic drawing of a large number of entities should be combined with the AutoCAD system.

AutoLISP is AutoCAD's built-in programming language. Unlike other computer languages, it can directly access the AutoCAD system. With AutoLISP, the user can write a command to draw a drawing entity or set a system variable. AutoLISP is defined so well that regular users of AutoCAD software can start working with it after a minimum of training. As a result, AutoLISP is the most suitable tool for AutoCAD users who are looking for a way to enhance and extend their use of AutoCAD.

In this research, a group of AutoLISP programs is to be developed and added to AutoCAD in order to simplify the addition of tolerances and GD&T specifications on a drawing.

3.4 The Application of GD&T Specifications in a New AutoCAD Menu

The development of new AutoLISP programs along with a new AutoCAD menu can accommodate both tolerance and GD&T specifications systematically. 120 AutoLISP programs can be loaded in the AutoCAD system. Each program corresponds to only one typical specification. A new AutoCAD menu called, "TK menu," (Figure 304) is applied to organize and run all AutoLISP programs. Due to the large number of programs, their specifications are divided into four groups: 1) conventional tolerance, 2) basic dimension, 3) datum, and 4) GD&T symbols. These groups are briefly discussed below.

1) Conventional tolerance:

The standard of conventional tolerance has been commonly used for a long time. Its format contains two parts. The first part is the nominal dimension and the second part is the tolerance. As illustrated earlier in Figure 101, 3.000±0.2000 is the conventional tolerance, where 3.000 is the nominal dimension and ±0.2000 is the tolerance. This example illustrates an equal tolerance because the upper tolerance (+0.2000) is equal to the lower tolerance (-0.2000). An unequal tolerance is available but it is not commonly used in tolerance analysis, therefore, this study concerns only with equal tolerances.

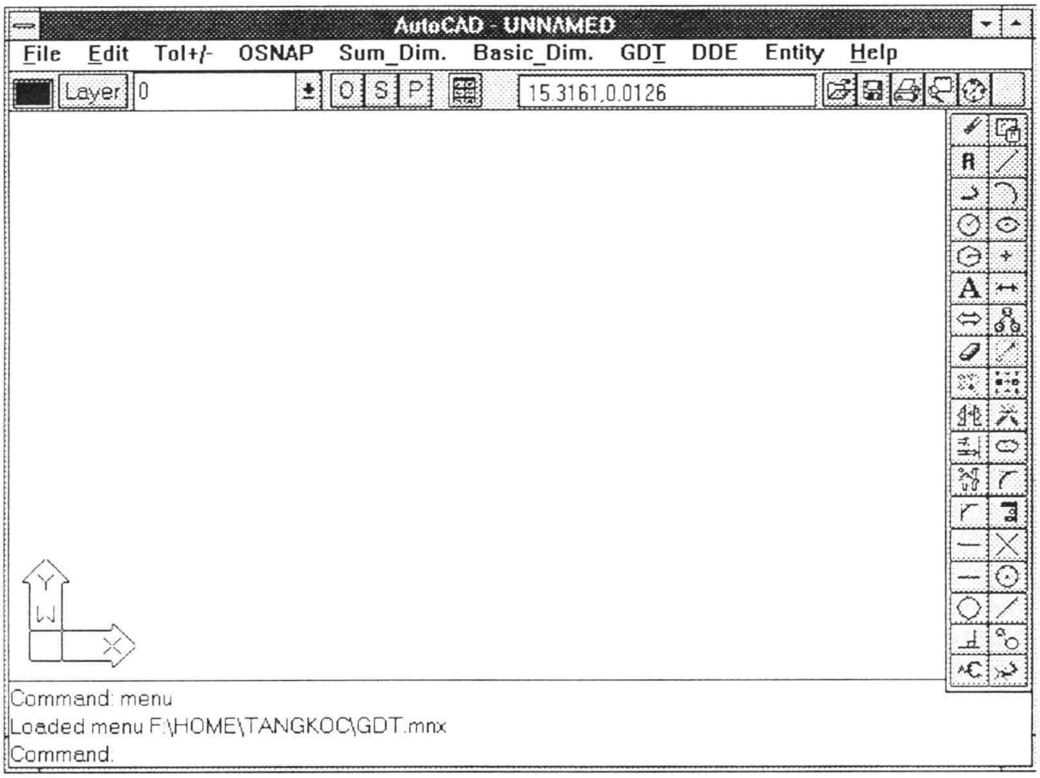


Figure 304. The New AutoCAD Menu Called, "TK menu."

2) Basic Dimension:

Basic Dimension is a theoretically exact value used to describe the exact size, profile, orientation, or location of a feature of the datum target[32]. It is used as the basis from which permissible variations are established by tolerances in feature control frames or on other dimensions or notes. An example is shown in Figure 305.

3) Datum:

A datum is the point of origin from which the location or geometric characteristics of features of a part are established. It is a theoretically exact point, axis, or plane derived from the true geometric counterpart of a specified datum feature[32]. Datum surfaces and datum references are actual part surfaces or features used to establish datums. They include all the surface or feature inaccuracies. Figure 306 illustrates the application of datum references in a part drawing.

4) GD&T symbols:

GD&T characteristics are classified into five types: form, profile, orientation, runout and location)[32], and

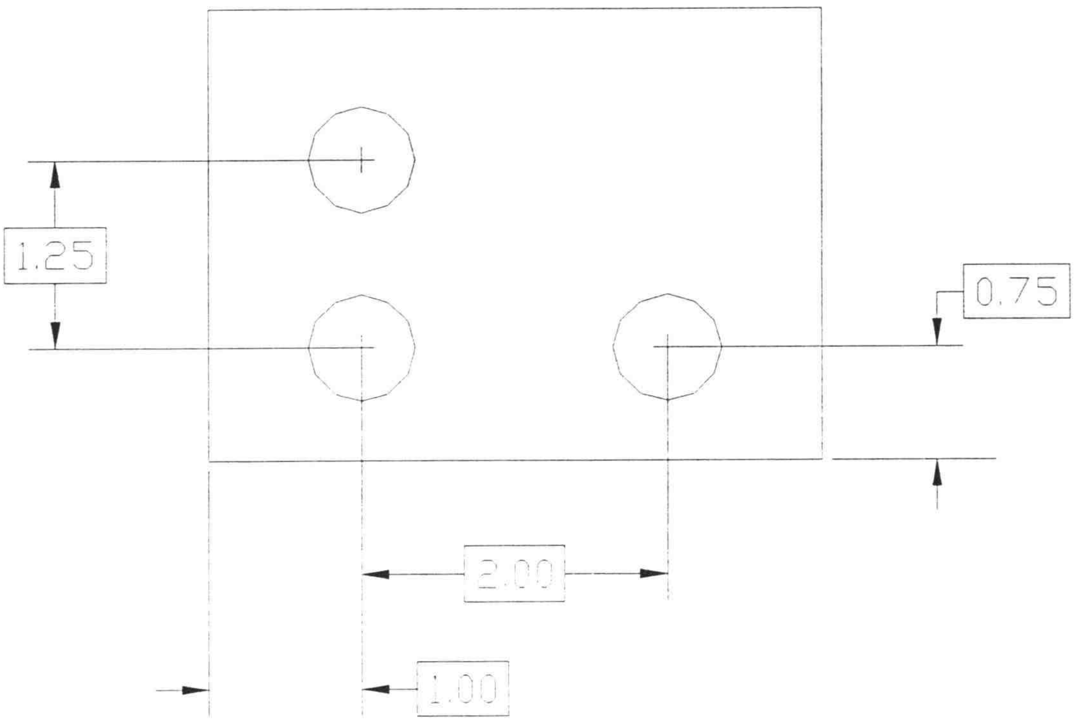


Figure 305. A Part Drawing Example with Basic Dimensions

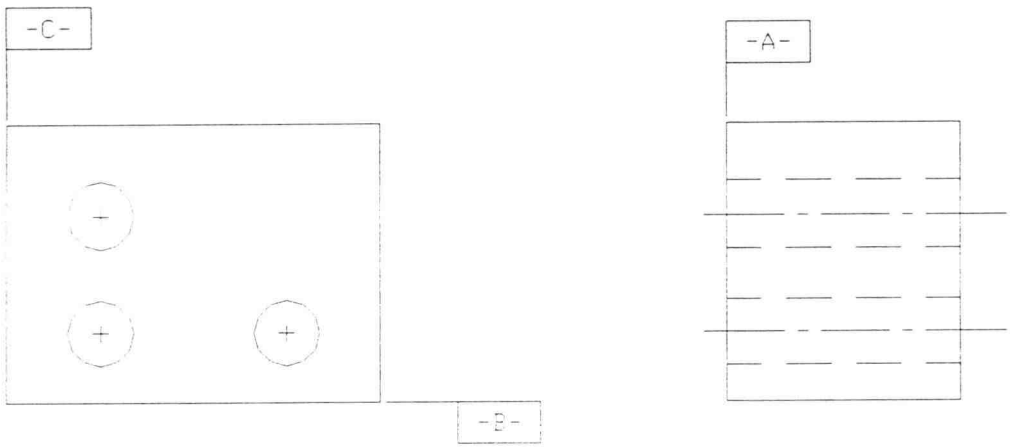


Figure 306. A Part Drawing Example with Datum References

each type has its own symbols, as illustrated in Figure 301. Each symbol may be of more than one format following the regulation and the application.

The new TK AutoCAD menu provides a pull-down menu which consists of a text string. To run an application, the user clicks at a pull-down menu by a pointing device, and makes a selection. Then a program is loaded and runs in the AutoCAD system. For example, Figure 307 shows the application of the TK menu in which GD&T specification symbols are drawn. During the time the program is running, the user should respond to supply parameters, expressed by text comments on the command line. The user prompts a point location, or types a value from the keyboard. After the user supplies all the information, the GD&T specifications or conventional tolerances will be drawn in the drawing. The application of TK menu is explained in Appendix A.

In addition to drawing an element, AutoLISP can be used for other applications. There is increasing interest in tolerance analysis in the AutoCAD system due to the need to discover tolerance and assembly problems in the design stage. For this reason, a tolerance chain generation application is introduced to be incorporated into the AutoCAD system.

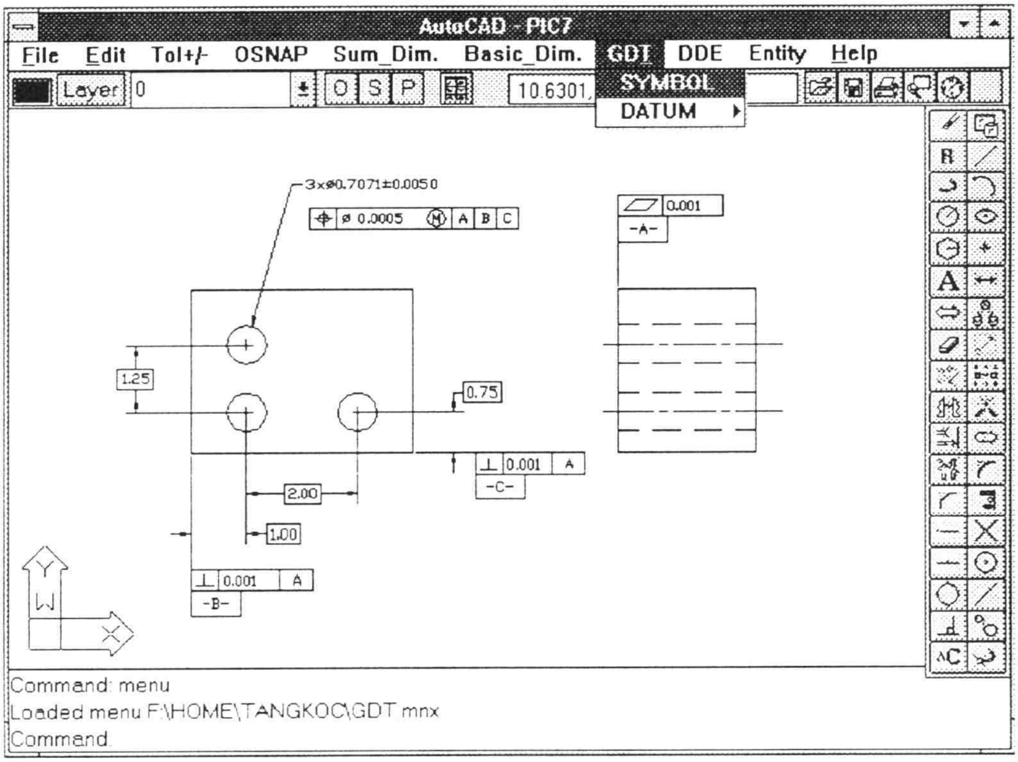


Figure 307. An Application of TK Menu for Creating GD&T Symbols

3.5 Tolerance Chain Generation

A tolerance analysis algorithm dealing with tolerance chains was developed in 1978 by Bjorke[6], who also defined "tolerance chain" in Computer-aided Tolerancing[6] as follows:

> A chain is a sequence of elements such that each element in the sequence has one endpoint in common with its predecessor in the sequence and its other endpoint in common with its successor in the sequence. A chain that does not encounter the same endpoint twice, is called elementary. A chain that does not use the same element twice, is called simple. All other chains are called interrelated.

The difference between elementary, simple, and interrelated tolerance chains is shown visually in Figure 308. According to a linearized fundamental equation, Equation 2.1's structure is associated with the simple chain. Each parameter (X_i) in Equation 2.1 is assigned independently and its value does not effect the others. Therefore, the sum dimension (X_Σ) can be simply determined from accumulating each individual dimension. If the value of an individual dimension effects the others, the interrelated chain is applied, resulting in the sum dimension becoming more complicated. This research, however, considers only the case of simple chains.

To show the geometric meaning of chain links, an assembly in Figure 309 displays each dimension as the vector in the space span. An AutoLISP program is written to create

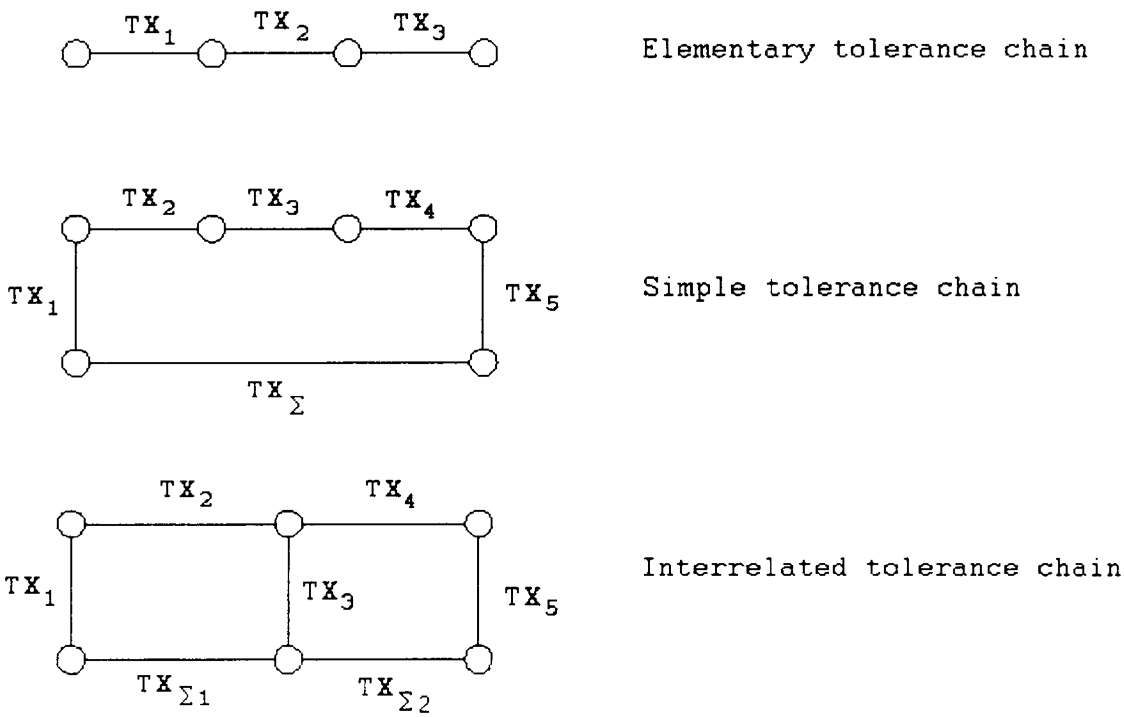


Figure 308. Tolerance Graphs (Bjorke, [6])

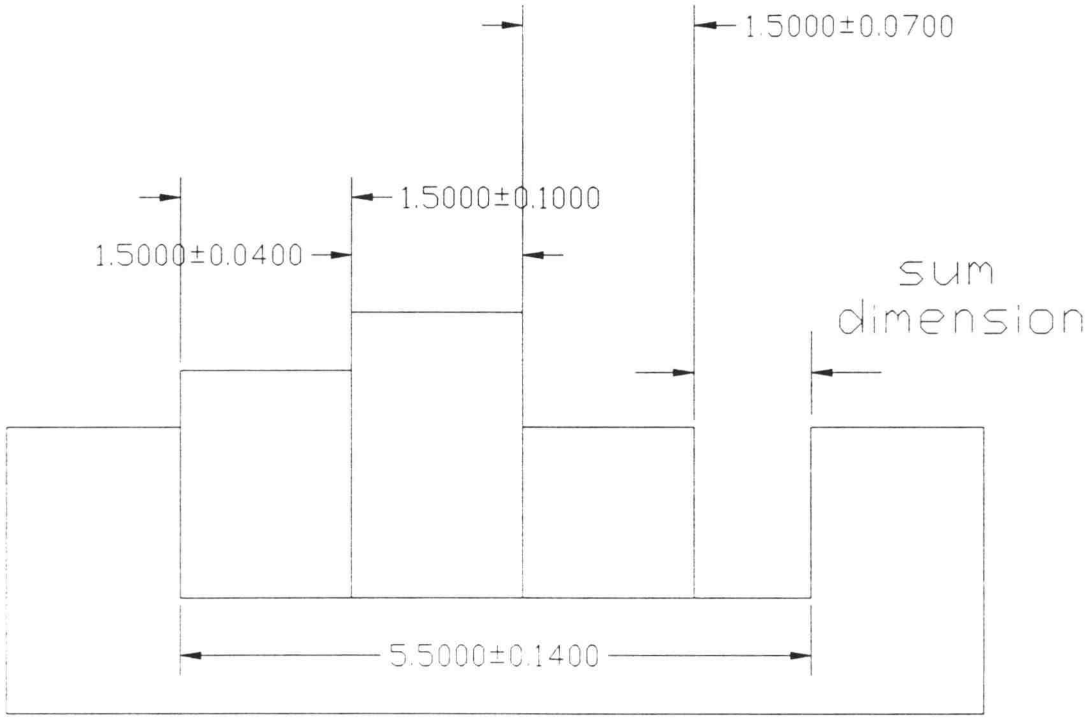


Figure 309. An Assembly Drawing Example

tolerance chains automatically related to the mating link of each part dimension. First, the unknown or sum dimension must be identified. Then a tolerance chain can be generated, starting at the left end-point of the unknown dimension and ending at the right end-point of the unknown dimension like a closed loop. All dimensions in the chain except one (the unknown dimension) must be specified, otherwise, the tolerance chain cannot be created due to the lack of the dimension's link. To avoid a conflict between tolerance chains and the assembly drawing, the tolerance chains are created in a different layer labeled, "*chain.*" The new TK AutoCAD menu provides a pull-down menu labeled, "Tol.Chain," to run the AutoLISP program. An example of this application is shown in Figure 310.

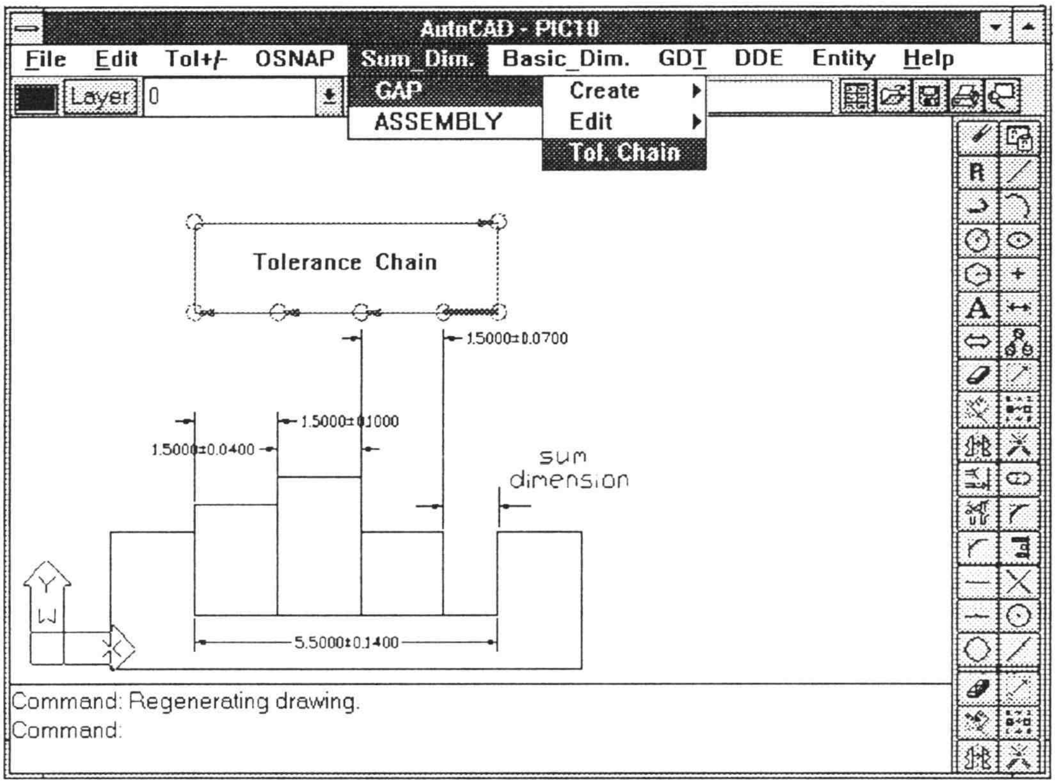


Figure 310. An Application of TK Menu for Generating a Simple Tolerance Chain

CHAPTER 4
A MODIFIED ASSEMBLY TOLERANCE ANALYSIS MODEL

4.1 Introduction

The existing assembly tolerance analysis models as previously discussed, are not useful under real manufacturing conditions. For example, using the Worst Case model for analyzing an assembly with a large number of components will greatly reduce the component tolerances resulting in higher cost of production. Also, the statistical models such as the Root Sum Squares model, the Modified Root Sum Squares model, and the Motorola's model [3] are limited by their assumptions involving statistical part dimension distributions. These models work only when part dimensions are normally distributed. In fact, part dimensions do not exactly follow normal distribution due to non-random factors such as tool wear and die wear. Due to non-random effects, the natural process tolerance ends up with a skewed distribution. Referring to the central limit theorem, the distribution of the sum dimension will be approximately normal regardless of the variation in the shapes of component distributions. However, this theorem will not work when the number of components is small. The existing assembly tolerance analysis models are designed to analyze tolerances only in ideal conditions and need to be modified.

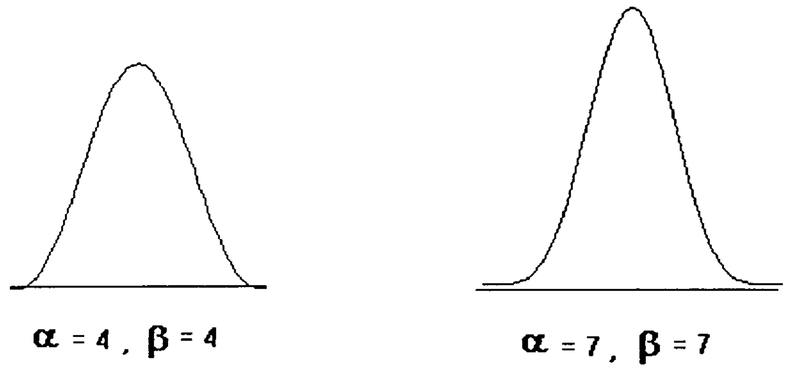
Beta distribution [6] has been chosen to develop a modified assembly tolerance analysis model. The beta distribution has the following advantages:

1. The beta distribution covers the actual range of distribution from normal to rectangular,
2. the beta distribution has a finite range, and
3. the beta distribution covers an asymmetrical case.

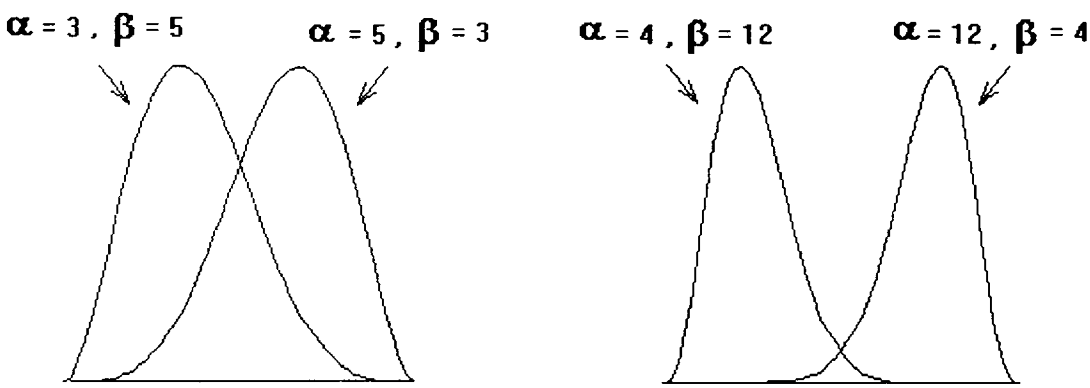
The beta distribution is more flexible than normal distribution for representing part dimensions.

The beta distribution is defined by four parameters: the lower limit (a), the upper limit (b), the alpha parameter (α), and the beta parameter (β). The alpha (α) and the beta (β) parameters control the distribution shape as shown in Figure 401. One should carefully evaluate all four parameters for each part dimension in order to understand the behavior of the sum dimension.

Referring to the central limit theorem, the assembly or sum dimension distribution is assumed to be approximately normal, nevertheless the variations in the shapes of component distributions are assorted. In the case of a small



a) Symmetric Beta Distribution



b) Asymmetric Beta Distribution

Figure 401. The Variation Shapes of the Beta Distribution, a) Symmetric and b) Asymmetric Shapes

number of parts, this theorem cannot be applied approximately. As displayed in Table 401[6], two different assumptions of the sum dimension distribution, normal and beta, are compared using the percentage of errors from the actual values. Comparing the data in Table 401, the sum dimension of the normal distribution model gives the percentage of errors approximately nine times greater than that of the beta distribution model. Therefore the assembly, or sum dimension distribution in this study is assumed to be the beta distribution. The objective of this chapter is to develop a new assembly tolerance analysis model based on beta distribution. In addition, the effectiveness of the flatness symbol, one of the GD&T specification symbols, is also studied and included in the new model, which helps to more accurately determine sum dimensions and tolerances.

Table 401. Computing Error by Summing Up Equal Rectangular Distributed Dimensions [6]. (Confidence Level 99.73%)

Number of dimensions	Normal distribution model	Beta distribution model
2	29.7%	3.0%
3	18.5%	2.0%

4.2 Visualization of the Part Dimension Including Geometric Dimensioning and Tolerancing (GD&T)

The existing assembly tolerance analysis models can accommodate only conventional dimensions (+/- system). A new tool to specify engineering design and drawing requirements, Geometric Dimensioning and Tolerancing (GD&T), is now commonly being used in manufacturing design. As illustrated in Figure 402, GD&T is applied in the part drawing.

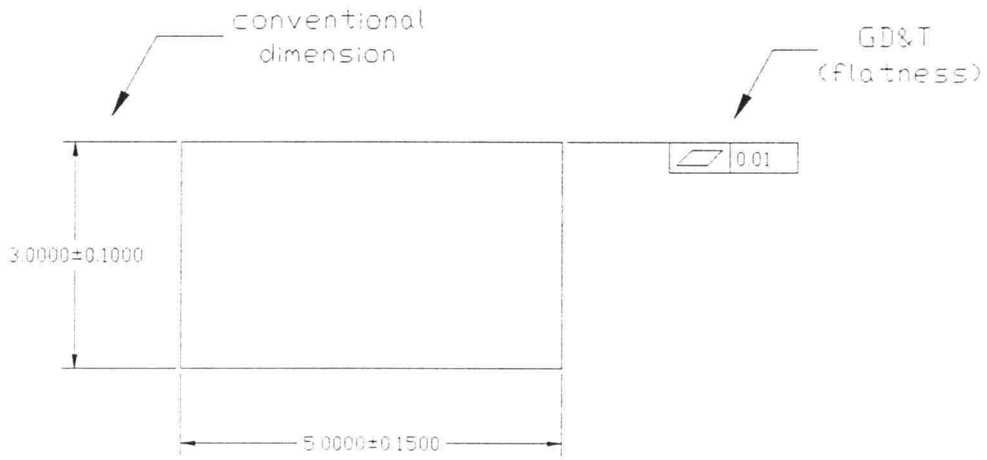


Figure 402. Part Drawing Including Both Conventional Dimension and GD&T (Flatness) Specification

Its purpose is to demonstrate not only the actual function but also the relationship between part features. There are two specifications displayed in the drawing. The first specification is the conventional dimension (+/-), and the second is the GD&T specification which is placed in a rectangular block. The GD&T symbols discussed in Chapter 3 are classified into five types: form, profile, orientation, runout and location. Each type has its own group of symbols. Moreover, each symbol has its own definition and application. Due to the scope of this research, only the flatness symbol is studied and included in the modified assembly tolerance analysis model.

The first specification in Figure 402, the conventional dimension, is assigned to limit the maximum and minimum size of the part dimension. This means that no part is allowed to exceed the specified tolerance limits. As a result, the part in Figure 403 should have dimension values between 2.9 and 3.1. In addition, the second specification is one of the GD&T symbols, flatness, which includes the value of the flatness tolerance zone, 0.01, in the rectangular box. Its application is to control the surface variation of the part. Following the rule and the application of the flatness symbol, the visual part feature (flatness) from Figure 402 is shown in Figure 403. When the flatness symbol is assigned, the part dimension is not only limited within the first specification (±0.1), but also limited within the value of the flatness tolerance zone, 0.01 or ±0.005. Usually, the first specification (conventional dimension) implies that the flatness is automatically controlled by the specified tolerance zone. For example, the first specification in Figure 402 automatically controls the flatness of the part

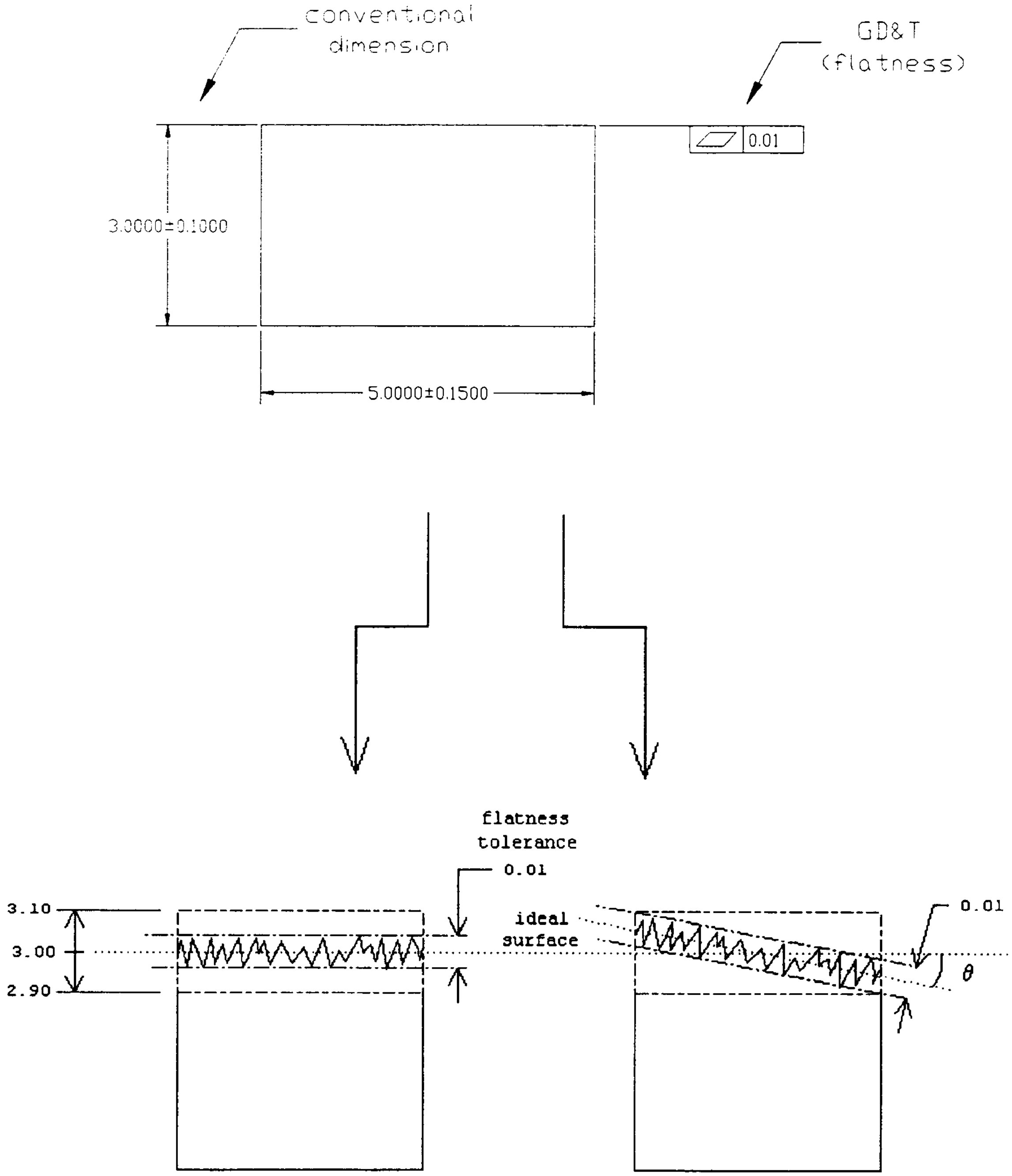


Figure 403. Visual Aspects of the Part Surface When the Flatness Symbol Is Applied

dimension within a 0.2 flatness tolerance zone. If the first specification is not sufficient to satisfy the function requirements of a part surface, the flatness symbol is necessary. Certainly, the flatness tolerance zone is always less than the specification tolerance zone. As a result, the flatness tolerance zone which is assigned in Figure 402 cannot be greater than 0.2.

When the flatness symbol is assigned to the surface of the part, there are two visual aspects as shown in Figure 403. The difference between the two aspects is the slope between two parallel planes and the ideal surface. On the left visual part, the slope between two parallel planes and the ideal surface is zero, whereas the slope between two parallel planes and the ideal surface on the right is not. In this research, it is assumed that the slope between two parallel planes and the ideal surface is zero. As a result, the left visual part in Figure 403 is considered to verify the various features of part dimension in the next section.

4.3 Development of a Simple Model to Determine the Nominal Dimension

The surface of the part dimension in Figure 403 is enlarged in Figure 404 to illustrate the variation of the flatness tolerance zones, when the flatness symbol is applied. Each pair of two parallel planes has its own middle point which varies within the specified tolerance. If the machine quality is more precise, the middle point of two parallel planes (flatness tolerance zone) will be located close to the middle point of specified tolerance. Otherwise, the middle point of two parallel planes will deviate from the middle point of the specified tolerance zone when the machine quality gradually reduces. Therefore, the quality factor (C_q) is generated to determine the middle point of the flatness tolerance zone. The center dimension (X_c) represents the middle range of the specified tolerance zone (L). The nominal dimension (X_n) represents the middle point of the flatness tolerance zone (T). All variables are shown in the part drawing in Figure 405. X_n varies from the minimum (X_c-L+T/2) to the maximum (X_c+L-T/2) depending on the quality factor (C_q). A simple model is developed as shown below:

$$X_n = X_c + C_q(L - \frac{T}{2}) \quad ; \quad -1 \leq C_q \leq 1. \qquad (4.1)$$

From Equation 4.1, it can be concluded that if the machine is of high quality, C_q becomes zero, therefore X_n is equal to X_c. On the other hand, the value of X_n varies between the minimum and the maximum range with the value of C_q , -1 and 1, respectively. A manufacturer can estimate the value of C_q

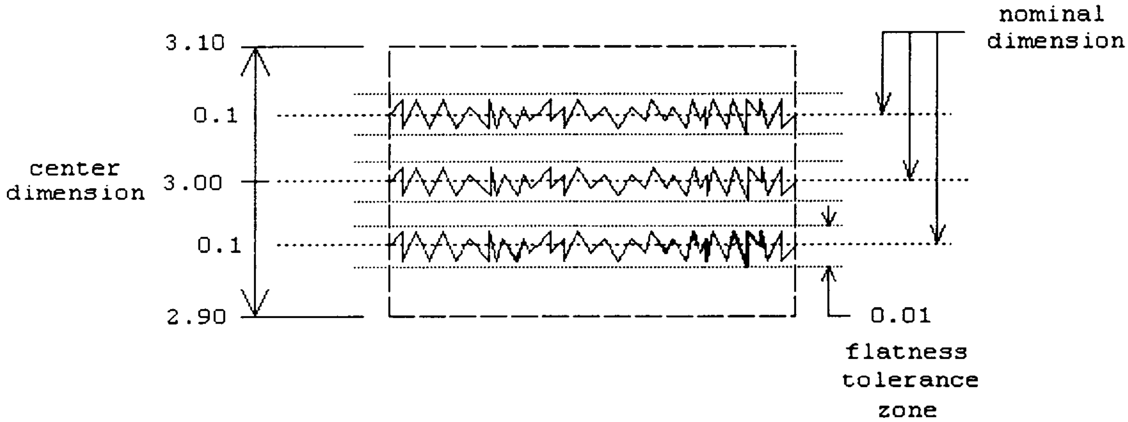


Figure 404. Variation of the Flatness Tolerance Zone on the Part Surface Depending on the Quality Factor(C_3)

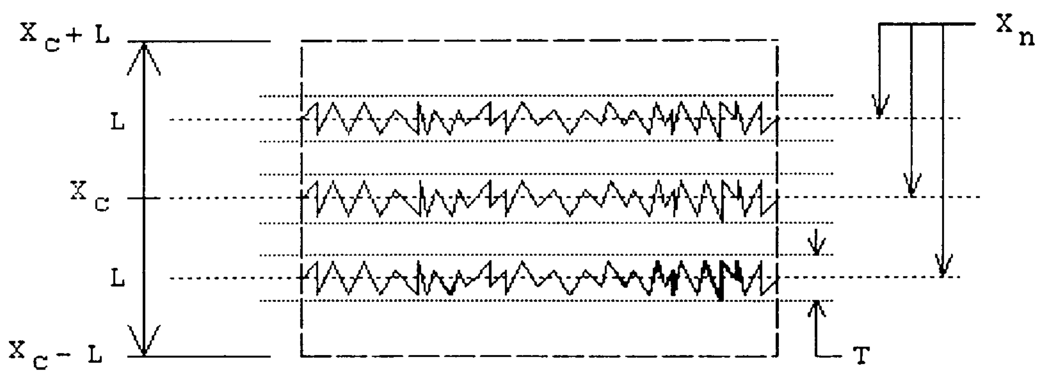


Figure 405. The Assigned Variables to the Part Dimension Specification to Determine the Nominal Dimension Depending on the Quality Factor (C_3)

by using his experience related to the behavior of the machine. Finally, Equation 4.1 can be developed in terms of part i. Let:

$X_{n,i}$ represent the nominal dimension of part i,

$X_{c,i}$ represent the center dimension of part i,

L_i represent the specified tolerance of part i,

T_i represent the flatness tolerance zone of part i, and

$C_{q,i}$ represent the quality factor of part i.

The simple model to determine the nominal dimension of part i is:

$$X_{n,i} = X_{c,i} + C_{q,i}\left(L_i - \frac{T_i}{2}\right) \quad ; \quad -1 \le C_{q,i} \le 1 \qquad (4.2)$$

After the nominal dimension of part i is specified, the flatness tolerance zone (T_i) automatically limits the upper and lower value of the actual part dimension. Now the part is not allowed to exceed $X_{n,i} \pm T_i/2$ (instead of $X_{c,i} \pm L_i$). Generally, the natural part tolerance is produced by the natural process tolerance ($\pm t_i$) depending on the type of machining process. Each type of machine gives a specific range of natural process tolerance. For example, a part processed by a broaching machine results in a natural process tolerance less than that processed by a drilling machine. If the natural part tolerance is less than or equal to one half of flatness tolerance zone ($\pm T_i/2$), the part will be acceptable provided the nominal dimension remains centered. Unfortunately, it is impossible to keep the nominal dimension centered during the production period because of the occurrence of the mean shift. The mean shift usually develops in part manufacturing due to uncontrollable factors such as tool wear or die wear when the machine produces a large number of parts (Figure 406). If the natural process tolerance ($\pm t_i$) is assigned to be equal to one half of the flatness tolerance zone ($\pm T_i/2$), as shown in Figure 407, the mean shift will cause parts rejection. To allow mean shift to occur, the natural process tolerance is always less than one half of the flatness tolerance zone, as displayed in Figure 407.

4.4 The Application of the Beta Distribution in Part Tolerance

An assembly consists of several components. In the case of a small number of parts, the central limit theorem is too imprecise to assume that the assembly sum dimension tolerance is normally distributed. As discussed in 4.1, the beta distribution in this research is applied not only to each part dimension, but also to the sum, or assembly dimension [6]. Before evaluating the assembly dimension and tolerance, the parameters assigned to each part dimension are considered.

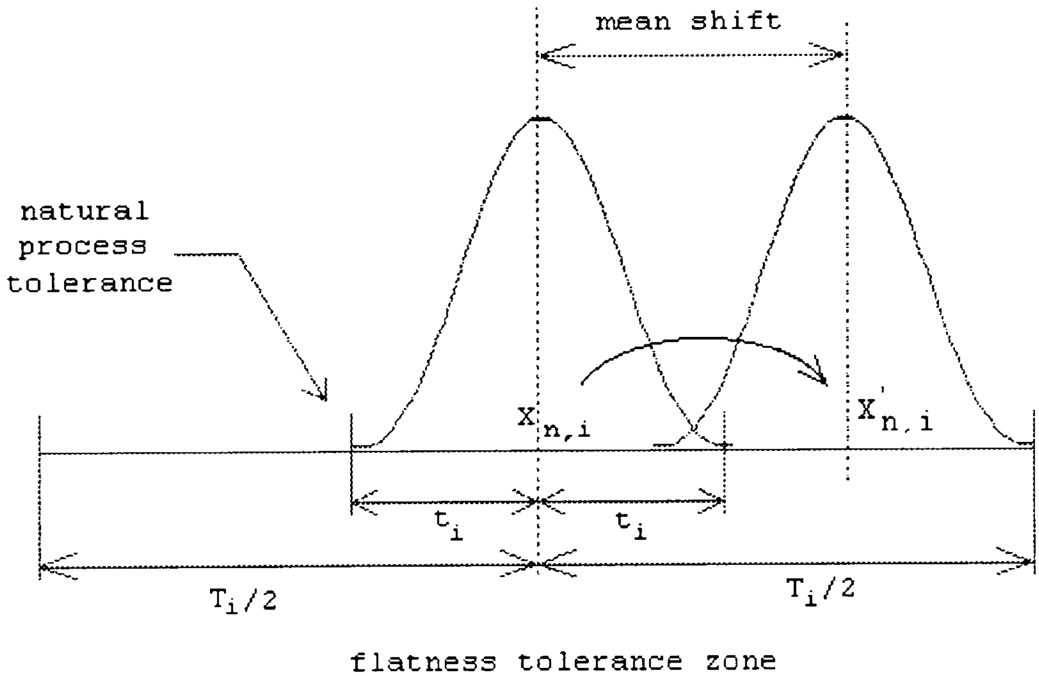


Figure 406. Mean Shift Occurs During the Manufacturing Process

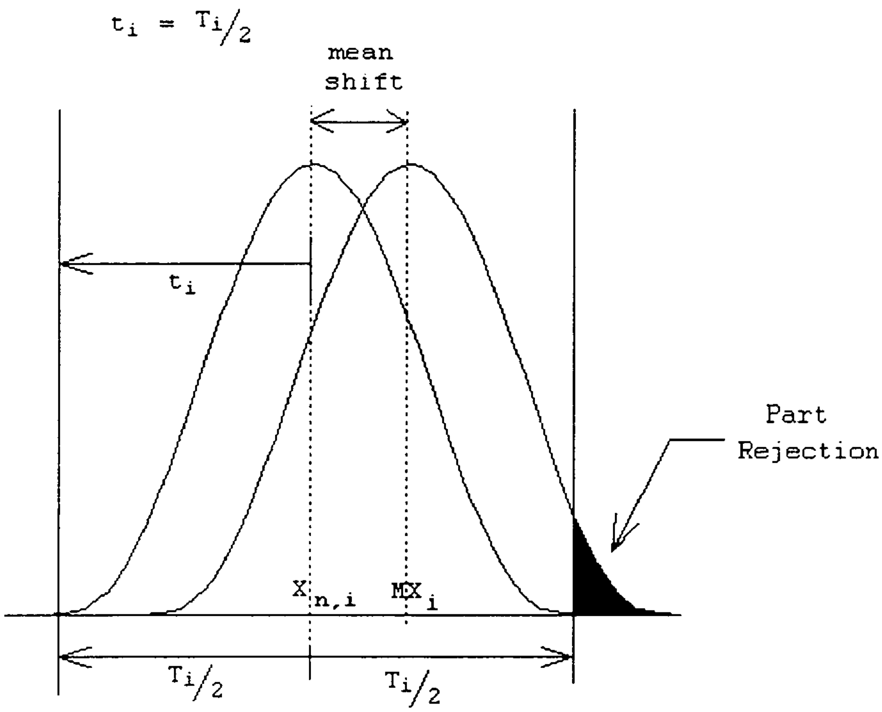


Figure 407. Part Rejection Occurs Due to the Mean Shift When the Natural Part Process Tolerance Is Equal to One Half of the Flatness Tolerance Zone (MX_i is the nominal dimension after including mean shift effect)

Each part dimension i is assigned as a group of variables as follows:

1. the specified tolerance ($\pm L_i$) is always greater than one half of the flatness tolerance zone ($T_i/2$),
2. the natural process tolerance ($\pm t_i$) is always less than one half of the flatness tolerance zone ($T_i/2$),
3. the part dimensions are assumed to have beta distribution; the distribution shape is allowed to be skewed depending on the alpha(α) and beta (β) parameters (Figure 401), and
4. the mean shift generally exists (Figure 408).

Actually, the actual part tolerance is directly produced by the natural process tolerance, and the middle point of the natural process tolerance (MX_i) is located in the middle of the natural process tolerance zone. In addition, MX_i is ideally located at the middle point of the flatness tolerance zone or the nominal dimension ($X_{n,i}$). However, the occurrence of the mean shift will gradually change the position of MX_i from $X_{n,i}$ as shown in Figure 408, so that the maximum mean shift for part i is limited to ($T_i/2 - t_i$). The midpoint of the natural process tolerance (MX_i) can be mathematically determined using the mean shift factor (f_i):

$$f_i = \frac{S_i}{(T_i/2)-t_i}$$

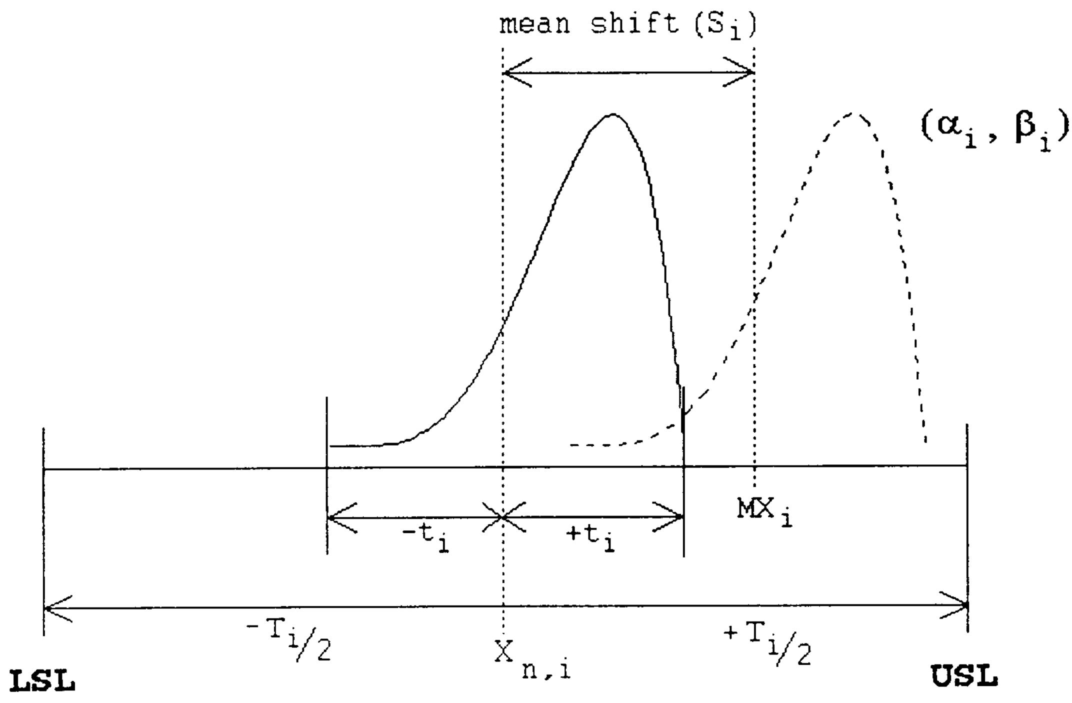


Figure 408. The Part Dimension Distribution and Mean Shift Behavior in the Proposed Model

where S_i is the mean shift for part i, and

f_i varies from 0 to 1 for part i.

$$C_i = \frac{1}{C_{p,i}} \quad \text{and} \quad C_{p,i} = \frac{T_i/2}{t_i}$$

where $C_{p,i}$ is the process capability index, and

C_i is the inverse process capability index.

C_i is a factor indicating how much process tolerance (t_i) covers one half of the flatness tolerance zone ($T_i/2$). Therefore, the midpoint of the natural process tolerance (MX_i) after accounting for the mean shift factor is given in Equation 4.3:

$$MX_i = X_{n,i} + f_i\,(1-C_i)\,\frac{T_i}{2} \qquad (4.3)$$

The shape of part dimension i's distribution is indicated by two parameters: the alpha parameter (α_i) and the beta parameter (β_i) as shown in Figure 401. Because of the variation of shapes available in the beta distribution, the distribution of the natural process tolerance should be simulated in terms of one specified shape value which means that the values of (α_i) and (β_i) must be carefully chosen for compatibility with the true part dimension's distribution.

In manufacturing, if the part dimension is normally distributed, the natural process tolerance is equal to 3 times the standard deviation (3σ) and the acceptance rate is 99.73%. When the symmetrical shape is simulated in terms of α and β parameters, the α and β parameters are always the same. As shown in Figure 401, there are various values of α and β parameters resulting in a symmetrical shape. The α and β parameters are simulated with the values 4 and 4 because these values make the shape look closely like normal distribution with 3 times the standard deviation equal to the natural process tolerances as proven in Appendix B. However, the acceptable rate is 100% (instead of 99.73% in normal case) because the range in beta distribution is finite. As a result, each part must be inspected before its assigned parameters: minimum dimension, maximum dimension, alpha and beta parameters, are evaluated. If all rejected parts were removed before they are sent to the assembly line, the result will be 100% acceptable assemblies.

The beta distribution can represent asymmetrical distribution. When the shape is asymmetric, the α and β parameters will be different. If the α parameter is greater than the β parameter, the shape will skew to the right side. On the other hand, if the β parameter is greater than the α parameter, the shape will skew to the left side, as shown in Figure 401. As shown in Figure 409, when the shape is symmetrical, the α and β parameters are defined with the same value (4,4). Next, when the shape begins to skew to the right side, the α parameter is assumed to increase, whereas the β parameter remains constant at 4.

Before determining the sum dimension, all eight parameters of part dimension i must be specified: the center dimension ($X_{c,i}$), the specified tolerance (L_i), the flatness tolerance zone (T_i), the quality factor ($C_{q,i}$), the mean shift factor (f_i), the natural process tolerance (t_i), and the α_i and β_i parameters. In addition, the nominal dimension ($X_{n,i}$) and the midpoint of the natural process tolerance (MX_i) can be determined by Equations 4.2 and 4.3, respectively. Finally, all ten parameters in each part dimension are used to determine the behavior of the sum dimension which is discussed in the following section.

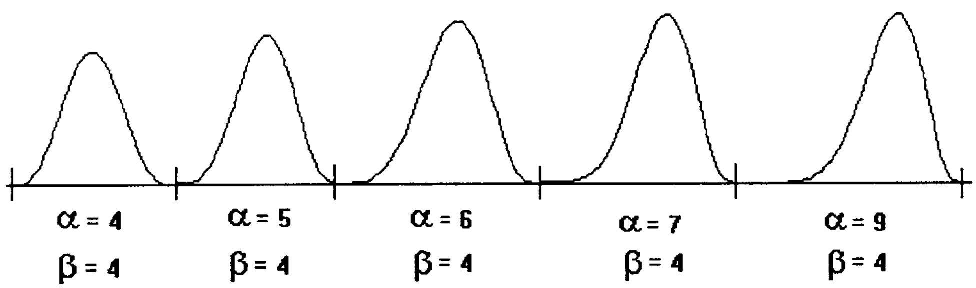


Figure 409. Gradual Skewness Developing in Beta Distribution as α Increases While β is Kept Constant at 4

4.5 Development of a New Modified Assembly Tolerance Model

Four parameters of the sum dimension are calculated as follows:

1. The middle range of the sum dimension ($MX_{\Sigma R}$)
2. The range of the sum dimension ($R\Delta X_{\Sigma}$)
3. The alpha parameter of the sum dimension (α_{Σ})
4. The beta parameter of the sum dimension (β_{Σ})

Using the four parameters above, the mean and the variance of the sum dimension are determined and two critical values of the sum dimension, the tolerance zone and the middle point in the tolerance zone, are calculated.

The beta distribution model of the sum dimension shown in Figure 410, displays the tolerance of the sum dimension (TX_{Σ}) represented by the hatched area, and the middle point in the tolerance zone (MX_{Σ}). Both TX_{Σ} and MX_{Σ} are easily calculated by using normalized dimensions:

$$TX_{\Sigma} = TW_{\Sigma}\sqrt{VAR\Delta X_{\Sigma}}$$

and

$$M\Delta X_{\Sigma} = MW_{\Sigma}\sqrt{VAR\Delta X_{\Sigma}}$$

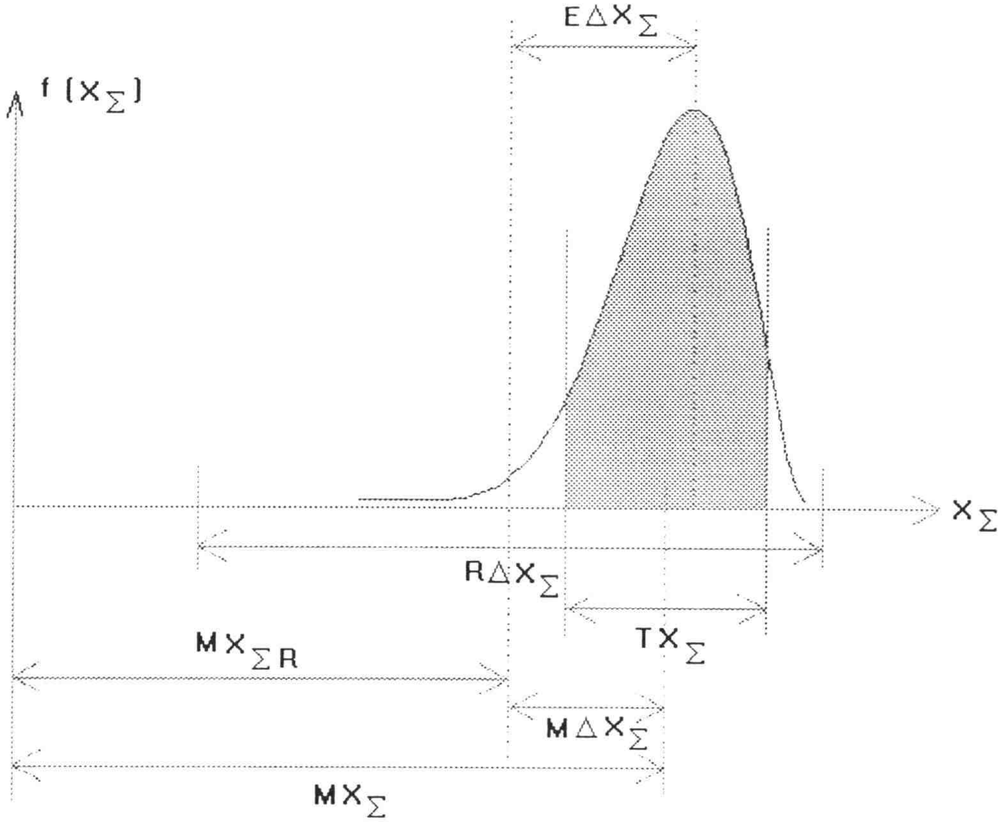


Figure 410. The Beta Distribution Model of the Sum Dimension

where

TW_Σ is the normalized tolerance,

MW_Σ is the normalized middle point movement,

$VAR\Delta X_\Sigma$ is the variance of the sum dimension, and

$M\Delta X_\Sigma$ is the middle point movement.

The middle point in the tolerance zone (MX_Σ) is given by

$$MX_\Sigma = MX_{\Sigma R} + M\Delta X_\Sigma,$$

where $MX_{\Sigma R}$ is the middle range of the sum dimension.

According to the normal distribution, the normalized tolerance (TW_Σ) is dependent only on the confidence level (Table 402), and the normalized middle point movement (MW_Σ) is always zero due to the symmetrical shape[6]. Thus, the middle point of the tolerance zone (MX_Σ) is equal to the middle range of the sum dimension as follows:

$$MX_\Sigma = MX_{\Sigma R} + M\Delta X_\Sigma$$

when

$MW_\Sigma = 0$, $M\Delta X_\Sigma = 0$ [6],

therefore

$MX_\Sigma = MX_{\Sigma R}$, for symmetrical beta distribution.

Table 402. Normalized Tolerance as a Function of Confidence Level based on the Normal Distribution Model[6]

Confidence level in %	Normalized tolerance TW_Σ
100	∞
99.99	7.78
99.9	6.58
99.73	6.00
99.5	5.60
99.0	5.15
95.0	3.92
90.0	3.29

In the beta distribution model, four parameters -the mean of the sum dimension, the variance of the sum dimension, the range of the sum dimension and the asymmetry of the sum dimension- are required to evaluate the critical values of the assembly dimension: assembly tolerance zone (TX_Σ) and the middle point in the assembly tolerance zone (MX_Σ). All four parameters can be calculated using four specified parameters in each part dimension: the lower limit (a), the upper limit (b), the alpha parameter (α), and the beta parameter (β). Moreover, TX_Σ and MX_Σ can be estimated by using normalized dimensions depending on the confidence level. In normalized beta distributions, the mean is 0 and the variance is 1. The normalized range of the sum dimension and the normalized asymmetry are tabulated resulting in the normalized assembly tolerance (TW_Σ) and the normalized middle point movement (MW_Σ). The normalized range of the sum dimension (RW_Σ) is given by Bjorke [6] as:

$$RW_\Sigma = \frac{R\Delta X_\Sigma}{\sqrt{VAR\Delta X_\Sigma}}$$

where $R\Delta X_\Sigma$ is the range of the sum dimension and the asymmetry (FW_Σ) is measured by:

$$FW_\Sigma = \frac{E\Delta X_\Sigma}{R\Delta X_\Sigma}$$

where $E\Delta X_\Sigma$ is the difference between the mean and the middle range of the sum dimension.

The normalized tolerance(TW_Σ) and middle point movement (MW_Σ) are tabulated as functions of RW_Σ and FW_Σ for different confidence levels. Appendix C [6] displays the normalized dimensions with six different confidence levels (Tables C.1, C.2, C.3, C.4, C.5, and C.6).

Appendix describes a procedure for determining alpha (α) and beta (β) parameters of the sum dimension. Using the sum mean (Equation d.3) and the sum variance (Equation d.4), as shown in Appendix D, the alpha parameter of sum dimension (α_Σ), the beta parameter (β_Σ) of sum dimension, the variation of sum dimension ($VAR\Delta X_\Sigma$), the mean of sum dimension [$E(X_\Sigma)$], the range of sum dimension ($R\Delta X_\Sigma$), and the middle range of sum dimension ($MX_{\Sigma R}$) are calculated. Using normalized dimensions, the normalized range (RW_Σ) and the asymmetry (FW_Σ) is computed resulting in the normalized tolerance (TW_Σ) and the normalized middle point movement (MW_Σ) depending on the confidence level. Finally, the tolerance zone (TX_Σ) of sum dimension and the middle point in tolerance zone (MX_Σ) are determined.

However, the sum dimension in the part drawing is a symmetrical tolerance specification so that the tolerance zone (TX_Σ) of the sum dimension and the midpoint of the tolerance zone (MX_Σ) can be easily transformed into symmetrical form as $MX_\Sigma \pm TX_\Sigma/2$, consistent with Figure 410.

4.6 An Application Example

The assembly shown in Figure 411 consists of five horizontal part dimensions. The center dimension, the specified tolerance and the flatness tolerance for each part is shown. All five part dimensions are assembled in linear relationship. Moreover, nine parameters for each part component: the center dimension ($X_{c,i}$), the specified tolerance (L_i), the flatness tolerance (T_i), the natural process

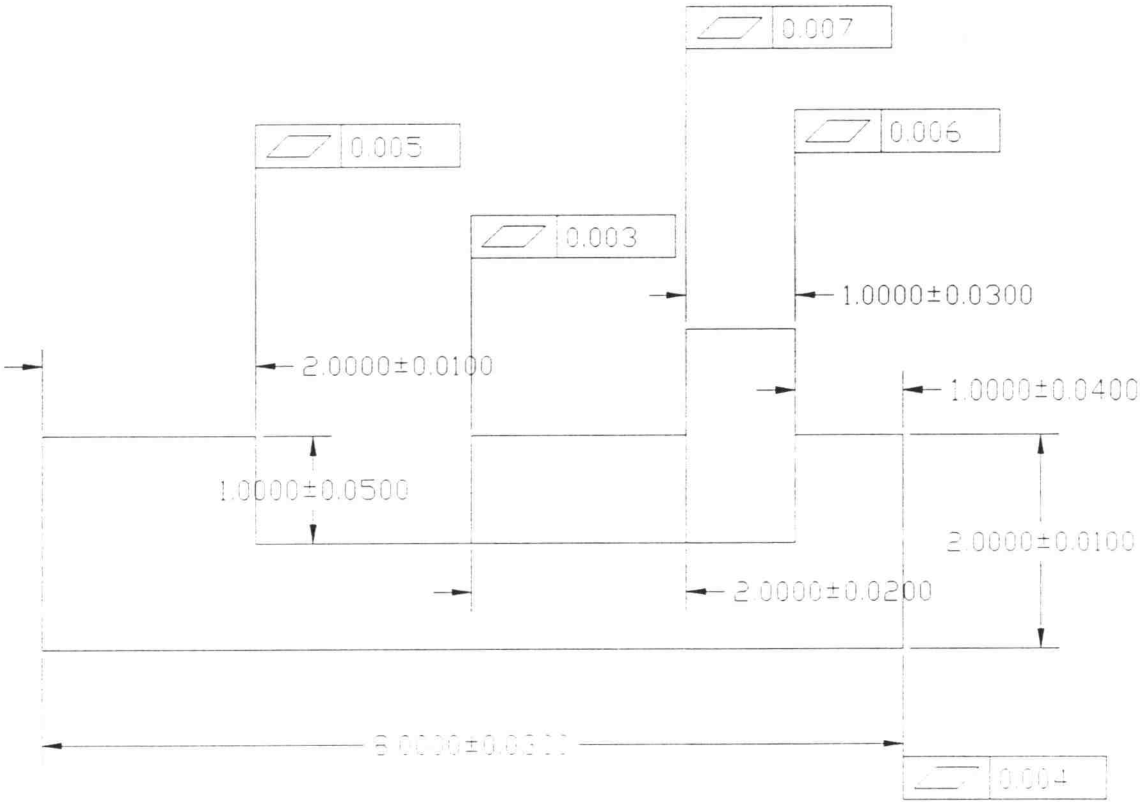


Figure 411. An Assembly Drawing with Conventional Tolerances and GD&T Symbols (Flatness)

tolerance (t_i), the quality factor ($C_{q,i}$), the mean shift factor (f_i), the inverse process capability index (C_i), the alpha parameter (α_i), and the beta parameter (β_i) are displayed in Table 403. The nominal dimension for each part ($X_{n,i}$) is determined following Equation 4.2 and shown in the last row of Table 403. Next, the middle range of part dimension (MX_i) and the modified variables for each part: q_i, t_i/q_i, t_i^2/q_i, t_i^2/q_i^2, and $t_i^2/(q_i+1)$ (following Appendix D) are shown in Table 404.

Four parameters of the sum dimension are evaluated as follows:

1. The middle range of the sum dimension ($MX_{\Sigma R}$)

$$
\begin{aligned}
MX_{\Sigma R} &= MX_2 - (MX_1 + MX_3 + MX_4 + MX_5) \\
&= 8.011568 - (1.99638 + 1.024384 \\
&\quad + 0.982422 + 2.00988) \\
&= 1.998465
\end{aligned}
$$

2. The range of the sum dimension ($R\Delta X_\Sigma$)

$$
\begin{aligned}
R\Delta X_\Sigma &= 2\sum_{i=1}^{5} t_i \\
&= 2*(0.0004 + 0.0012 + 0.0004 \\
&\quad + 0.0003 + 0.0006) \\
&= 0.0058
\end{aligned}
$$

Table 403. Data for The Application Example

Component Dimension	X_1	X_2	X_3	X_4	X_5
Center Dimension ($X_{c,i}$)	2.00	8.00	1.00	1.00	2.00
Specified Tolerance (L_i)	0.01	0.03	0.04	0.03	0.02
Flatness Tolerance (T_i)	0.005	0.004	0.006	0.007	0.003
Natural Process Tolerance (t_i)	.0004	.0012	.0004	.0003	.0006
Quality Factor($C_{q,i}$)	-0.7	0.4	0.6	-0.7	0.5
Mean shift factor (f_i)	0.78	0.46	0.84	0.31	0.71
Inverse Process Capability index (C_i)	0.16	0.6	.1333	.0857	0.4
Alpha parameter (α_i)	8	7	9	4	10
Beta parameter (β_i)	4	4	4	4	4
Nominal Dimension ($X_{n,i}$)	1.9947	8.0012	1.0222	0.9814	2.0092

Table 404. The Evaluated Variables for Each Part Dimension.

Part number	X_1	X_2	X_3	X_4	X_5
MX_i	1.996388	8.011568	1.024384	0.982442	2.00988
t_i	.0004	.0012	.0004	.0003	.0006
q_i	12	11	13	8	14
$\frac{t_i}{q_i}$	3.33E-5	1.09E-4	3.08E-5	3.75E-5	4.29E-5
$\frac{t_i^2}{q_i}$	1.33E-8	1.31E-7	1.23E-8	1.13E-8	2.57E-8
$\frac{t_i^2}{q_i^2}$	1.11E-9	1.19E-8	9.47E-10	1.41E-9	1.84E-9
$\frac{t_i^2}{(q_i+1)}$	1.23E-8	1.2E-7	1.14E-8	1E-8	2.4E-8

3. The alpha parameter of the sum dimension (α_Σ)

Before calculating the alpha parameter (α_Σ), the modified variables (Appendix D): A, B, C, J, K ,and M are determined as follows:

$$J = \sum_{i=1}^{5} \frac{t_i}{q_i}$$

$$= 3.33\times10^{-5} + 1.09\times10^{-4} + 3.08\times10^{-5} + 3.75\times10^{-5} + 4.29\times10^{-5}$$

$$= 0.000254$$

$$A = \sum_{i=1}^{5} \frac{t_i^2}{q_i}$$

$$= 1.33\times10^{-8} + 1.31\times10^{-7} + 1.23\times10^{-8} + 1.13\times10^{-8} + 2.57\times10^{-8}$$

$$= 1.94\times10^{-7}$$

$$B = \sum_{i=1}^{5} \left(\frac{t_i}{q_i}\right)^2$$

$$= 1.11\times10^{-9} + 1.19\times10^{-8} + 9.47\times10^{-10} + 1.41\times10^{-9} + 1.84\times10^{-9}$$

$$= 1.72\times10^{-8}$$

$$C = \sum_{i=1}^{5} \frac{t_i^2}{q_i+1}$$

$$= 1.23\times10^{-8} + 1.2\times10^{-7} + 1.14\times10^{-8}$$

$$+ 10^{-3} + 2.4\times10^{-8}$$

$$= 1.78\times10^{-7}$$

$$M = \sum_{i=1}^{5} t_i$$

$$= 0.0004 + 0.0012 + 0.0004 + 0.0003 + 0.0006$$

$$= 0.0029$$

$$K = 5A-4B-5C$$

$$= 5\times1.94\times10^{-7} - 4\times1.72\times10^{-8} - 5\times1.78\times10^{-7}$$

$$= 1.01\times10^{-8}$$

Finally, the alpha parameter (α_Σ) is evaluated as follows:

$$\alpha_\Sigma = \frac{(M-4J)^2 J-(M-4J)K}{KM}$$

$$= 30.18324$$

4. The beta parameter of the sum dimension (β_Σ)

$$\beta_\Sigma = \frac{4J}{M-4J}\alpha_\Sigma$$

$$= 16.23288$$

After calculating all four parameters of the sum dimension, the mean and the variance of the sum dimension are determined as follows:

1. The mean of the sum dimension $E(X_\Sigma)$

$$E(X_\Sigma) = MX_{\Sigma R} + \left[\frac{\alpha_\Sigma}{\alpha_\Sigma + \beta_\Sigma} - \frac{1}{2} \right] R\Delta X_\Sigma$$

$$= 1.998465 + \left[\frac{30.18}{30.18+16.23} - \frac{1}{2} \right] 0.0058$$

$$= 1.999337$$

2. The variance of the sum dimension ($VAR\Delta X_\Sigma$)

$$VAR\Delta X_\Sigma = \frac{(R\Delta X_\Sigma)^2 \alpha_\Sigma \beta_\Sigma}{(\alpha_\Sigma + \beta_\Sigma + 1)(\alpha_\Sigma + \beta_\Sigma)^2}$$

$$= \frac{0.0058^2 \times 30.18 \times 16.23}{(30.18+16.23+1)(30.18+16.23)^2}$$

$$= 1.61 \times 10^{-7}$$

The normalized asymmetry (FW_Σ) and the normalized range of the sum (RW_Σ) are determined as follows:

$$FW_\Sigma = \frac{E\Delta X_\Sigma}{R\Delta X_\Sigma}$$

$$= \frac{E(X_\Sigma) - MX_{\Sigma R}}{R\Delta X_\Sigma}$$

$$= \frac{1.999337-1.998765}{0.0058}$$

$$= 0.150275$$

$$RW_\Sigma = \frac{R\Delta X_\Sigma}{\sqrt{VAR\Delta X_\Sigma}}$$

$$= \frac{0.0058}{\sqrt{1.61\times 10}}$$

$$= 14.43947$$

The normalized middle point (MW_Σ) and the normalized tolerance (TW_Σ) are chosen from Table C.5 (Appendix C) as a function of normalized asymmetry (FW_Σ) and the normalized range of the sum dimension (RW_Σ). With a 99.73% confidence level, the normalized middle point movement (MW_Σ) is 1.24 and the normalized tolerance (TW_Σ) is 5.81. Consequently, the tolerance of the sum dimension (TX_Σ) and the middle point movement ($M\Delta X_\Sigma$) are both evaluated as follows:

$$M\Delta X_\Sigma = MW_\Sigma \sqrt{VAR\Delta X_\Sigma}$$

$$= 1.24 \times \sqrt{1.61 \times 10^{-7}}$$

$$= 0.000498$$

$$TX_\Sigma = TW_\Sigma \sqrt{VAR\Delta X_\Sigma}$$

$$= 5.81 \times \sqrt{1.61 \times 10^{-7}}$$

$$= 0.002334$$

The middle point in the tolerance zone (MX_Σ) is determined and the sum dimension distribution is displayed in Figure 412.

$$MX_\Sigma = MX_{\Sigma R} + M\Delta X_\Sigma$$

$$= 1.998465 + 0.000498$$

$$= 1.998963$$

Finally, all sum dimension parameters are shown in the assembly drawing (Figure 413). The assembly tolerance is one half of the tolerance zone (TX_Σ) so that the assembly tolerance is equal to 0.001167.

To show the improvement of the new model, the tolerances of the sum dimension using the WC and RSS models are determined and compared with each other.

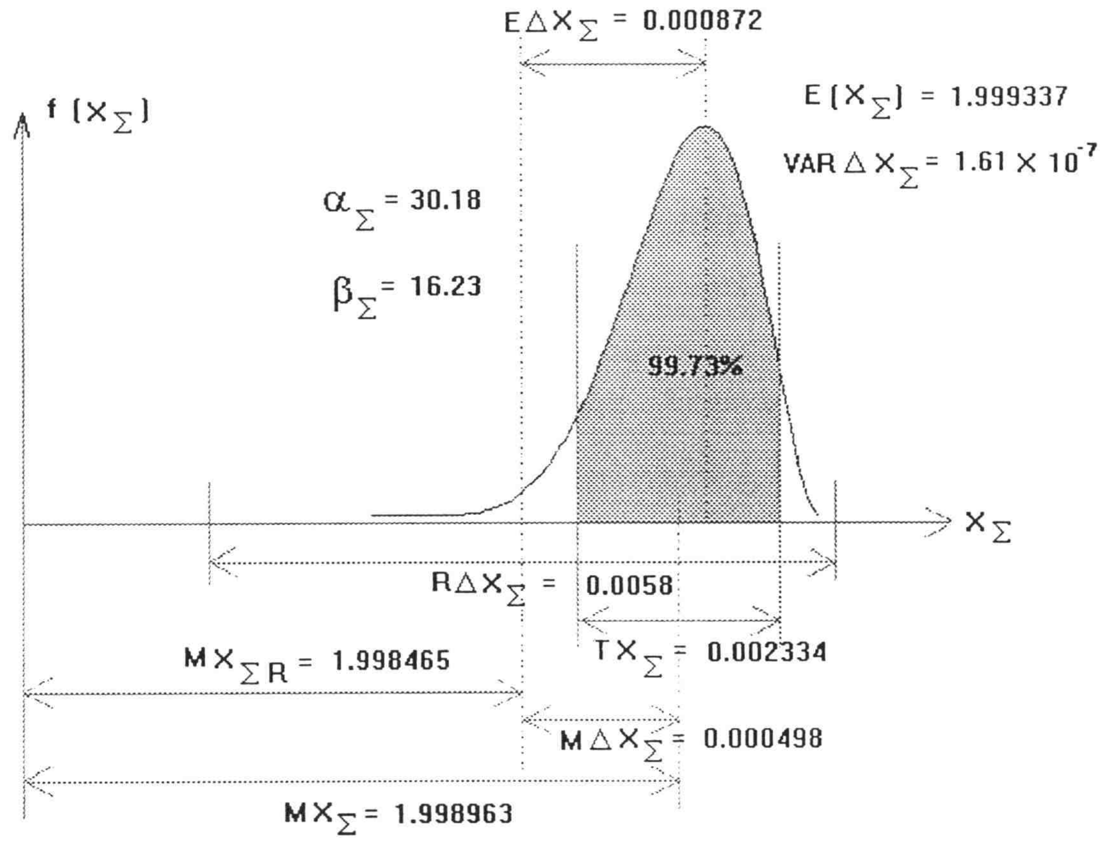


Figure 412. Parameter Values of the Sum Dimension in the Application Example

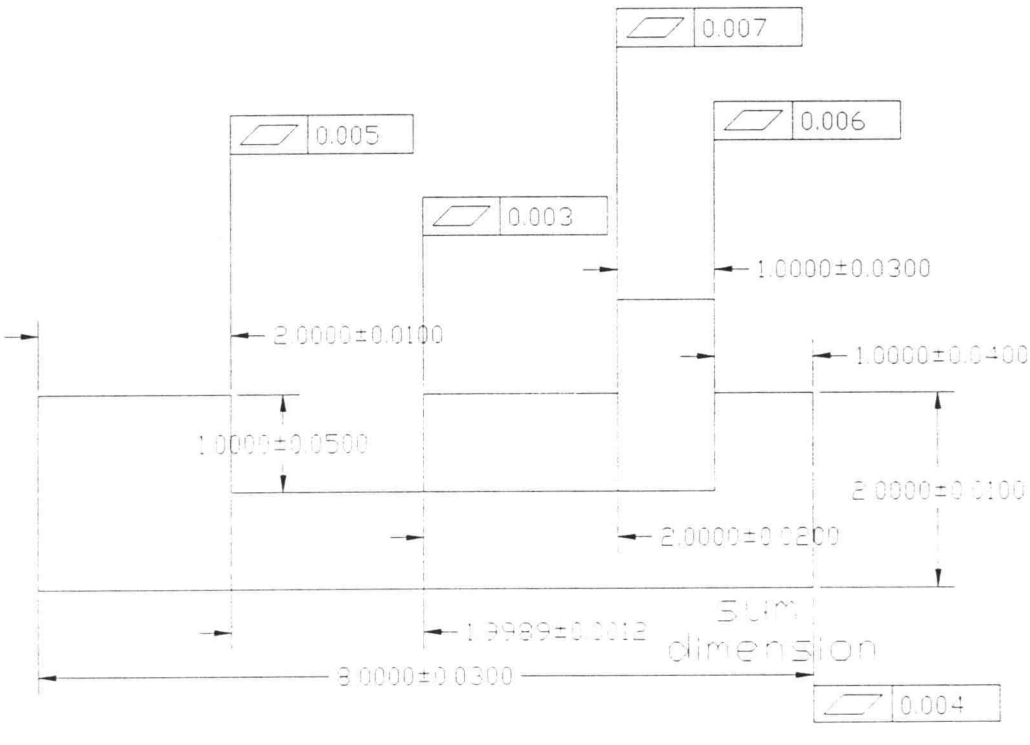


Figure 413. The Sum Dimension in Terms of Symmetrical Specified Tolerance

The Worst Case Model:

$$Tol_{asm} = \sum_{i=1}^{5} t_i$$

$$= 0.0004 + 0.0012 + 0.0004 + 0.0003 + 0.0006$$

$$= 0.0029$$

The RSS Model with 99.73% confidence level:

$$Tol_{asm} = \sqrt{\sum_{i=1}^{5} t_i^2}$$

$$= [0.0004^2 + 0.0012^2 + 0.0004^2 + 0.0003^2 + 0.0006^2]^{1/2}$$

$$= 0.001487$$

Comparing the assembly tolerances from the WC model (0.0029) and the RSS model (0.001487), the assembly tolerance of the modified model (0.001167) is the least of all three.

The determination of critical assembly dimensions and tolerances from the modified assembly tolerance analysis model is tedious because this model involves a large number of variables and demanding calculations. To facilitate calculation procedures, a computerized integrated system (discussed in the next chapter) is incorporated into the model.

CHAPTER 5
SYSTEM INTEGRATION

5.1 Introduction

Designers and manufacturers treat tolerance analysis in different ways. In the design stage, tolerance analysis is used to control dimension limits for each part without violating the assembly dimension limits. On the other hand, tolerance analysis in the manufacturing process is applied to handle the production quality, indicated by process capabilities and mean shift behavior.

Integration of tolerance analysis from both stages is a great challenge in the field of Integrated Manufacturing Engineering. However, an integrated system must deal with several support systems. For example, designers might employ assembly tolerance analysis models to analyze a critical assembly tolerance. In contrast, to produce sufficient information for tolerance analysis in the manufacturing process, manufacturing tolerance databases are used to measure process capabilities and control mean shifts. Therefore, integration of tolerance analysis is extremely tedious. It requires not only demanding calculations, but also a user interface to display tolerance information. To facilitate the application of the integrated system, a computerized integrated system, as discussed in this chapter, is incorporated to handle four support systems; a drawing system, a GD&T specifications system, an assembly tolerance analysis model, and a manufacturing tolerance database. Four main objectives of the integrated system are:

1) Drawing a part or an assembly in a CAD system,
2) Drawing GD&T specifications in the part or assembly drawing,
3) Determining the critical assembly tolerance of the assembly drawing using assembly tolerance analysis models, and
4) Dynamic support of the tolerance database during the design stage.

This integrated system concurrently applies all four objectives in a user-friendly interface. The advantage of this system is that the user can perform advanced tolerance analysis and design at an early stage, resulting in cost reduction and production quality improvement.

5.2 Conceptual Framework

As discussed in Chapter 2, a number of computer programs have been developed in tolerance analysis. Most of them are designed to solve one specific problem and run under a single overhead. For example, Karolin and Ahluwalia[13] developed an independent computer aided tolerance control (CATC) program based solely upon the tolerance-chart technique. Martino[17] used a simple linear programming approach to generate design functions based on the use of solid models and variational geometry. The difficulty in using existing tolerance analysis programs is the lack of flexibility to accommodate different problem aspects. To extend their abilities, they require several different support systems, resulting in larger and more complex programs. As a result, the productivity of the user remains low. The integration of different support systems in a single computer application has been greatly needed.

During the last several years, the development of hardware and operating systems, such as Microsoft Windows 3.1 and IBM OS/2.x, has made personal computers (PCs) more powerful. Specifically, Microsoft Windows 3.1 provides an advanced feature -- the ability of mutitasking in a single unified system. Using this feature, various software applications can be run simultaneously under the Windows environment. In addition, a new linking method, the Dynamic Data Exchange (DDE), is available to transfer data between two applications, resulting in an integrated system. Under Windows 3.1, the applications that support DDE can establish links between several other applications and exchange data or pass remote commands dynamically. Thus, DDE is considered a key feature in developing a computerized integrated system.

5.3 System Components

To approach the objectives of the integrated system in this research, all four support systems (drawing, GD&T specifications, assembly tolerance analysis model, and manufacturing tolerance database) have been implemented under Windows. Each support system requires a suitable software package to operate as an individual user interface.

Most commercial software packages, such as spread sheets, databases, or CAD are established for serving the Windows system. Some of these are Excel for Windows, Paradox for Windows, and AutoCAD for Windows, respectively. Most of them support DDE. In this research, two software packages are included in the integrated system: AutoCAD for Windows and Microsoft Visual Basic Version 3.0 for Windows.

AutoCAD for Windows is chosen to handle two support systems; the drawing and GD&T specifications support systems. The drawing capability of AutoCAD software is well known among design and manufacturing engineers. A part or an assembly can easily be drawn by using the AutoCAD standard menu. In addition, a group of AutoLISP programs has been developed to create the ability of GD&T specifications in the AutoCAD system, as stated in Chapter 3. The AutoCAD software also supports DDE, so that the data in a part drawing can be transferred to other applications, or vise versa. However, the AutoCAD system is not able to exchange data with the other support systems, such as the tolerance database system. A user interface design software package is greatly needed to handle this. Microsoft Visual Basic Version 3.0 for Windows has been used for this purpose because it can easily handle implementations such as databases or calculations. To build a visual basic application, the user can create the user interface by "drawing" controls, such as text boxes and command buttons on a form, illustrated in Figure 501. Next, all properties of form and controls are freely set to such values as caption, color, or size. The source code written in Basic enables the application to function. Under the Windows environment, the user can open the designed form in a manner similar to that of a user interface. The capability of the designed form is freely activated depending on the user assignment. As a result, the user can apply his work

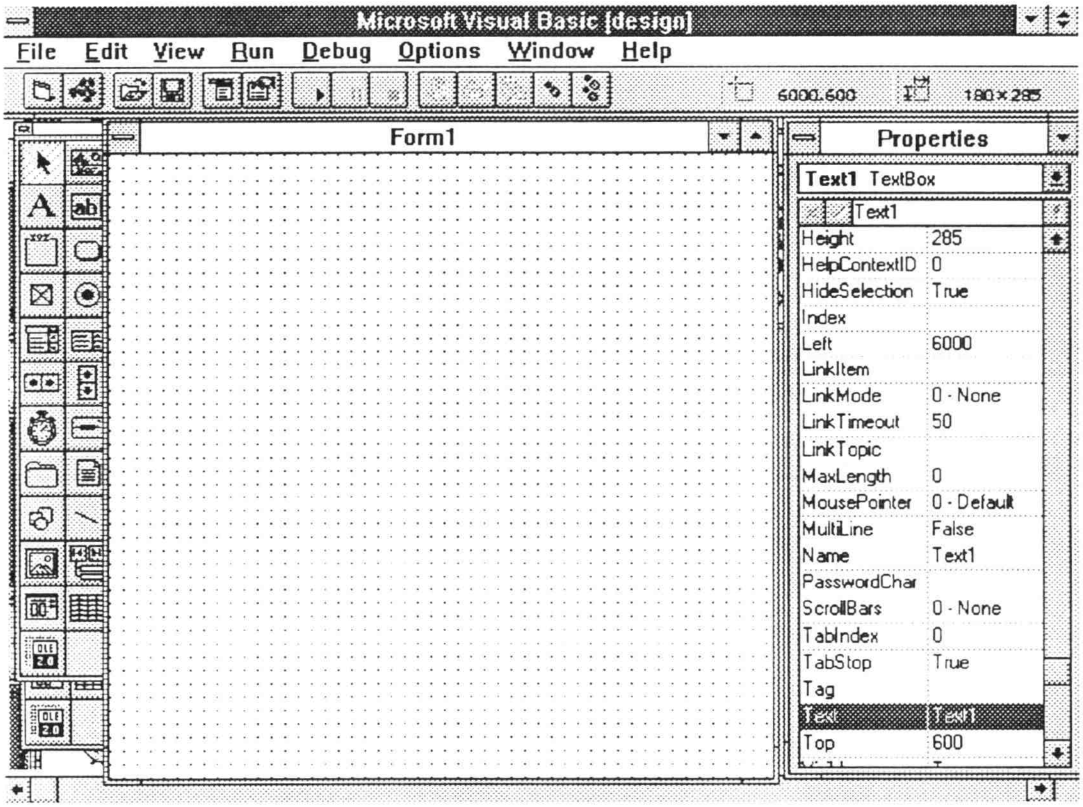


Figure 501. A Design Form for Developing a User Interface in Microsoft Visual Basic

visually with this software. For these reasons, Microsoft Visual Basic Version 3.0 for Windows was chosen to process and display the tolerance database support system.

There are three system components: the CAD drawing program, the Assembly tolerance analysis program, and the tolerance database program, operated in the integrated system. DDE links all three system components in the Windows environment which is graphically summarized in Figure 502. The system components are described in the following paragraphs:

1) CAD Drawing Program

CAD drawing program is incorporated by AutoCAD for Windows, handling two support systems; the drawing and the GD&T specifications support system. The standard AutoCAD menu is applied to help the user to develop a part or an assembly drawing. In addition, a new AutoCAD menu using AutoLISP programming has been developed to create GD&T specifications for drawing. The inclusion of GD&T specifications has enhanced the capabilities of standard AutoCAD. The AutoCAD screen can be set like a user interface in the Windows, so that the user can apply other user interfaces concurrently without violating the AutoCAD system. Additionally, the data from the drawing can be transferred to other user interfaces dynamically through DDE.

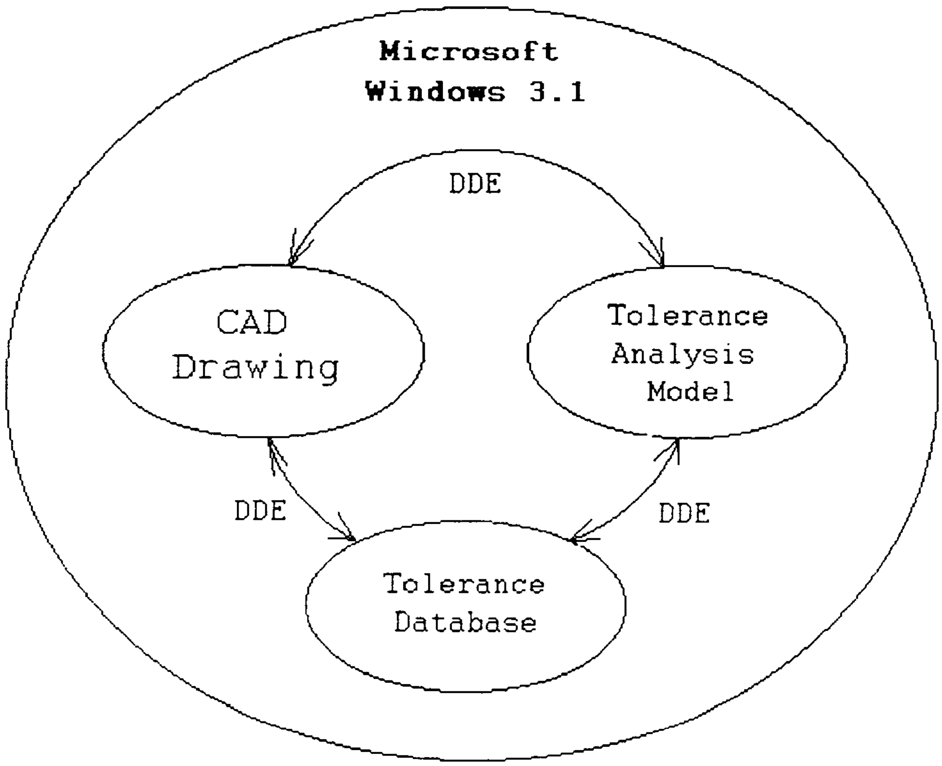


Figure 502. The System Components

2) Assembly Tolerance Analysis Program

Determining a critical assembly tolerance is an essential application to provide maximum and minimum limits of the assembly dimension. Assembly tolerance analysis models have been developed to calculate a critical assembly tolerance by accumulating each part dimension. The integrated system includes only three assembly tolerance analysis models; the Worst Case(WC), the Root Sum Squares(RSS), and the new model as described in Chapter 4. Due to their simplicity of calculations, the first two assembly tolerance analysis models (WC and RSS) can be operated in the AutoCAD system. In contrast, the new model, as stated in Chapter 4, employs a number of input variables, such as flatness tolerance zone, process tolerance, quality factor, mean shift factor, and the alpha and beta parameters. Therefore, the determination of a critical assembly tolerance with the new model is complex. AutoCAD system itself can not handle this application. Two user interfaces, as shown in Figure 503, have been developed using Microsoft Visual Basic Version 3.0 to handle the requirements of the new model. Using the assembly tolerance analysis program, the part dimensions in the drawing are dynamically extracted and a critical assembly tolerance can be determined and inserted in the drawing automatically. Though the critical assembly tolerance with the modified model is determined in a user

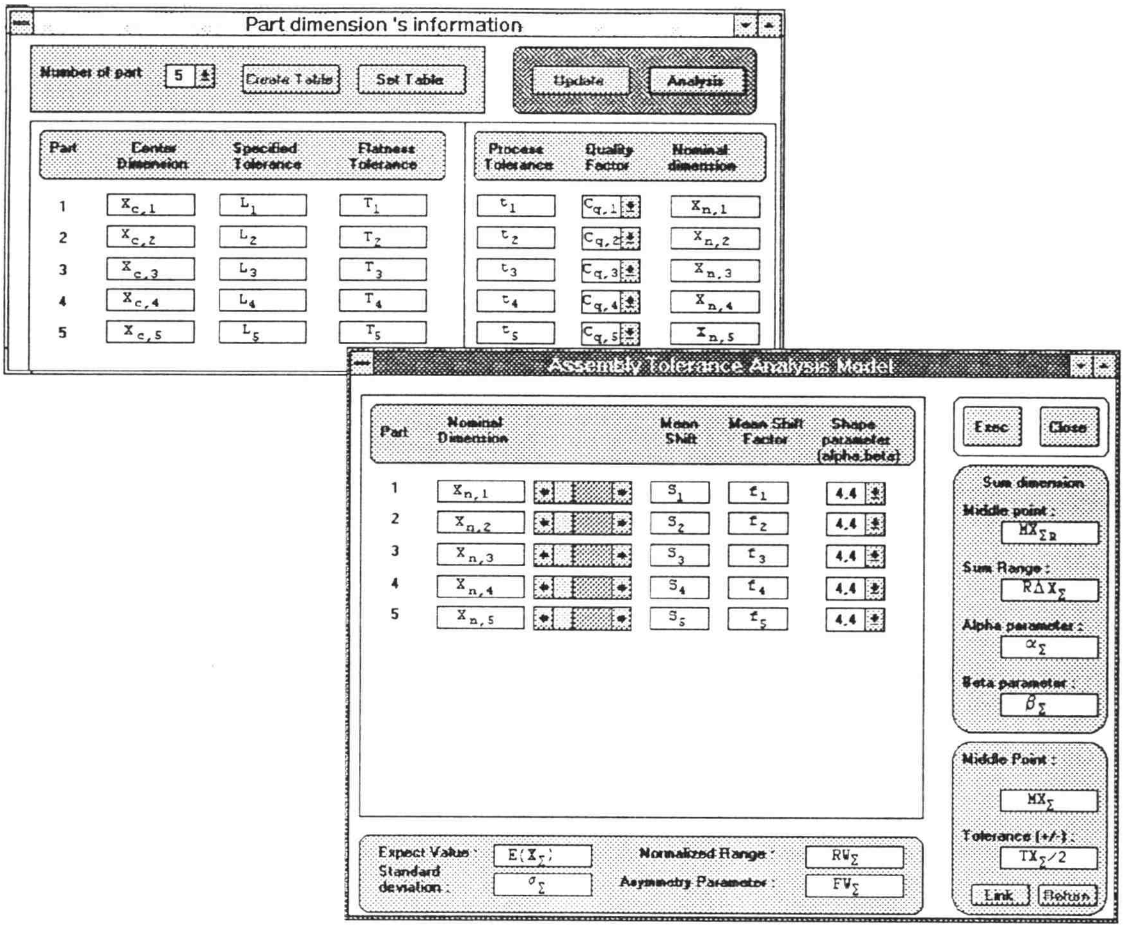


Figure 503. Two User Interfaces for the Modified Assembly Tolerance Analysis Model

interface (outside the AutoCAD system), its value can be transferred and activated in the AutoCAD system by DDE, as shown in Figure 504.

3) Tolerance Database Program

During the design stage, the designer needs some general tolerance information to specify manufacturable tolerances and not violate the assembly dimension limits. The integration of a tolerance database in the tolerance analysis system is greatly needed.

The database of conventional tolerance selected for this study consists of two parameters: the range of part dimension and the machining process. The tolerance value varies depending on both parameters. The tolerance database used in this research is shown in Figure 505 [37]. In each range of part dimension and machining process, the conventional tolerance is actually limited in a specified range. For example, the range of dimension between 0.600 to 0.999 inches, processed by a broaching machine has the conventional tolerance between ±0.00025 to ±0.001 inches.

The user interface of the tolerance database was developed in Microsoft Visual Basic Version 3.0 for Windows. The form of this application shown in Figure 506 consists of text boxes, scroll bars, and command buttons. The objective

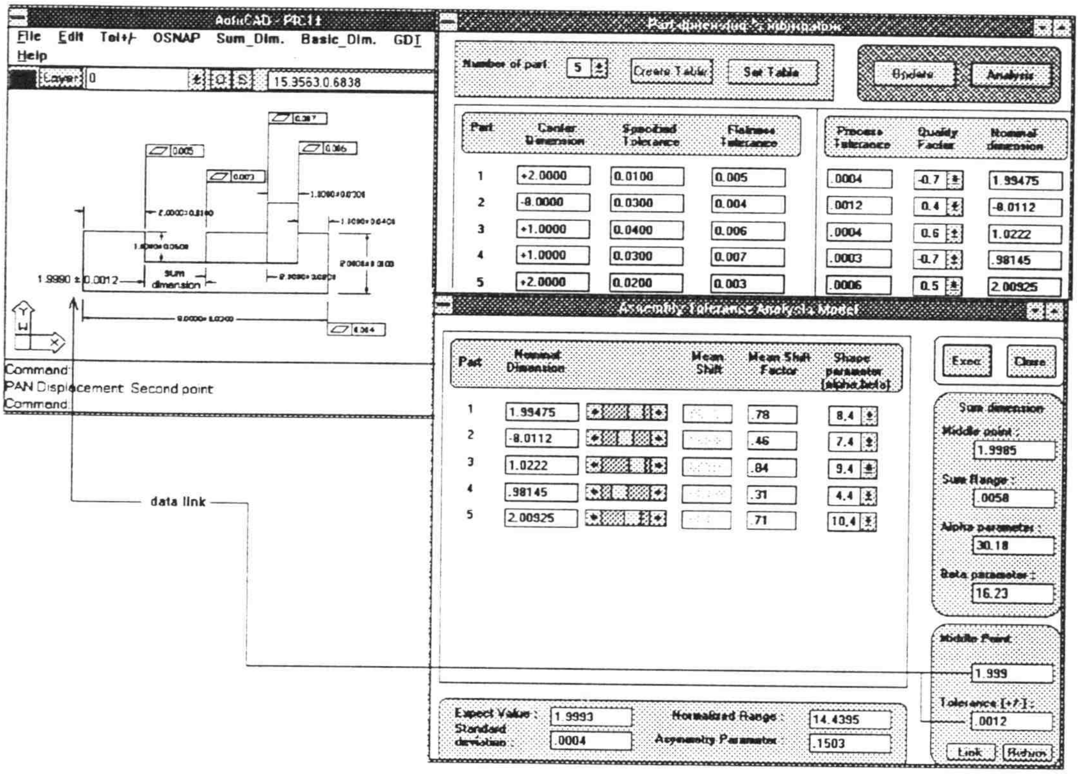


Figure 504. The Data Link between the User Interfaces of the New Assembly Tolerance Analysis Model and the Assembly Drawing in the AutoCAD System

RANGES OF SIZES		TOLERANCES ±								
FROM	TO									
0. 000	0. 599	0. 00015	0. 0002	0. 0003	0. 0005	0. 0008	0. 0012	0. 002	0. 003	0. 005
0. 600	0. 999	0. 00015	0. 00025	0. 0004	0. 0006	0. 001	0. 0015	0. 0025	0. 004	0. 006
1. 000	1. 499	0. 0002	0. 0003	0. 0005	0. 0008	0. 0012	0. 002	0. 003	0. 005	0. 008
1. 500	2. 799	0. 00025	0. 0004	0. 0006	0. 001	0. 0015	0. 0025	0. 004	0. 006	0. 010
2. 800	4. 499	0. 0003	0. 0005	0. 0008	0. 0012	0. 002	0. 003	0. 005	0. 008	0. 012
4. 500	7. 799	0. 0004	0. 0006	0. 001	0. 0015	0. 0025	0. 004	0. 006	0. 010	0. 015
7. 800	13. 599	0. 0005	0. 0008	0. 0012	0. 002	0. 003	0. 005	0. 008	0. 012	0. 020
13. 600	20. 999	0. 0006	0. 001	0. 0015	0. 0025	0. 004	0. 006	0. 010	0. 015	0. 025

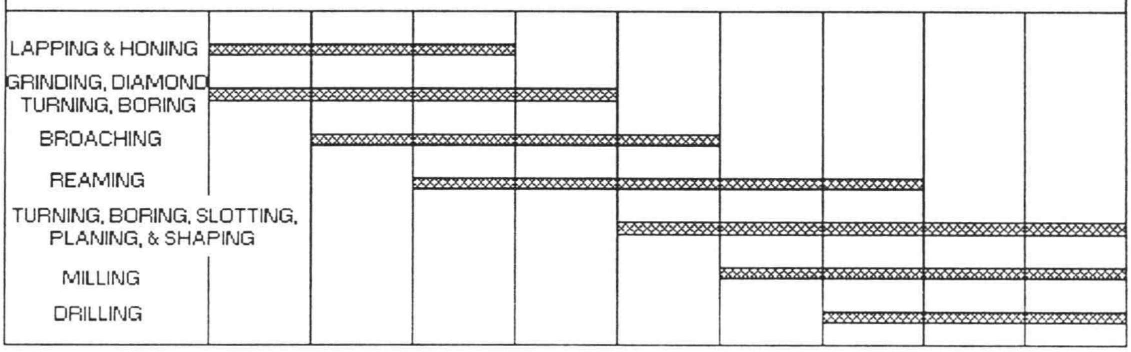


Figure 505. Manufacturing Tolerance Database [37]

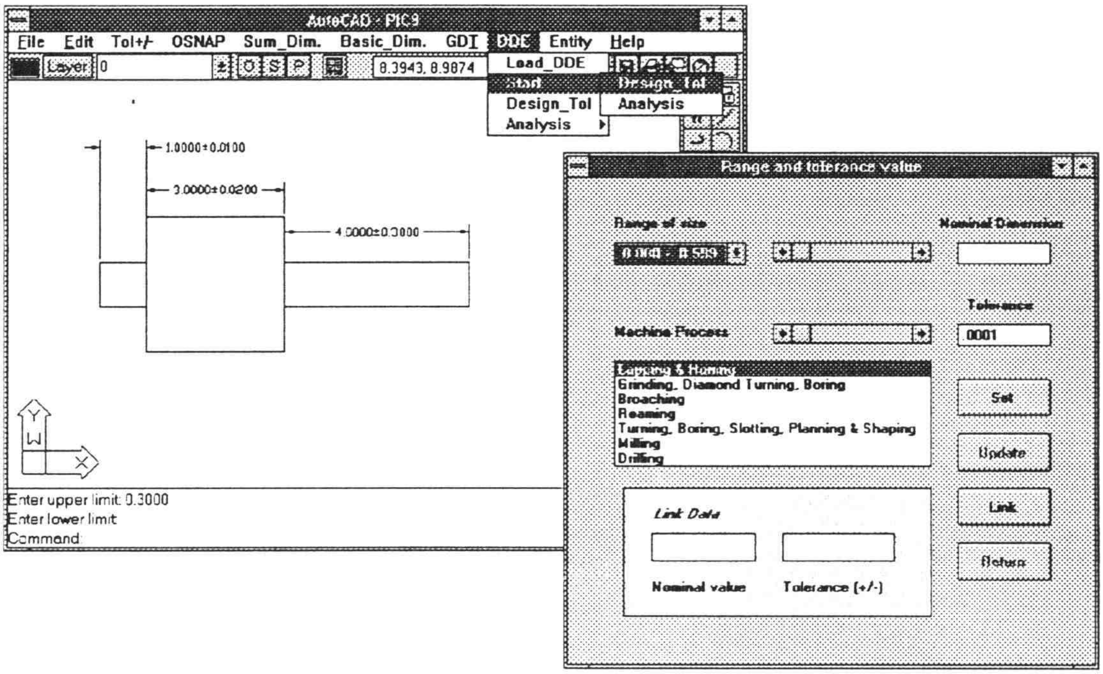


Figure 506. Application of the Pull-Down Menu "Start" to Open the User Interface of Tolerance Database

of this user interface is to handle the requirements of the tolerance database support system. The tolerance database program can link both CAD drawing and assembly tolerance analysis programs. For example, the user can visualize the tolerance database and also automatically update a part dimension in CAD drawing program, as shown in Figure 507. With the application of the modified assembly tolerance analysis model, the tolerance databases can also be activated to introduce process tolerance as dynamic information with a user interface, as displayed in Figure 508.

5.4 The Integrated System Application Procedure

The application of the integrated system is controlled by the new TK AutoCAD menu, as developed and described in Chapter 3. The TK menu is used to load AutoLISP programs and operate all three system components: the CAD drawing, assembly tolerance analysis and tolerance database programs. To run a program, a number of pull-down menus are provided to be clicked upon by a pointing device. The functions of pull-down menus are explained in Appendix A. When a program is running, a series of text strings emerges in the command line (of the AutoCAD system) to let the user follow the program procedures. The details of the program procedures, including examples, are described in Appendix A.

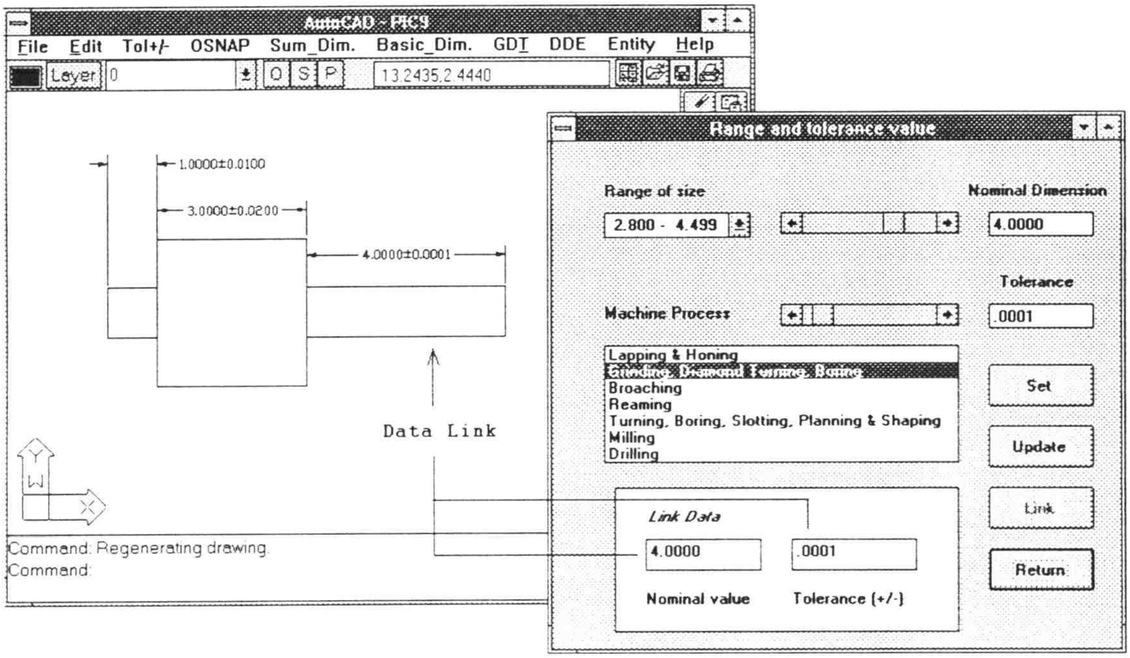


Figure 507. The Data Link Between the User Interfaces of the AutoCAD System and the Tolerance Database

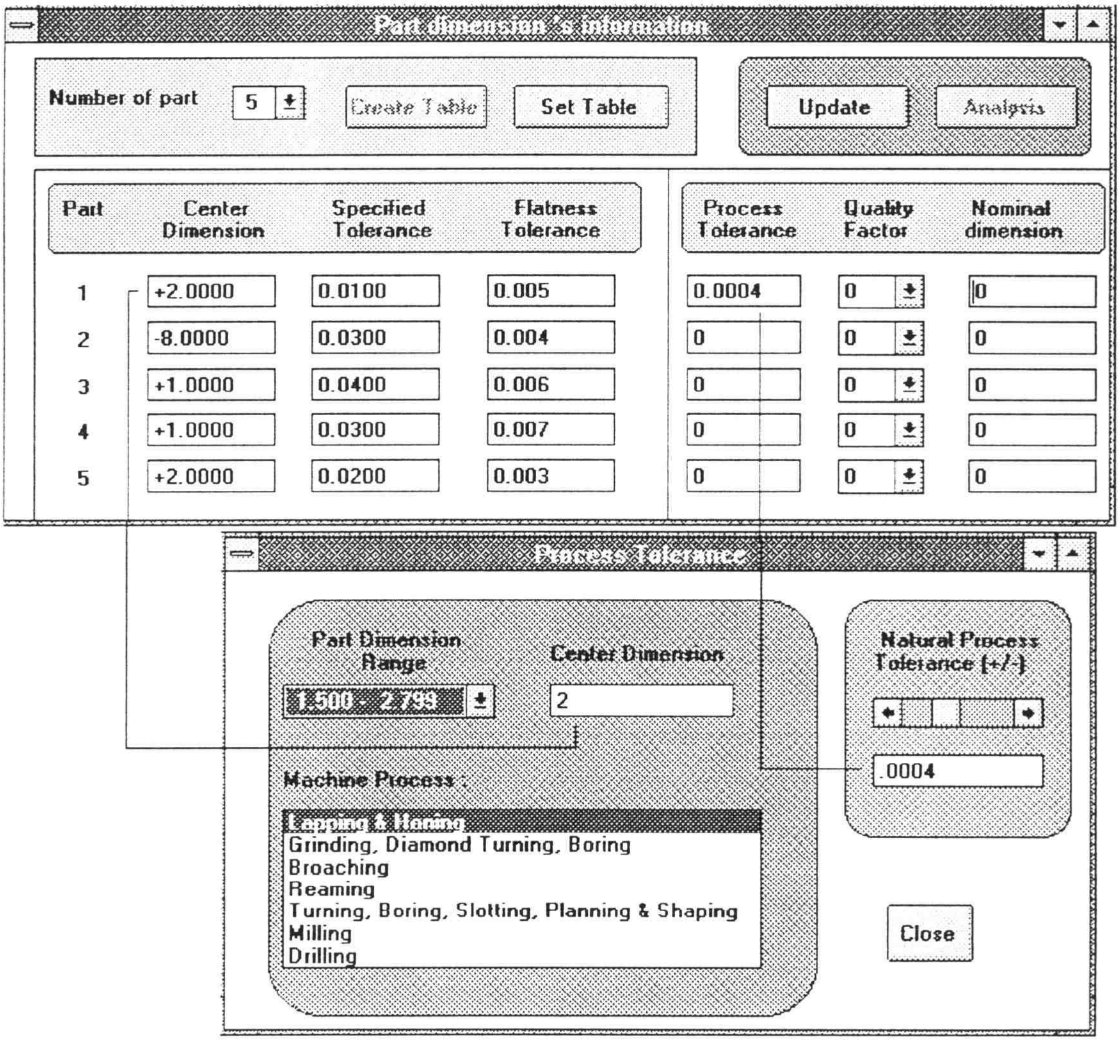


Figure 508. The Data Link Between the User Interfaces of the Modified Assembly Tolerance Analysis Model and the Tolerance Database

5.5 An Application Example

To present the application of the integrated system, an assembly drawing and the TK menu are provided in the AutoCAD system (Figure 509). All AutoLISP programs should be loaded in the computer following the installation procedures explained in Appendix A. The application procedures are:

1) Inserting a conventional tolerance in the drawing

To insert a conventional tolerance, a pull-down menu "Tol+/-" is designed to provide various types of dimension features such as a horizontal line, a vertical line, or a curved line. The program procedures require two end points of a selected line and the location of the conventional tolerance. In addition, the value of conventional tolerance is freely set by typing in the keyboard. Figure 510 displays an assembly drawing with conventional tolerances.

2) Inserting a GD&T specification in the drawing

GD&T specification is divided into three categories: 1) basic dimension, 2) datum, and 3) GD&T symbols. The pull-down menu "Basic_Dim", "Datum", and "symbol" are used to insert a basic dimension, a datum, and a GD&T symbol, respectively. Due to the various types of GD&T symbols, a

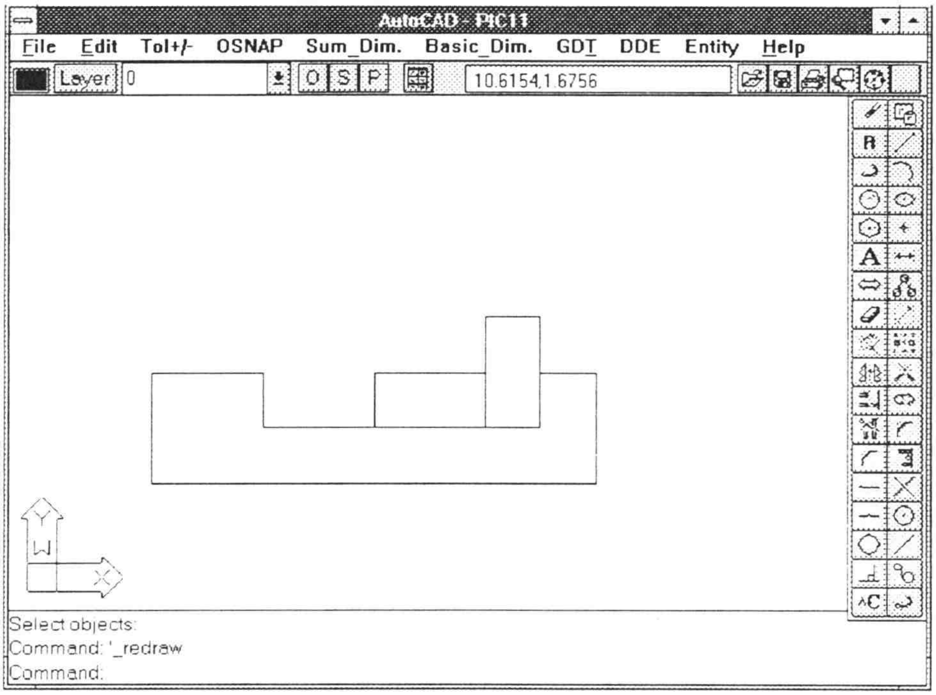


Figure 509. The Assembly Example with New TK AutoCAD Menu

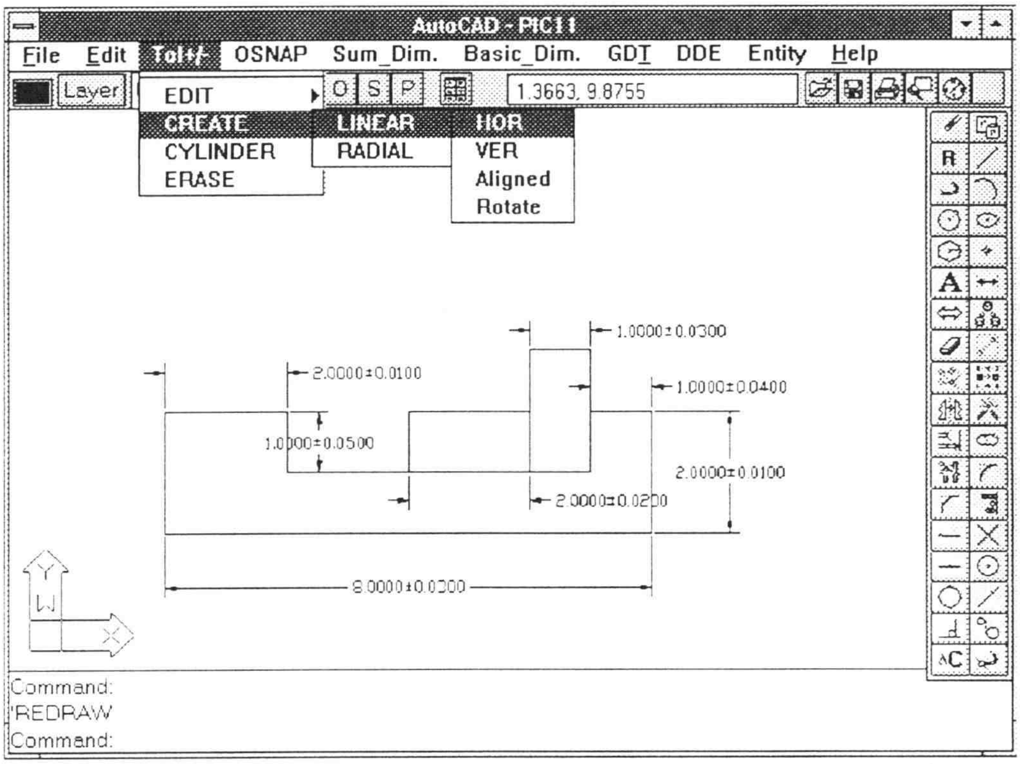


Figure 510. The Function of "Tol+/-" Pull-Down Menu for Creating Conventional Tolerances in the Assembly Drawing

group of symbol slides has been designed, as shown in Figure 511. To specify a GD&T symbol, the user selects a choice from the slides and supplies the required parameters during the procedures. In Figure 512, only the flatness symbol is inserted in the drawing.

3) Adjusting a conventional tolerance with a tolerance database

A user interface of the tolerance database has been designed, as shown in Figure 506. To open the user interface, the user selects "DDE," "START," and "Design_tol," respectively. To run the program, the user selects "DDE" and "Design_Tol." For example, the selected conventional tolerance "8.0000±0.0300" is transferred to the user interface, as shown in Figure 513. Its value can be adjusted following the tolerance database, and then updated in the drawing dynamically. As shown in Figure 514, the conventional tolerance "8.0000±0.0300" is changed to "8.0000±0.0100."

4) Determining a critical assembly tolerance

Three assembly tolerance analysis models are included in the integrated system: 1) the Worst Case model (WC), 2) the Root Sum Squares model (RSS), and 3) the new model. The WC

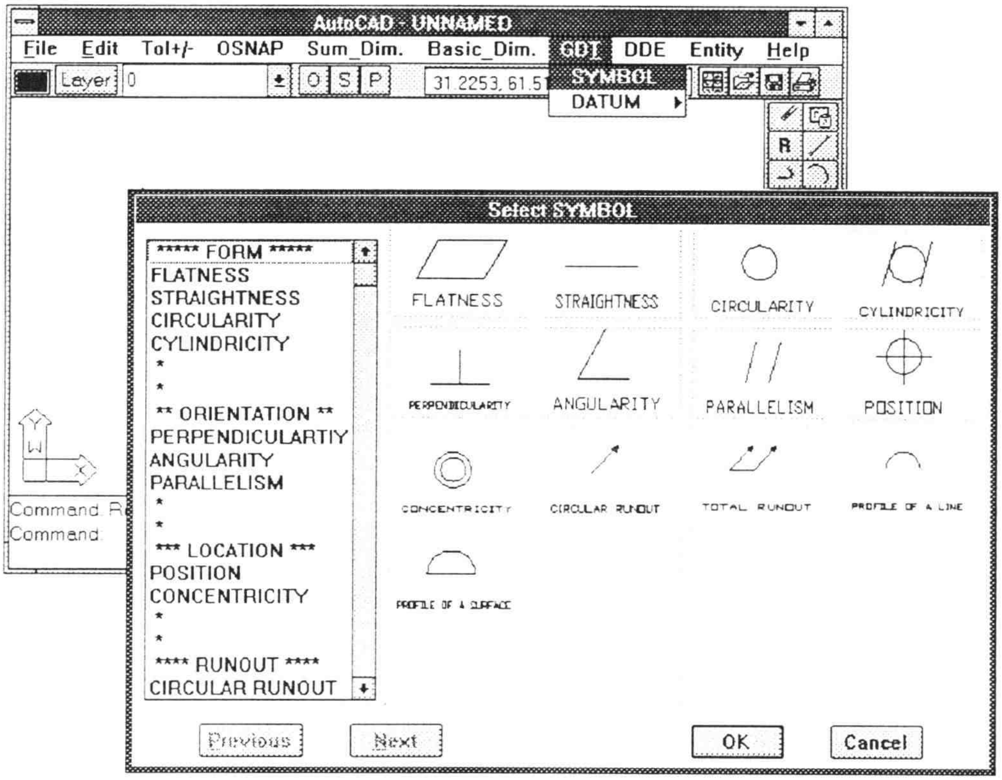


Figure 511. A Group of GD&T Symbol Slides

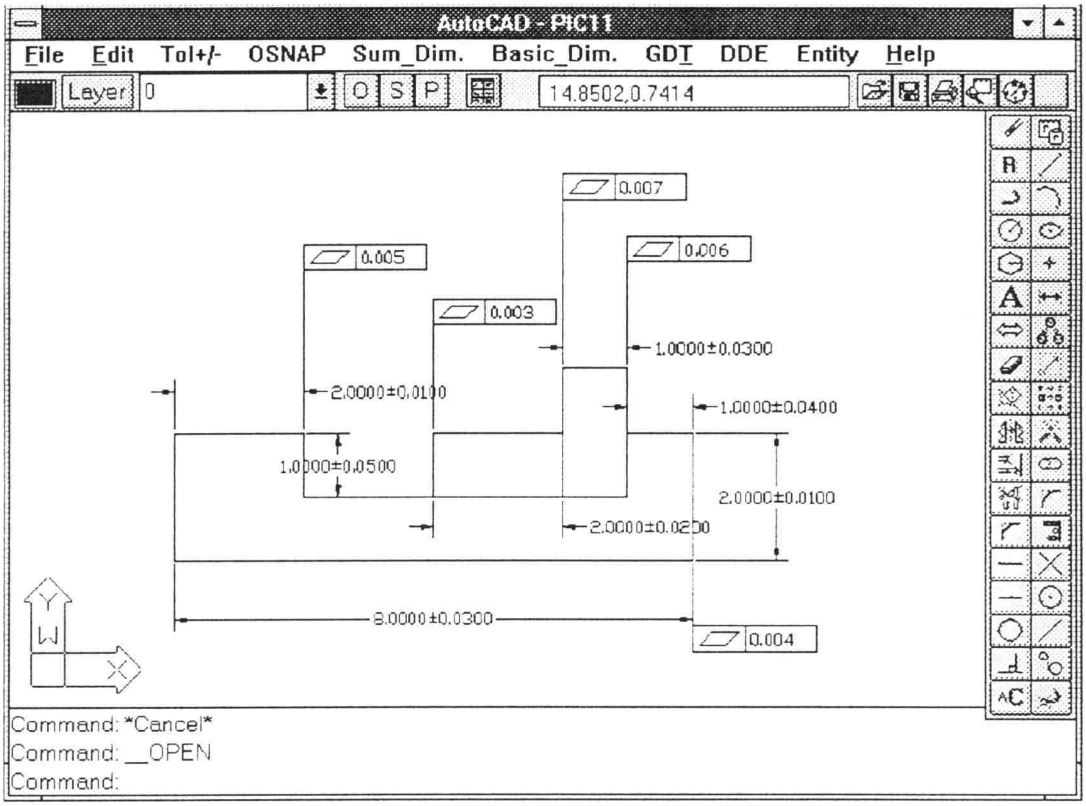


Figure 512. The Assembly Example with Flatness Symbols

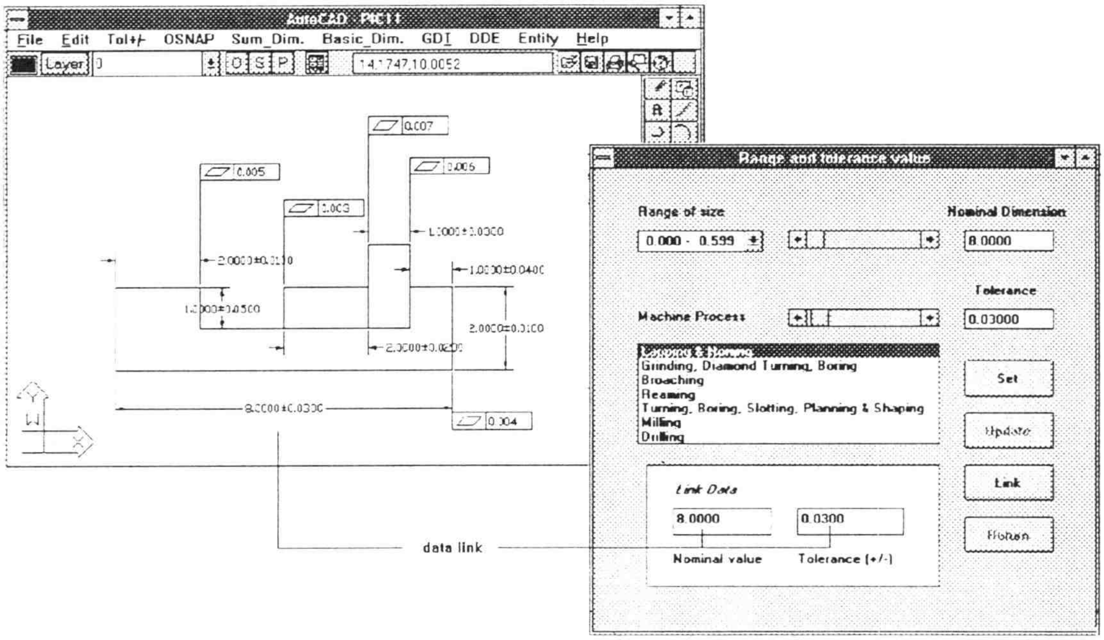


Figure 513. Transferring Conventional Tolerance from AutoCAD System to the User Interface

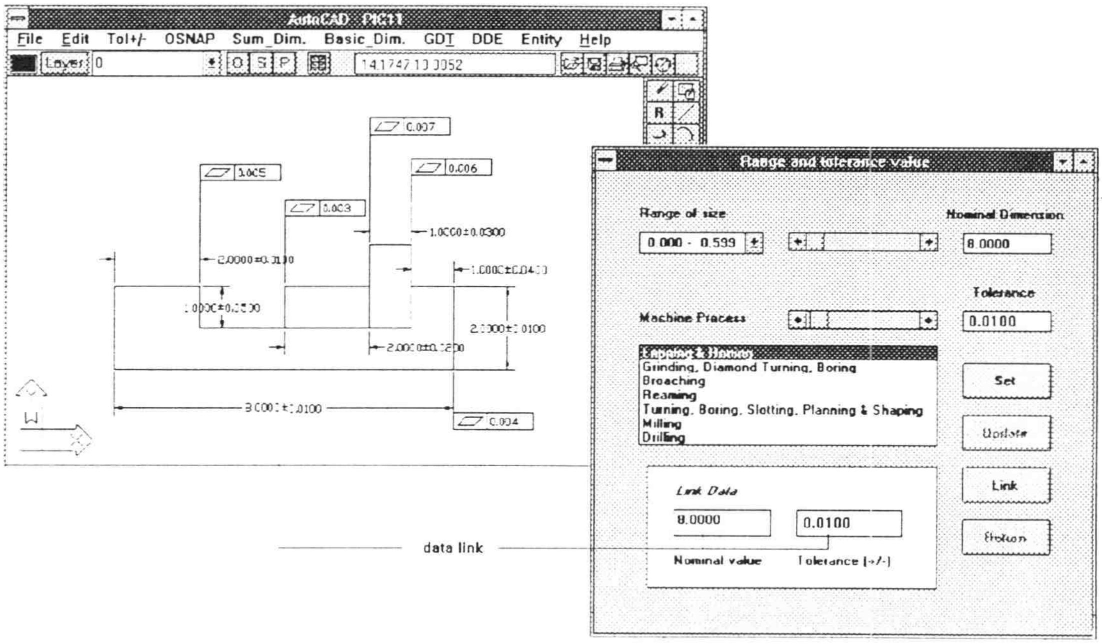


Figure 514. Updating Conventional Tolerance in the AutoCAD Drawing

and RSS models are operated into the AutoCAD system. The user chooses the pull-down menu, "Sum_Dim," and selects either "WC" or "RSS." The assembly dimension and tolerance will be determined following the selected models (WC or RSS) and inserted in the drawing, as shown in Figures 515 and 516. Due to demanding calculations of the new model, its determination is applied in a user interface. All data from the part drawing are transferred to the user interface. In addition, all supply variables can be freely set by typing in the desired text box. All data in the user interface, as shown in Figure 517, follows the data in Table 403. Finally, the assembly dimension and tolerances "1.999±0.0012" are determined and inserted in the drawing, as shown in Figure 518. Moreover, the value of text box "process tolerance" in the user interface of the new model can be assigned automatically by connecting with a user interface of a tolerance database. To open the tolerance database window, the user clicks twice at the selected text box "Process tolerance." In the tolerance database window, the user adjusts the value of process tolerance, and then clicks the command button on "Close." The adjusted process tolerance will be transferred to the user interface of the modified model and the tolerance database will be closed automatically, as shown in Figure 519.

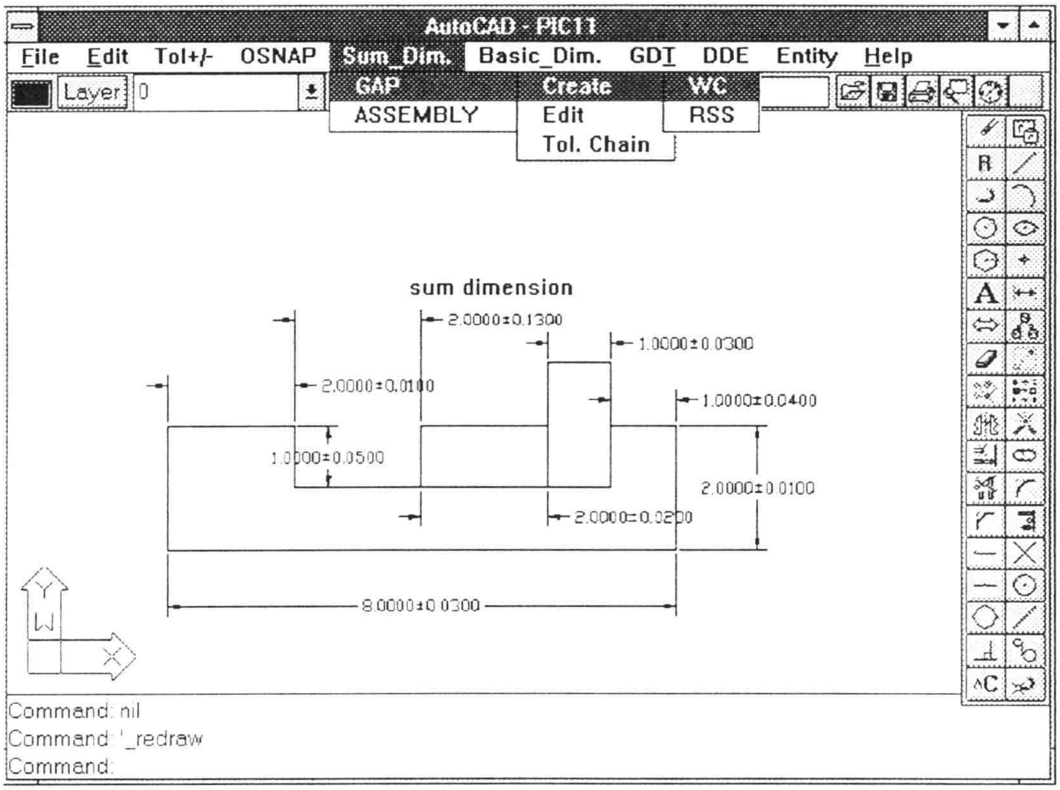


Figure 515. The Application of Pull-Down Menu "WC" for Determining Sum Dimension with Worst Case Assembly Tolerance Analysis Model

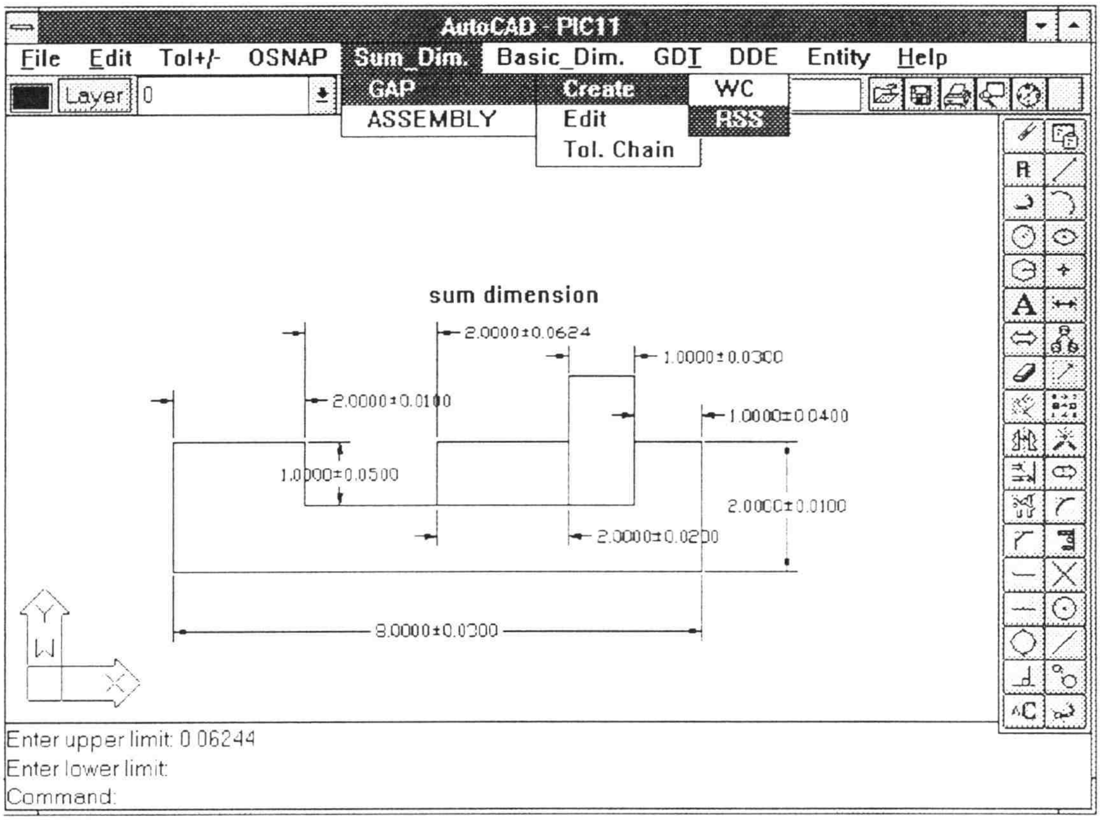


Figure 516. The Application of Pull-Down Menu "RSS" for Determining Sum Dimension with Root Sum Squares Assembly Tolerance Analysis Model

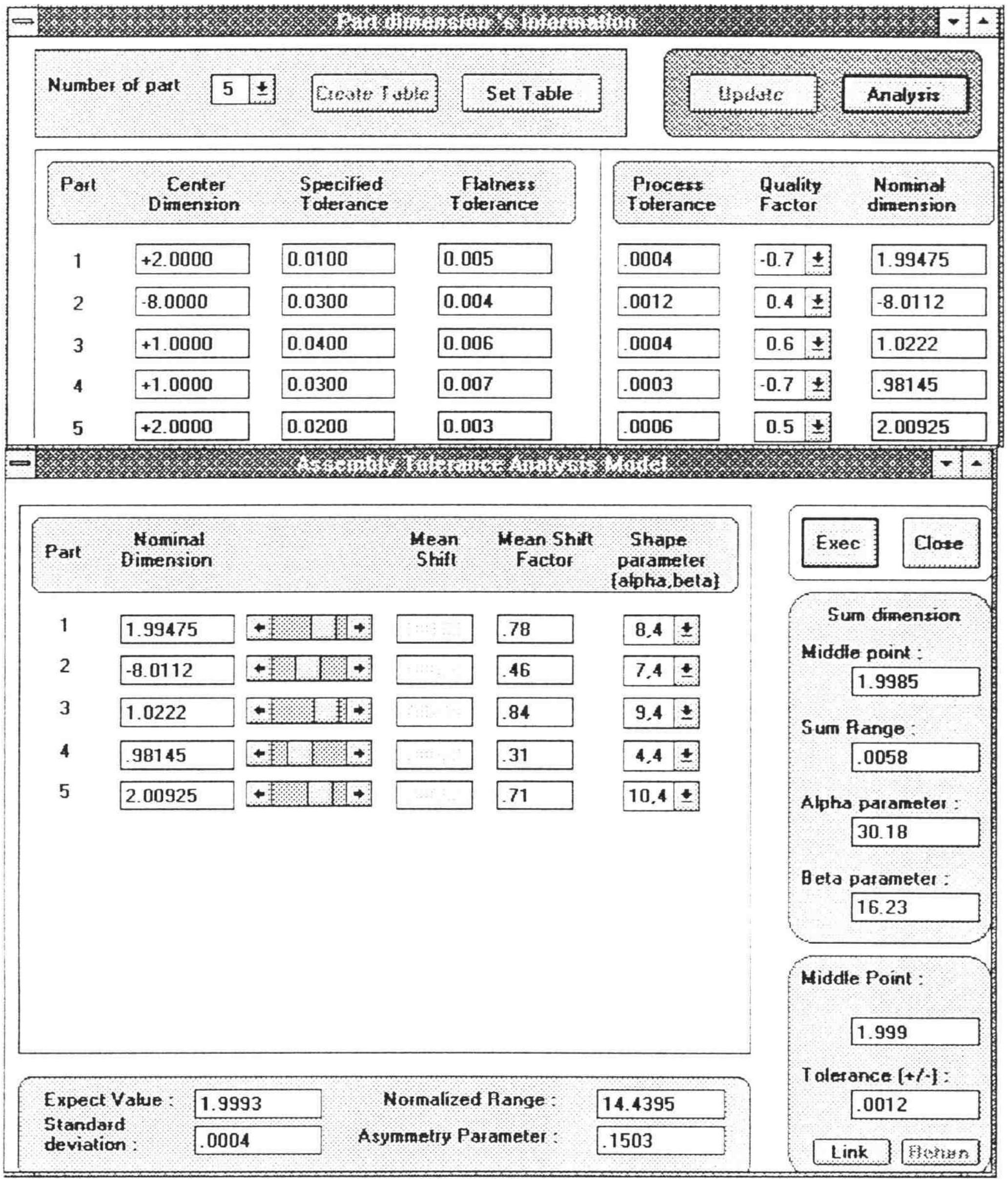


Figure 517. User Interface of the Modified Model Including All Supply Variables of the Assembly Example Following Assigned Values in Table 403

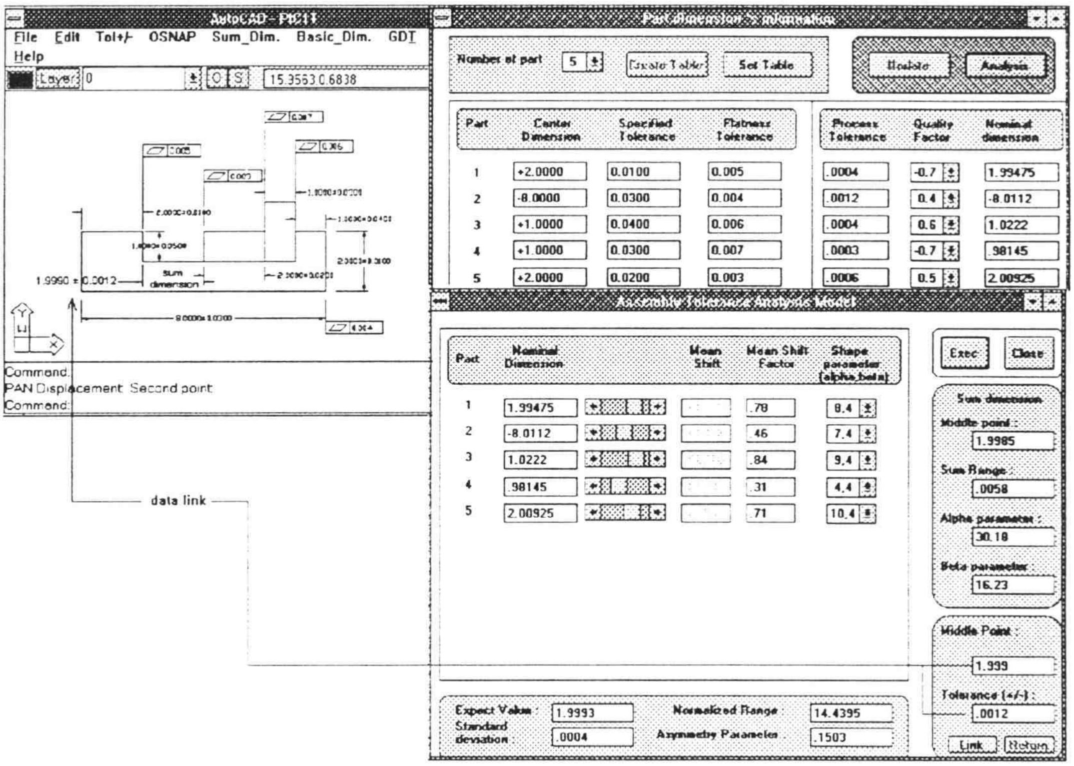


Figure 518. Transferring and Inserting the Assembly Dimension and Tolerance, "1.999±0.0012," from the User Interface into the Assembly Drawing

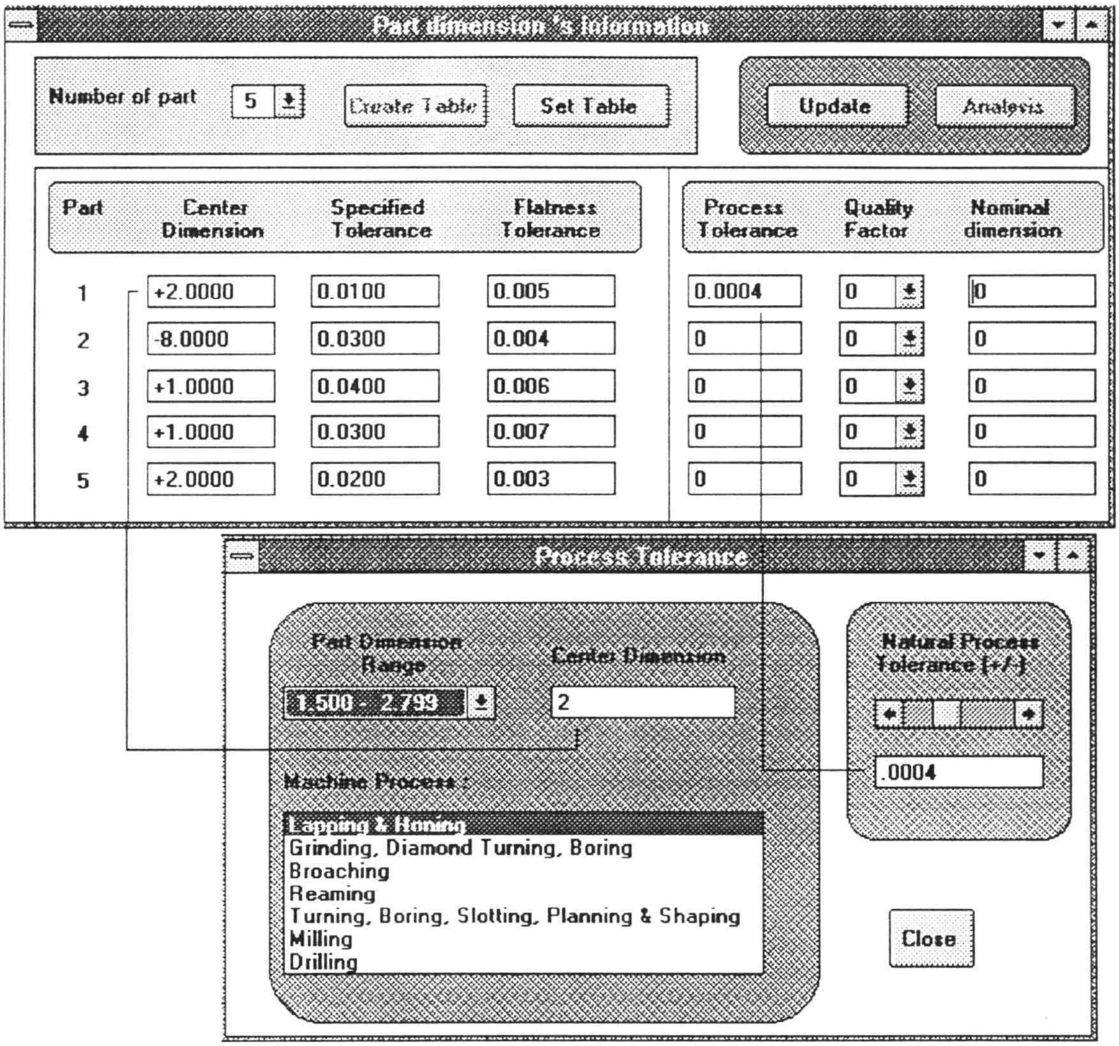


Figure 519. The Application of the Tolerance Database for Providing a Process Tolerance into the User Interface of the New Model

Integration of four support systems (drawing, GD&T specifications, assembly tolerance analysis models, and tolerance databases) has been developed in a single computer application. Windows 3.1 software is applied to address and operate the three system components (CAD drawing, assembly tolerance analysis, and tolerance database programs). Using DDE, all three programs can communicate and transfer their data, making the integrated system more powerful and able to solve tolerance analysis problems in a large scale software package. All source files are developed and stored in diskettes 1, 2, and 3. The program installation is explained in Appendix A. It is important to note that adequate RAM and hard disk capacity is required, since the working files have substantial memory requirements to access system integration functions.

CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

In any manufacturing process, the manufactured part dimensions will always deviate from their nominal design data due to random and/or systematic effects. Tolerance data is an important part of design information, as expressed in a part or an assembly drawing. During the design stage, however, the specification of tolerance constraints can have major consequences for product quality and cost. Tighter tolerances result in better product quality, but they increase manufacturing costs considerably. In addition, the tolerance constraints on part dimensions control the assembly dimension and tolerance which should not exceed the relevant design constraints.

Critical assembly dimensions and tolerances are often evaluated by using existing assembly tolerance analysis models, such as the Worst Case(WC) and Root Sum Squares(RSS) models. These models can handle only conventional tolerance. Rather than conventional tolerance, a new standard, the "geometric dimensioning and tolerancing," or, "GD&T," is defined by ANSI standard Y14.5-1982 in order to give an unambiguous interpretation for each geometric feature. It consists of rules, symbols, and their representations. To improve the applicability of the existing assembly tolerance analysis models, a new assembly tolerance analysis model was developed to include the effects of the flatness symbol (one of the GD&T specifications symbols). In addition, the part dimensions are assumed to have beta distributions, making their distribution shapes vary from normal to skewed. However, the calculation of the modified model is highly complicated because it involves a large number of variables. These variables are divided into two groups. The first group of variables must follow the data in the part drawing, such as the center dimension, specification tolerance and flatness tolerance zone. The other group is dependent upon information provided by the user or extracted from a database. For example, the process tolerance variable is extracted from the manufacturing tolerance database. Therefore, tolerance analysis in the design stage requires different support systems.

A computer integrated system has been developed to reduce the application complexity. The operation of the system deals with four support systems: 1) a drawing system, 2) a GD&T specifications system, 3) an assembly tolerance analysis model, and 4) a manufacturing tolerance database system. To take advantage of the multitasking feature, the integrated system runs under the Windows environment. Using the Dynamic Data Exchange(DDE), all three system components -- the CAD drawing program, the assembly tolerance analysis program, and the tolerance database program, can transfer information dynamically. The CAD drawing program allows the allocation of conventional tolerances and GD&T specifications in the AutoCAD system. This application uses computer programs written in AutoLISP. Using the assembly tolerance analysis program, the user can access several commonly used assembly tolerance analysis models as well as the modified model developed in this thesis. The manufacturing tolerance database was prepared such that it could be applied like a user interface to provide dynamic support of tolerance information to the integrated system. A tolerance chain generation is also included in the system to initiate a closed loop of dimension links around a critical assembly dimension. All application programs are controlled by a new AutoCAD menu named "TK menu" using a pointing device. Finally, the application of the integrated system improves the capability of tolerance analysis concurrently with CAD modeling while making use of a manufacturing process database.

6.2 Recommendations for Future Research

This research not only covers a tolerance analysis methodology, but also improves the ability of AutoCAD computer application. However, four additional research areas have been identified for further refinement of the methodology and for the extensions of its applications as follows:

1). Application of the Tolerance Analysis Model Associated with Interrelated Tolerance Chains

Only the simple tolerance chains are dealt with in this thesis. Each individual part dimension is independent of the others in an assembly, whereas those of the interrelated tolerance chains are not. In fact, some assemblies result in an interrelated tolerance chain if each part dimension is directly affected by the others. Certainly, tolerance analysis becomes more complicated and needs to be explored.

2). Determination of Appropriate Alpha and Beta Parameters for Individual Part Dimensions

Although the exact distribution of part dimensions are not known, the distribution of part dimensions in this research is assumed to be the beta. The part distribution varies depending on the beta and alpha parameters. If the estimated alpha and beta parameters are close to the exact distribution of the part dimension, the tolerance analysis will be more precise and meritorious. Therefore, some studies for finding appropriate values of alpha and beta parameters need to be initiated. These studies are laborious because they are associated with a great deal of data collection and analysis.

3). Determination of Appropriate Natural Parameters for Individual Part Dimensions

The modified model includes significant natural factors such as the quality factor and the mean shift factor. However, the expected values of these factors still need to be explored because their values are independent of the actual manufacturing process. Some experiment designs are recommended for future works dealing with the actual conditions of manufacturing.

4). Application of the Tolerance Analysis Model with Other GD&T Symbols

The modified model developed in this thesis includes only the flatness symbol. The rules and applications of the flatness symbol are discussed and applied in the modified model. Other GD&T symbols, such as straightness, circularity and cylindricity, need to be considered and included in the tolerance analysis model.

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APPENDICES

APPENDIX A
USER'S GUIDE OF COMPUTER APPLICATION

Setting Up

All source codes are stored in diskettes 1, 2, and 3. Before you install all source files in a hard disk, make sure that the computer meets the requirements as follows:

- Any IBM-compatible make with an 80386 processor or higher
- A hard disk (drive C)
- A 3 1/2" floppy drive (drive A)
- A super VGA or compatible display
- One megabyte of memory in addition to run AutoCAD program
- A mouse
- Microsoft Windows 3.1 or later
- AutoCad for Windows Release 12 or later

After checking the system requirements, the user should open the File Manager in Windows and follow the steps below:

- Select drive C and create directory named JOB
- Insert Disk 1 in drive A
- Copy all files from drive A to C:\JOB\

- Insert Disk 2 in drive A
- Copy all files from drive A to C:\JOB\
- Select drive C and create directory named PLAY
- Insert Disk 3 in drive A
- Copy all files form dive A to C:\PLAY\

After processing, all research files are kept in a specific directory (C:\JOB\ and C:\PLAY\).

Getting Started

This user's guide assumes that the user knows how to use AutoCad for Windows and the basic techniques to work with Windows. First, the AutoCad system must be initiated and an assembly drawing must be completed by using the standard AutoCAD menu or drawing commands. To run an AutoLISP program, the completed assembly drawing must be saved in directory C:\JOB\ because all AutoLISP programs in this research are saved in directory C:\JOB\. A new AutoCAD menu named TK menu has been developed to group and initiate all application programs in the integrated system. The new AutoCAD menu must be called and loaded as follows:

- Type "menu" in the command line
- Select C:\JOB\TK.mnu

The functions of the TK menu are briefly displayed in Figure A.1. The following sections describe the application of the TK menu to serve as a complementary tool for the application of the integrated system.

The Application of the Integrated System

The application of the integrated system is divided into five sections. These sections are described as follows:

1. Creating a Conventional Tolerance
2. Creating a GD&T specification
3. Creating a Simple Tolerance Chain
4. Determining a Critical Assembly Tolerance
5. Dynamic Support with a Tolerance Databases

All sections are controlled by an individual group of pull-down menus. The following paragraphs describe the functions of the pull-down menu for each section.

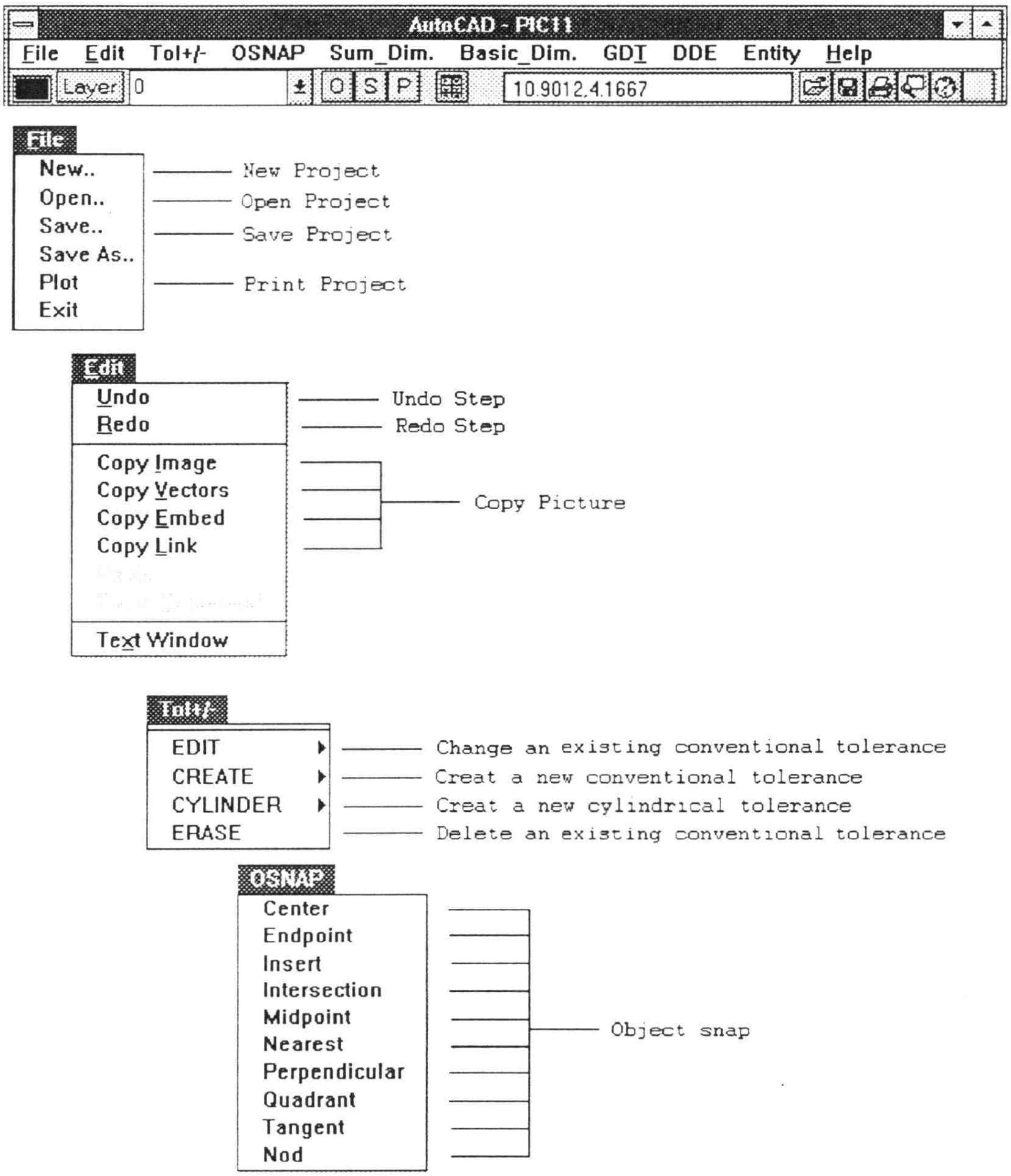

Figure A.1. The Describtion of TK AutoCAD Menu

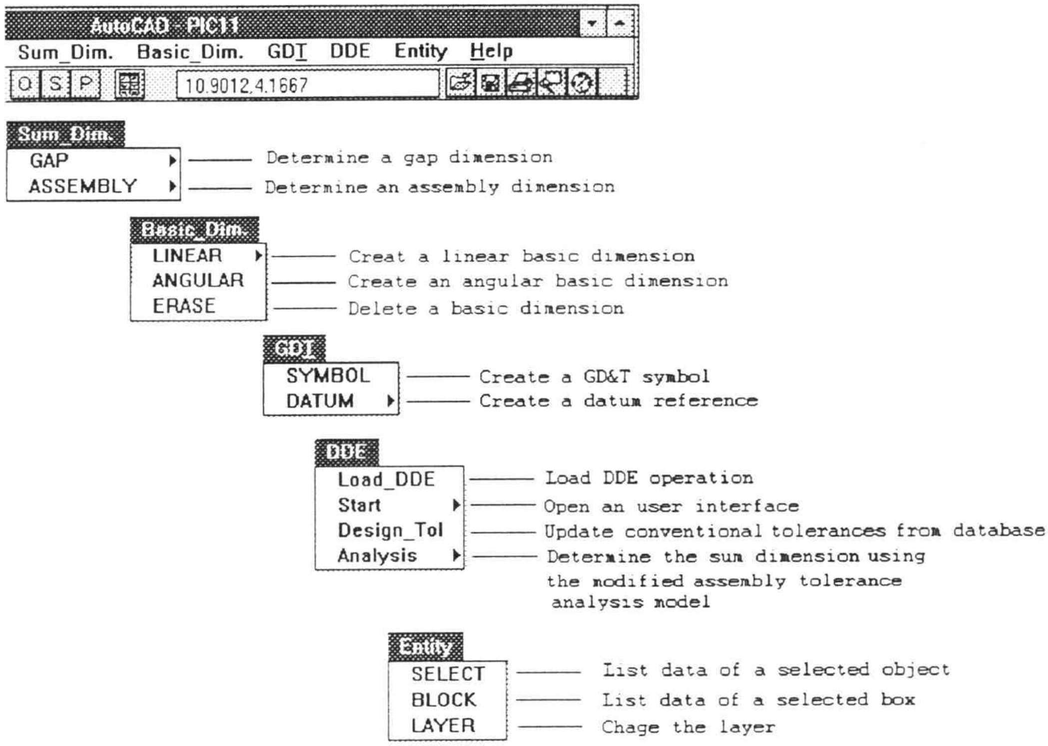

Figure A.1. The Describtion of TK AutoCAD Menu (continued)

Creating a Conventional Tolerance

In the TK menu, a group of AutoLisp programs for crating a conventional tolerance is initiated by clicking the pull-down menu named "Tol+/-". There are different types of dimension lines associated with conventional tolerances such as a vertical line, a horizontal line, or a circular line. A group of pull-down menus is designed to serve all types of dimension lines and their description is shown in Figure A.2. After clicking at a choice menu, the AutoLISP program associated with the selected choice will be initiated and run immediately. The user follows the comments at the command line, and finally, a conventional tolerance will be inserted, related to the selected dimension. Only symmetrical conventional tolerances are included in the system.

Creating a GD&T Specifications

In the TK menu, GD&T specifications are divided into three categories: basic dimensions, GD&T symbols, datum references. Figure A.3 displays the application of the pull-down menu "Basic_Dim" which can draw a basic dimension depending on the type of dimension line (horizontal, vertical, or circle). In addition, the pull-down menu "GDT" can establish both GD&T symbols and datum references as

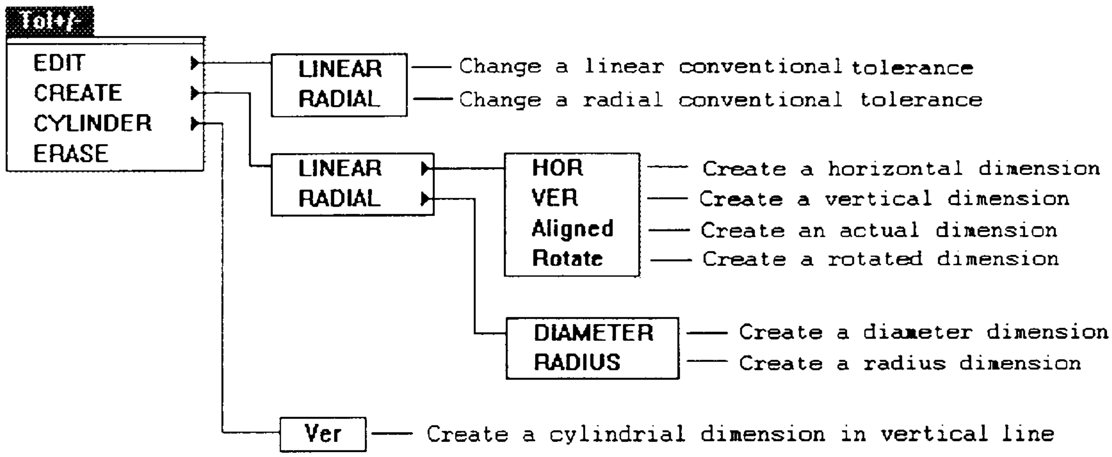

Figure A.2. Application of the Pull-Down Menu "Tol+/-"

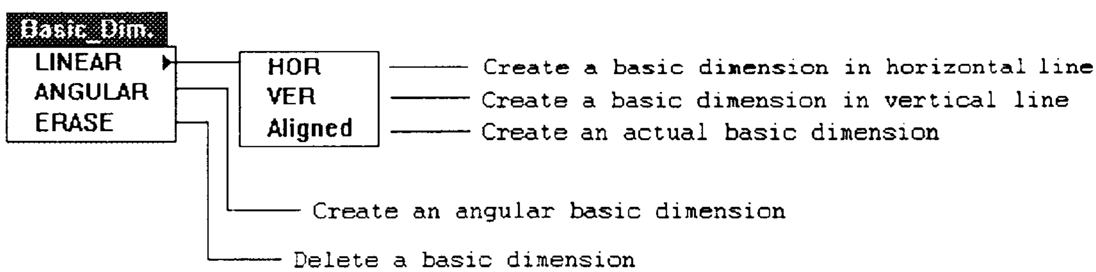

Figure A.3. Application of the Pull-Down Menu "Basic_Dim" described in Figure A.4. To crate a GD&T symbol, the user clicks the pull-down menu "symbol", and then the figure frames of GD&T symbols will be visualized (Figure A.4). Next, the user selects the choice and actives following the comments in the command line. Finally, A datum reference or a GD&T symbol will be inserted in the drawing.

Creating a Simple Tolerance Chain

A simple tolerance chain can be generated in different types of the sum dimension. The sum dimension in the TK menu is divided into two features: assembly and gap dimensions. An assembly dimension is considered as an accumulated dimension from all part dimensions. In contrast, a gap dimension is the unknown sum dimension located between two part sections. Both types of sum dimensions are displayed in Figure A.5.

The pull-down "Sum_Dim" is divided into two sections as shown in Figure A.6. The user chooses the pull-down menu named Tol_chain, and then specifies two end points of the unknown sum dimension. A simple tolerance chain will be created following the successive link between each part dimension, as displayed in Figure A.7. Moreover, the simple tolerance chain will be located in a new layer named "chain". However, all part dimensions except one (the unknown sum

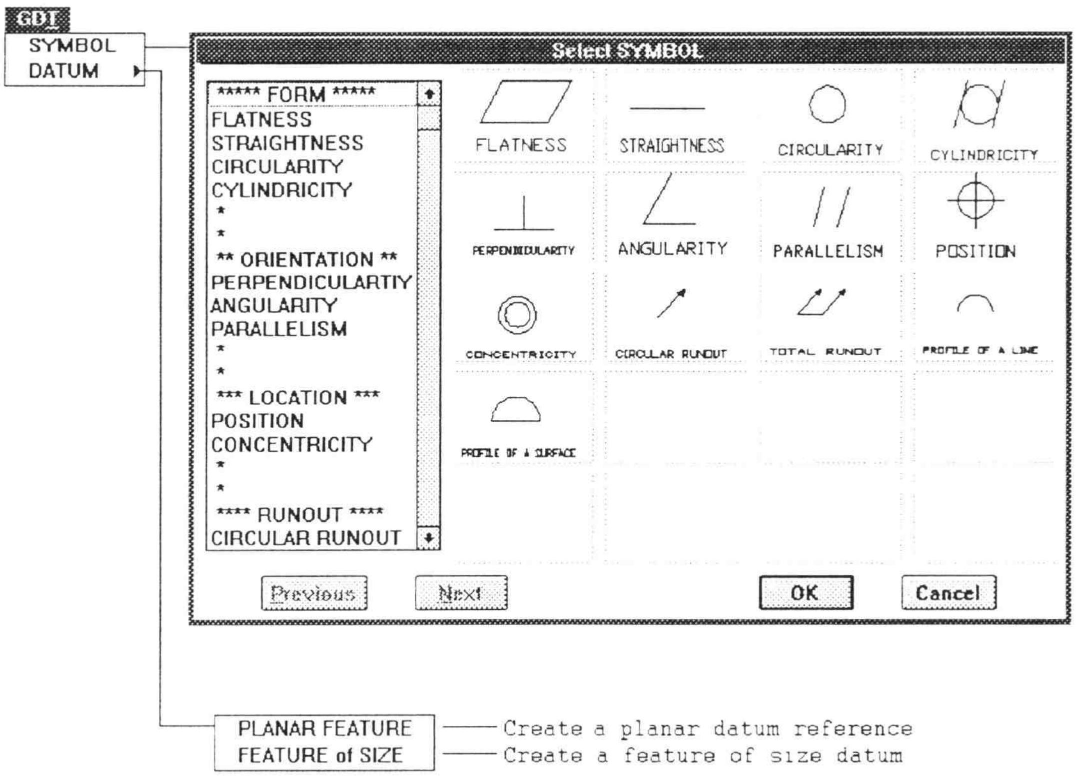

Figure A.4. Application of the Pull-Down Menu "GDT"

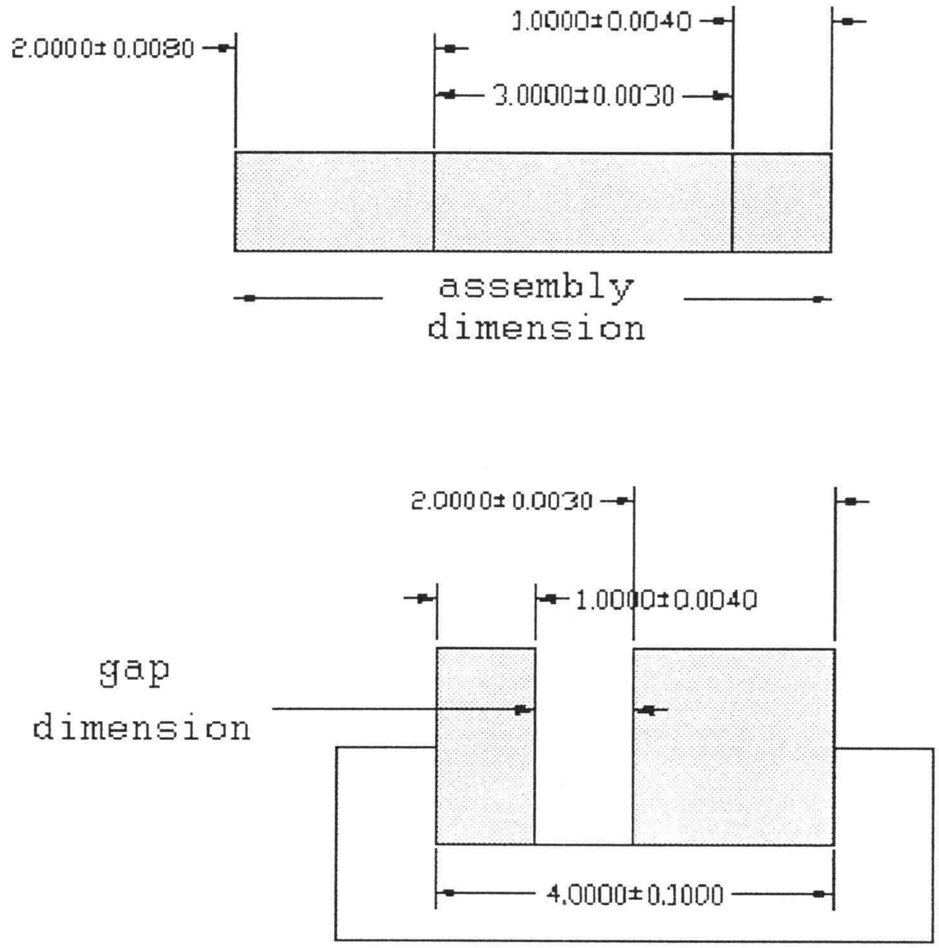

Figure A.5. Two Types of the Sum Dimension; Assembly Dimension and Gap Dimension

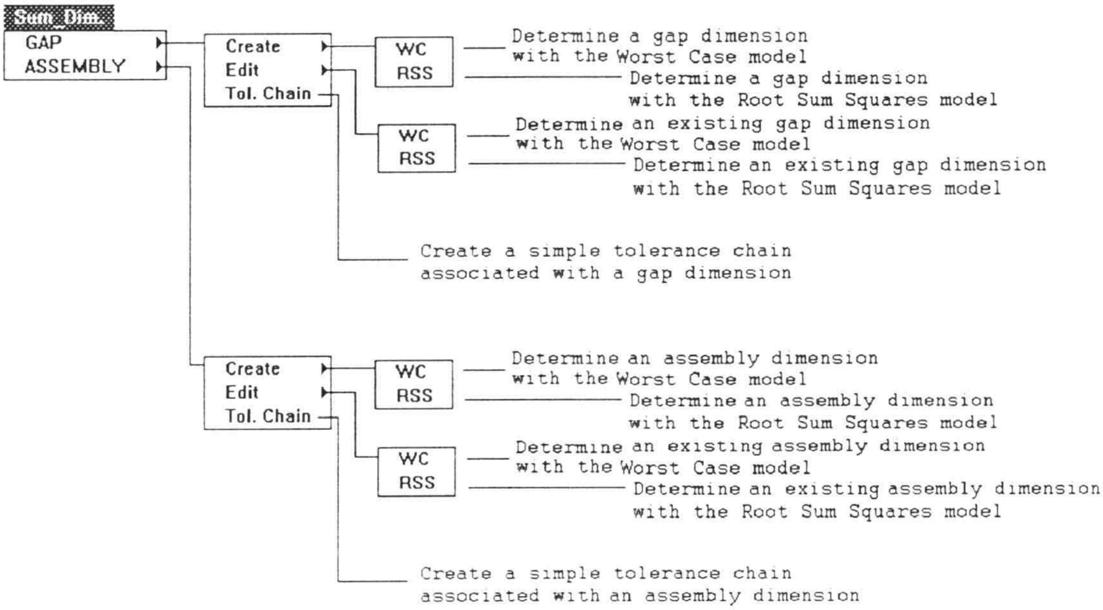

Figure A.6. Application of the Pull-Down Menu "Sum_Dim"

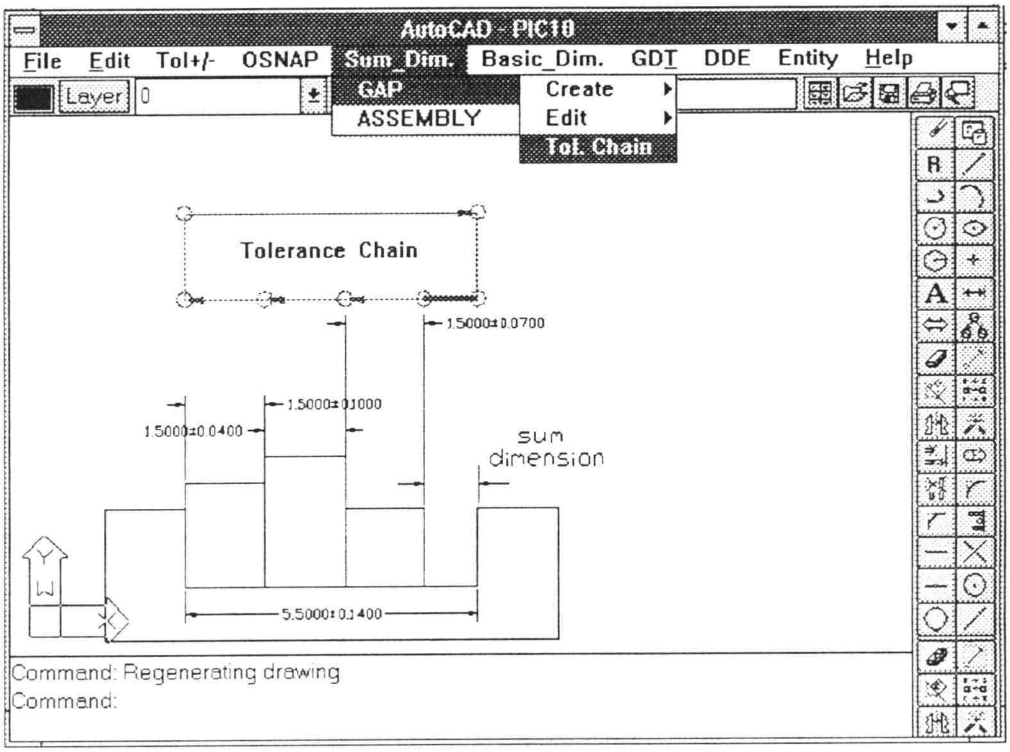

Figure A.7. Application of the Pull-Down Menu "Tol.Chain" dimension) must be specified. Otherwise, the program cannot find the link between two successive part dimensions resulting in an unsuccessful application.

Determining a Critical Assembly Tolerance

Assembly tolerance analysis models are used to determine a critical unknown sum dimension of an assembly. Only three assembly tolerance analysis models are available in the integrated system. The first two models are the Worst Case and Root Sum Squares models. Both assembly tolerance analysis models can operated within AutoCad system due to their simple calculations. Choosing the pull-down menu as shown in Figure A.6, the sum dimension from a selected model, either the Worst Case or the Root Sum Squares, is determined and inserted in the part drawing. However, the third assembly tolerance analysis model or the modified assembly tolerance analysis model, described in Chapter 4, is too complicated and associated with a large number of input variables. Therefore, the modified model should be used outside the AutoCAD system. Microsoft Visual Basic was chosen to serve in the execution of this third model as a user interface under Windows environment. In addition, Dynamic Data Exchange (DDE) links the data between both AutoCAD system and the user interface of the modified assembly tolerance analysis model.

To determine the sum dimension using the modified assembly tolerance analysis model, the pull-down menu "DDE" is used and its application is displayed in Figure A.8. The assembly drawing in Figure 501 of Chapter 5 is selected to illustrate the application of the user interface. To examine this illustration, the user should follow these steps:

1. Make sure that the assembly drawing and the TK menu are already initiated in the AutoCAD system
2. Click the toolbar "DDE" and select the pull-down menu "Start"
3. Select "MODEL", then the user interface of the modified assembly tolerance analysis model will be loaded and opened in the form of a window named "Part dimension's information" as shown in Figure A.9.
4. Click "Create Table" command button, and rows of table boxes will be created corresponding to the number in the combobox "Number of part". Referring to the assembly drawing in Figure 501, five part dimensions are involved so that five rows of text boxes will be created as shown in Figure A.10.

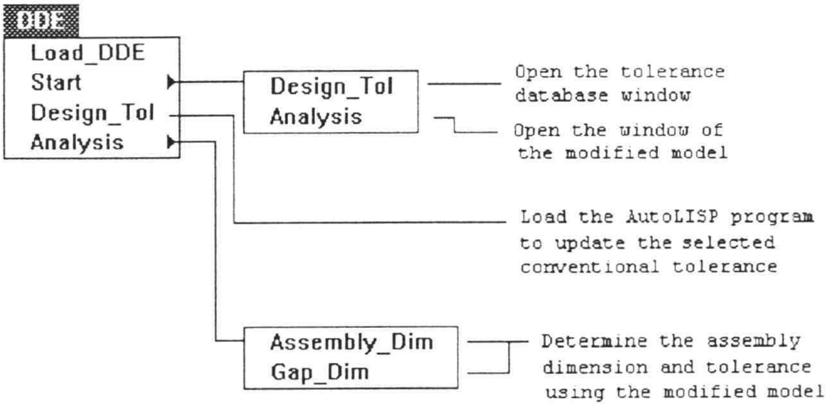

Figure A.8. Application of the Pull-Down Menu "DDE"

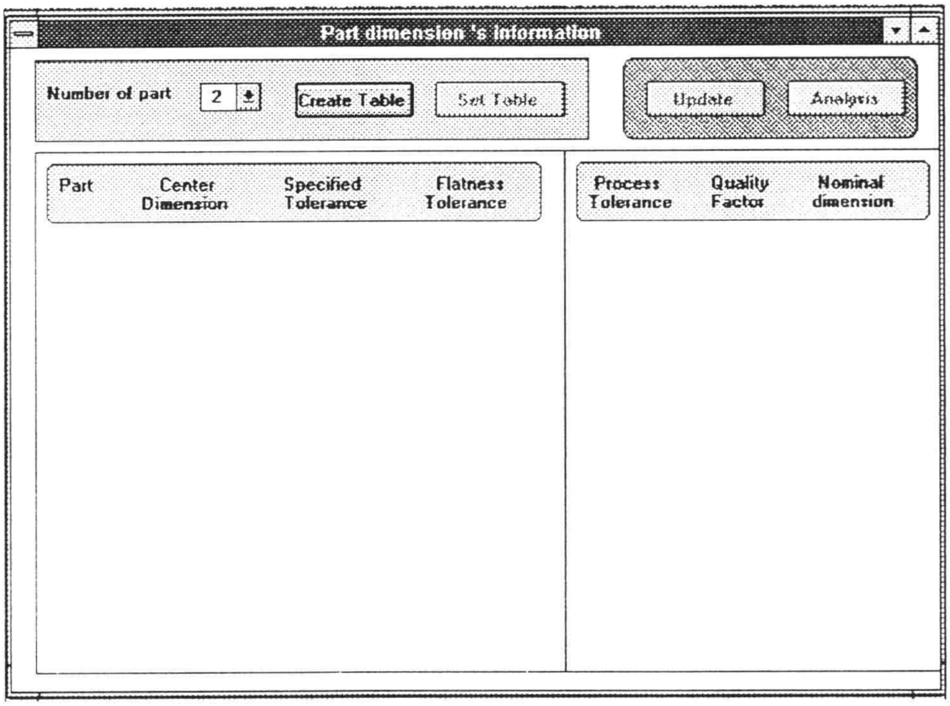

Figure A.9. The User Interface of "Part dimension's information"

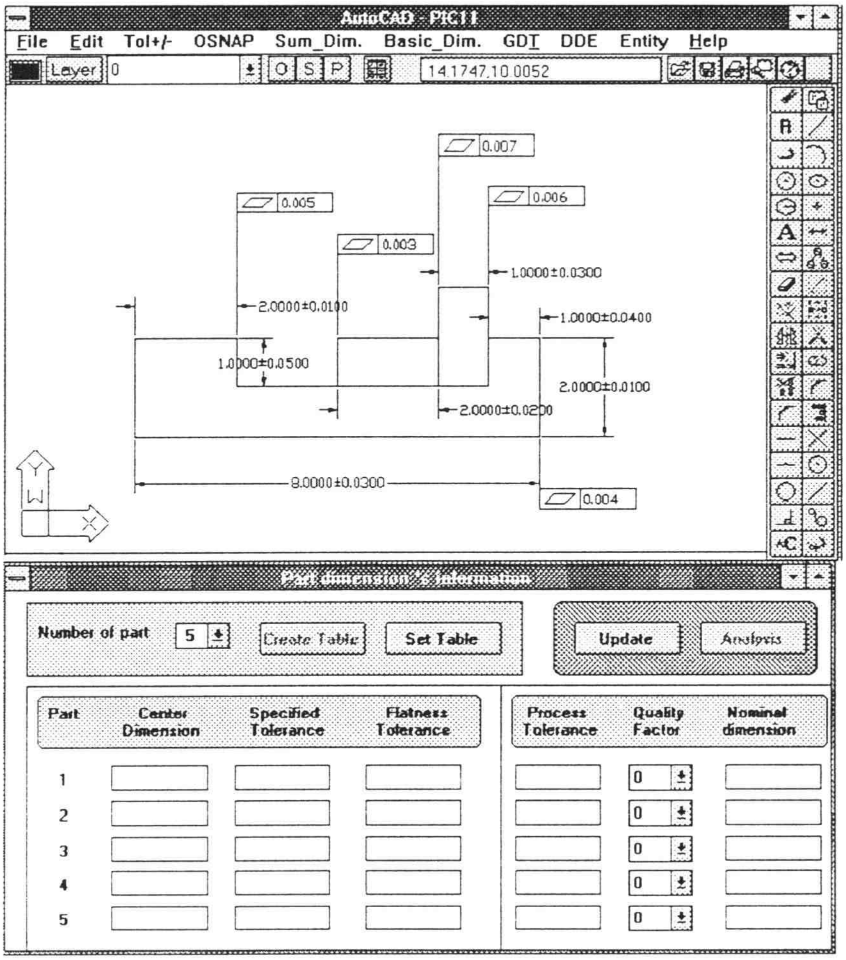

Figure A.10. Application of Command Button "Create Table" to Initiate Five Rows of Text Box

5. Click the toolbar "DDE" and select the pull-down menu "Load_DDE". This process establishes a link between both application windows.
6. Click the pull-down menu "Analysis" and select either "Assembly_Dim" or "Gap_Dim" depending on the types of the sum dimension. Following the drawing example, "Gap_Dim" is selected.
7. An Autolisp program will be loaded and run.
8. After following the comments in the command line, all part dimensions' data from the part drawing such as: center dimension ($X_{c,i}$), specification dimension (L_i), and flatness tolerance zone (T_i) are copied and transferred to the text boxes as shown in Figure A.11. In addition, the plus and minus signs (+/-) in front of the center dimension indicate the direction of the part dimension in a closed dimension loop, associated with the simple tolerance chain (Figure 310).
9. In "Part dimension's information" window, the user should assigned the natural process tolerance (t_i) and the quality factor ($C_{q,i}$). In the text boxes of the natural process tolerance, the user can directly type the value in the text box.
10. Click "Update" command button, all nominal dimensions ($X_{n,i}$) are calculated and appear in the text boxes. Moreover, if the assigned values are violated the computer generates an error message. For example, the

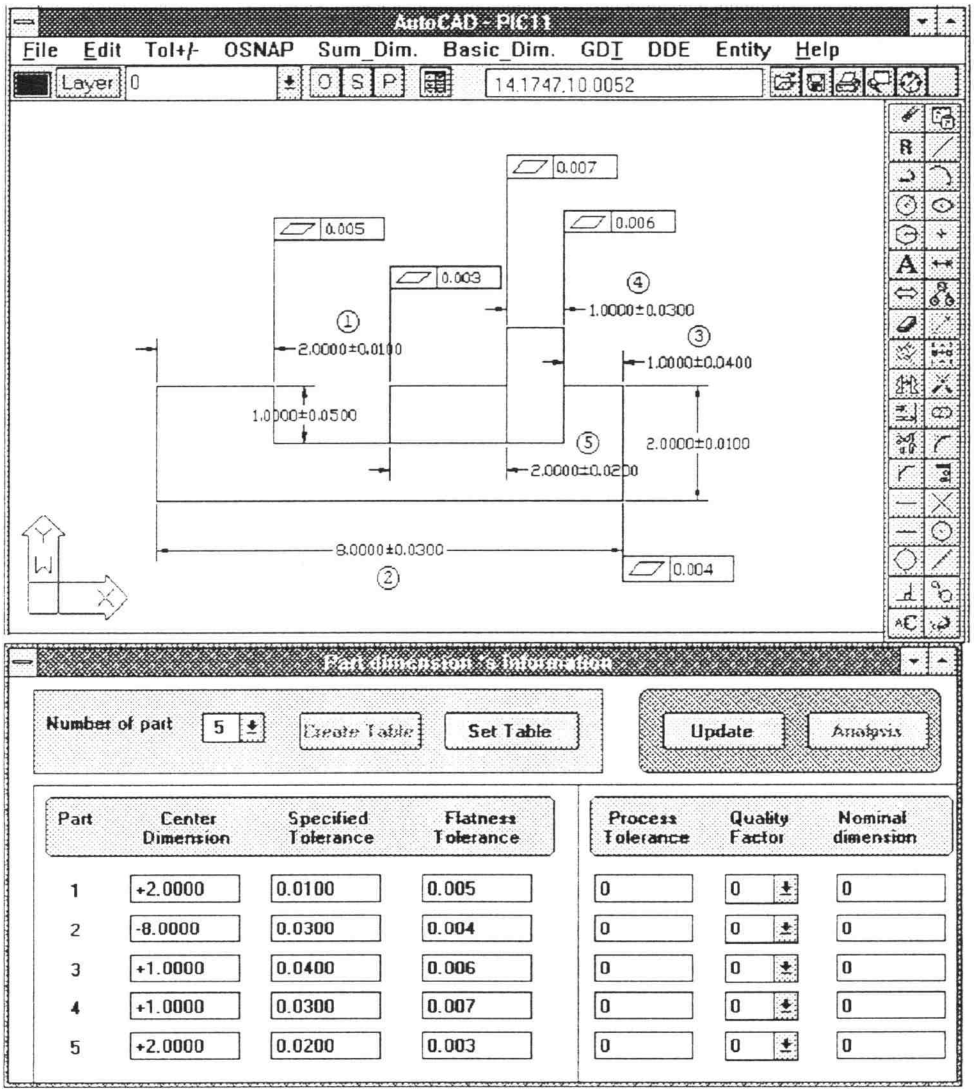

Figure A.11. Transferring Data from the AutoCAD Drawing into the Text Boxes of the User Interface natural process tolerance is greater than one half of the flatness tolerance zone or the specified tolerance is less than one half of the flatness tolerance zone. The nominal dimensions are not determined and a message error related to the error value will be displayed.

11. Click "Analysis" command button, the "Assembly Tolerance Analysis Model" window will appear (Figure A.12). The user adjusts the mean shift factor by moving the scroll bar or typing in the text box directly. Furthermore, the alpha and beta parameters are needed to be set.
12. Click "Exec" command button, all sum dimension parameters are determined as displayed in Figure A.13.
13. Click "Link" and then click "Return", the assembly dimension and tolerance are transferred to AutoCad system.
14. Go back to AutoCad system and follow the comments in the command line to specify the location of the sum dimension. Finally, the estimated sum dimension and tolerance are inserted in the drawing.

Dynamic Support with a Tolerance Database

The user interface of a tolerance databases [37] is developed to support the integrated system in two system processes; the drawing process, and the assembly tolerance

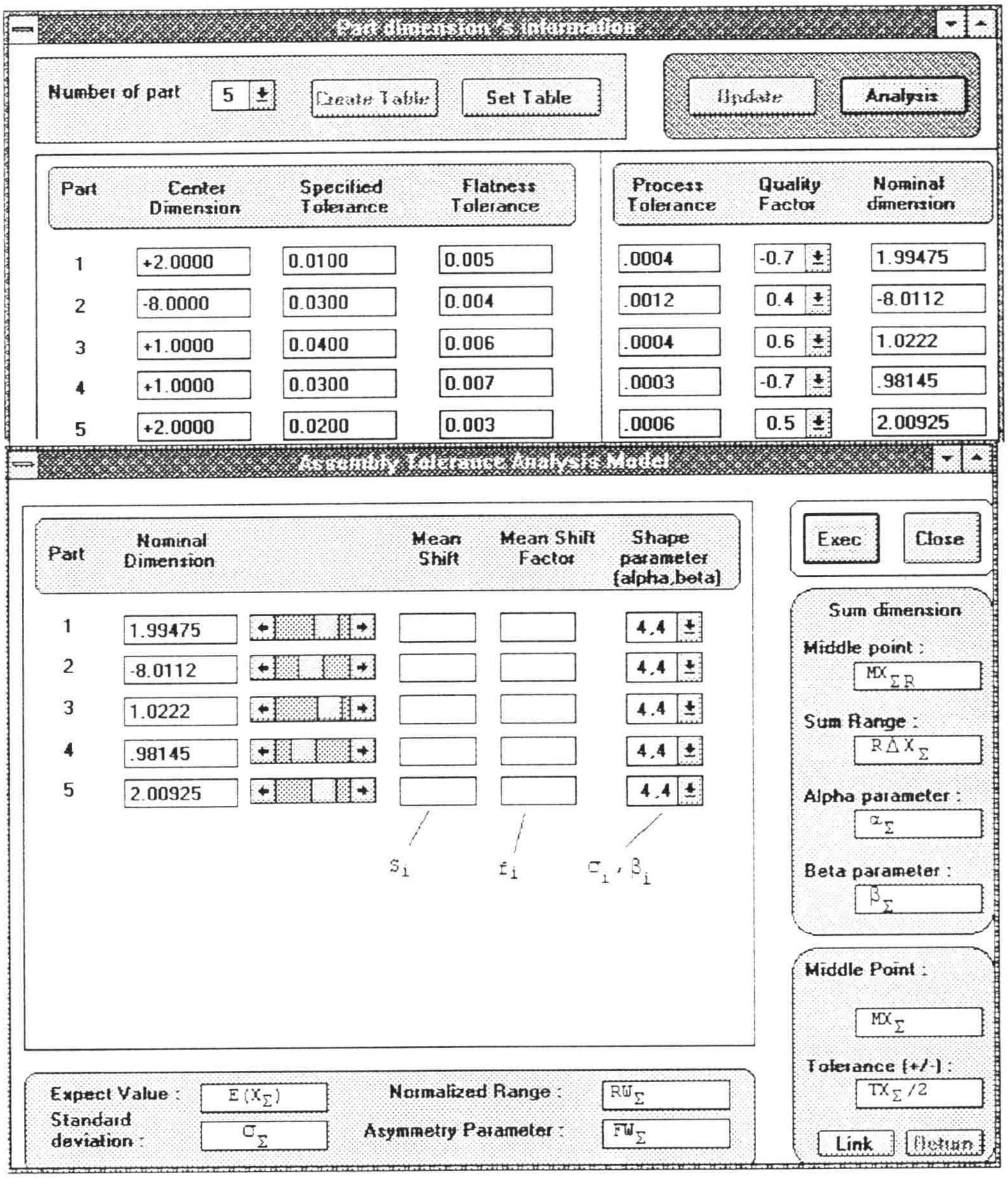

Figure A.12. Application of the Command Button "Analysis" to Open the User Interface "Assembly Tolerance Analysis Model"

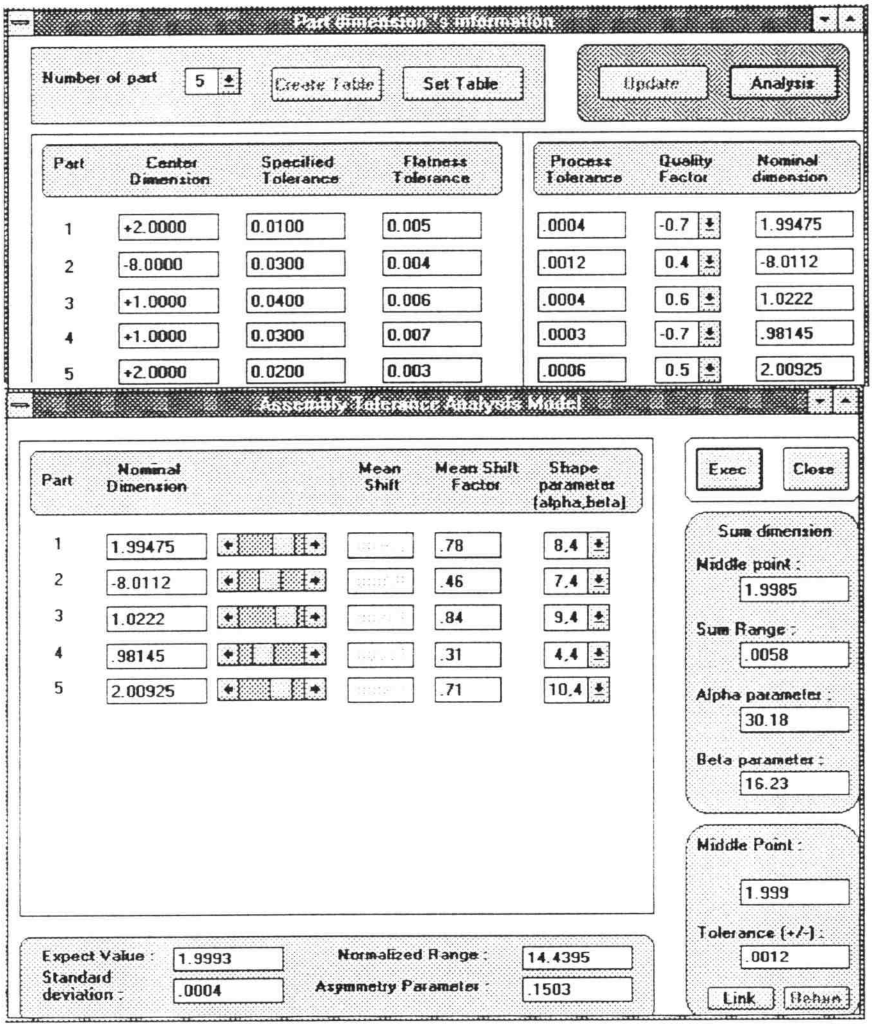

Figure A.13. Application of the Command Button "Exec" to Determine the Unknown Assembly Dimension and Tolerance Based on the Modified Model analysis process. Two user interfaces are created to use in two different processes. The application of the user interfaces in the two processes are described as follows:

1) Drawing Process

During the drawing process, a tolerance database is needed to help the designer assign a realistic manufacturing tolerance. A user interface of tolerance database (Figure A.14) is developed to link the AutoCAD system. Its main application is to pull a dimension from the drawing and update dynamically. To apply this user interface, the user should follow the steps below:

1. Make sure that a drawing and the TK menu is already initiated in the AutoCAD system
2. Click the pull-down menu "DDE".
3. Click "START" and then click "Design_Tol" to open the user interface of tolerance database (Figure A.15)
4. Click "Load_DDE" to establish a link program
5. Click the pull-down menu "Design_Tol" (Figure A.16), an AutoLISP will be loaded and run.
6. Select a dimension in the drawing, then the nominal and tolerance values of the selected dimension will transfer and appear in the appropriate text boxes, in the *Link*

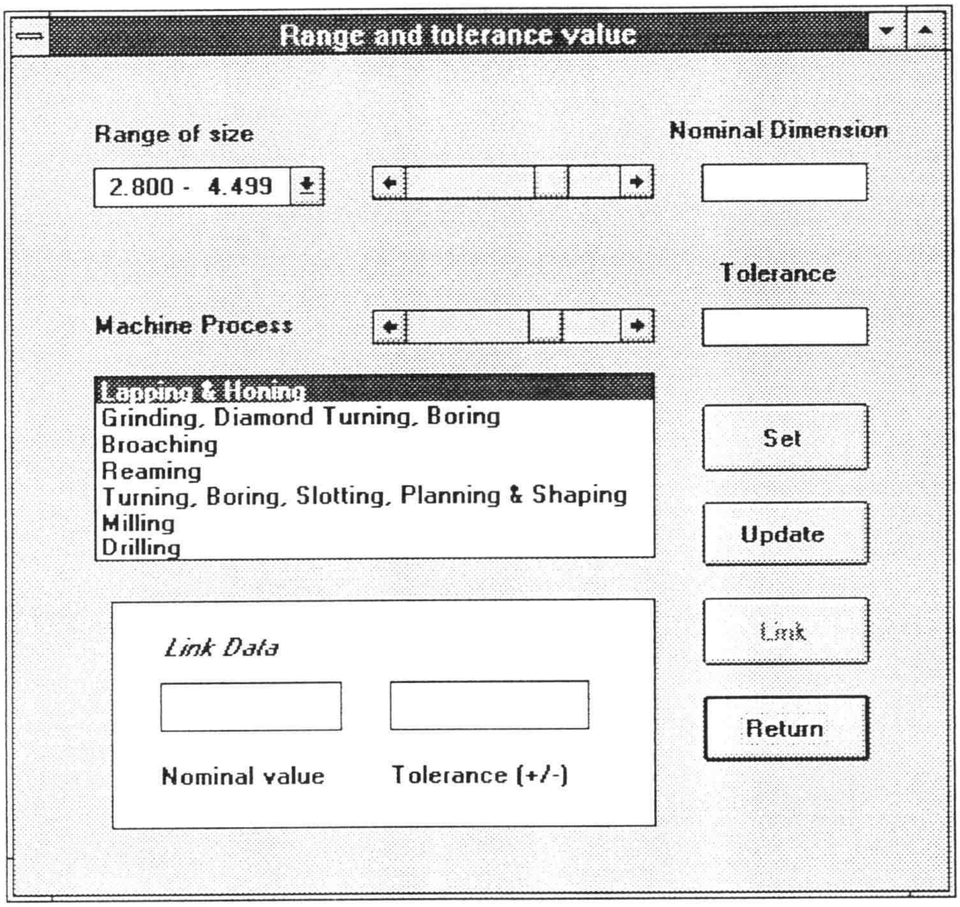

Figure A.14. The User Interface of the Tolerace Database

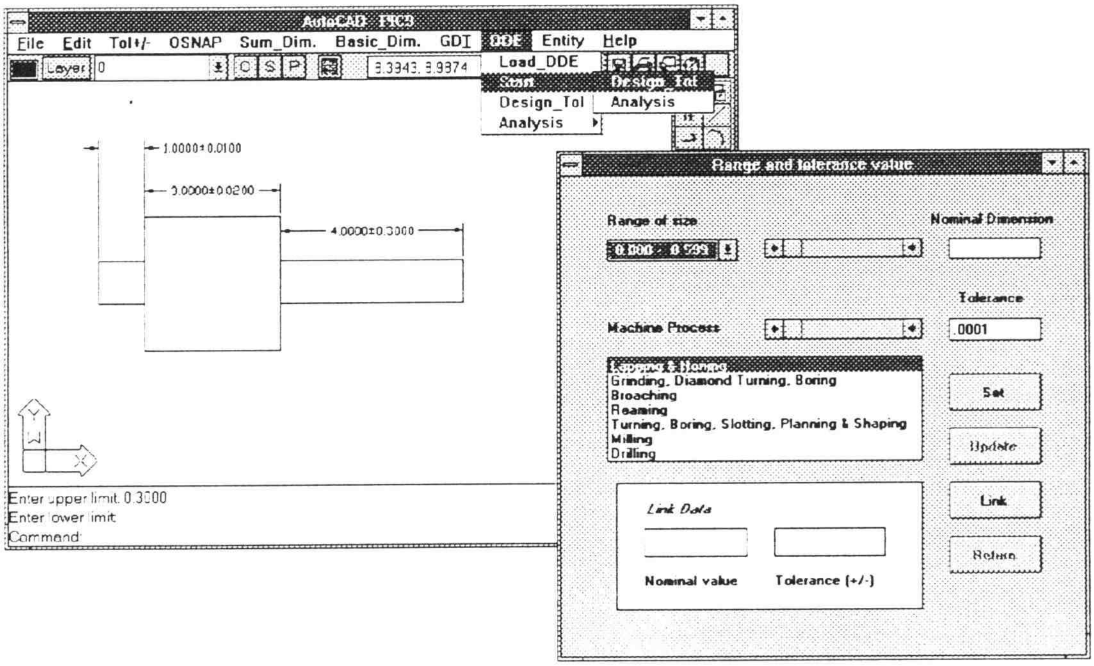

Figure A.15. Application of the Pull-Down Menu "Start" to Open the Tolerance Database Window during Drawing Processes

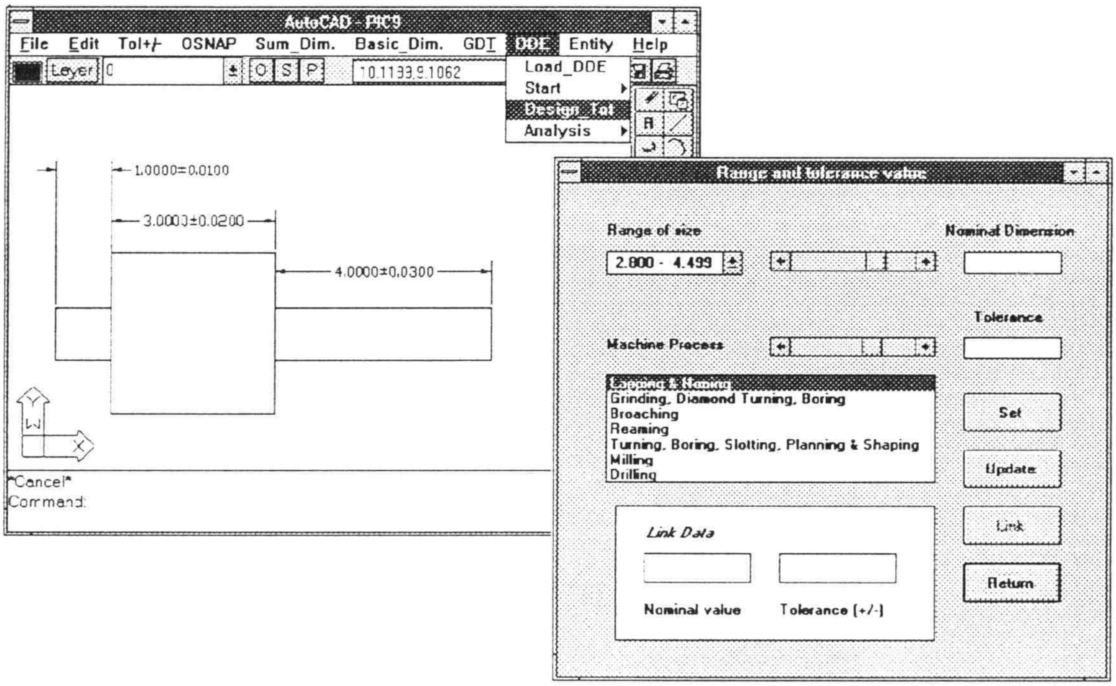

Figure A.16. Application of the Pull-Down Menu "Design_Tol" to Update the Selected Conventional Tolerance "4.0000±0.0300"

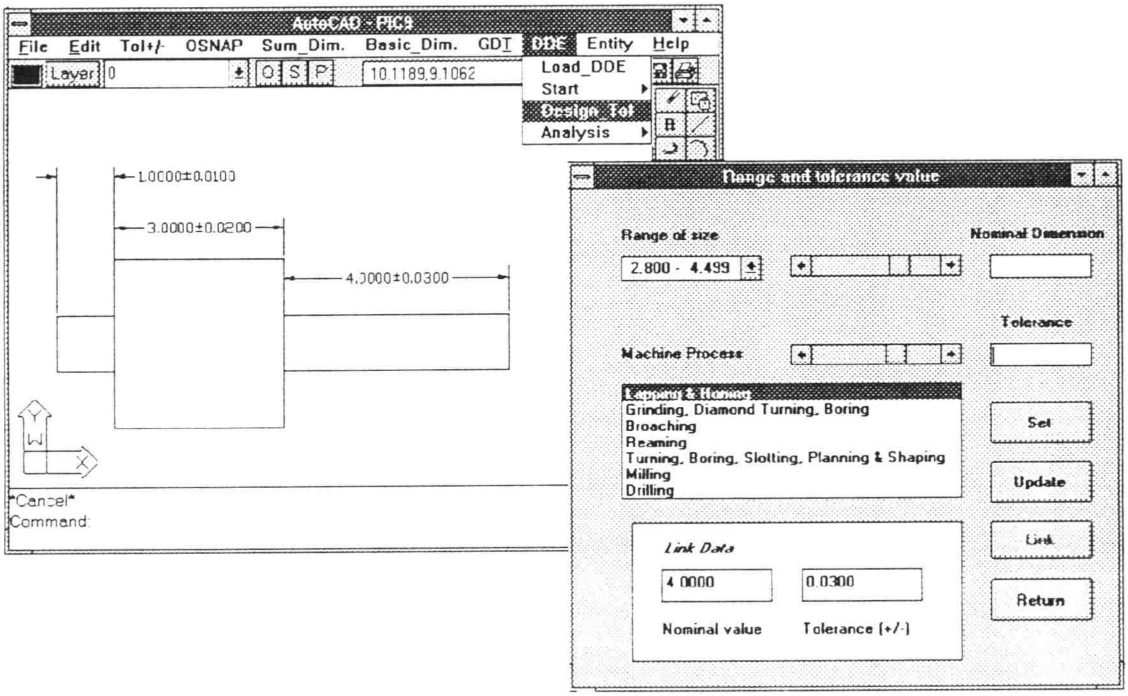

Figure A.17. Transferring "4.0000±0.0300" from the AutoCAD Drawing into the Text Box of "Link Data" Frame

Data frame. As shown in Figure A.17, the dimension 4.000±0.0300 is selected and its values appear in the text boxes.

7. Click "set" command button in the user interface, the nominal and tolerance values from the *Link Data* frame will transfer to the text boxes named *Nominal Dimension* and *Tolerance* in the upper right corner of the window, as displayed in Figure A.18.
8. Select the types of machines in *Machine Process* list box
9. Move the scroll bar on the right side of the *Machine Process* label and the tolerance value in the text box will change from the minimum to the maximum value following the tolerance database in Figure 504. In this example, 4.000±0.0300 is changed to 4.000±0.0001.
10. Click "Update" command button, the updated tolerance will appear in the *Link Data* frame (Figure A.19).
11. Click "Link" command button, making a linking object between two application windows.
12. Click "Return" command button, the selected dimension will be changed from 4.000±0.0300 to 4.000±0.0001, as displayed in Figure A.20.

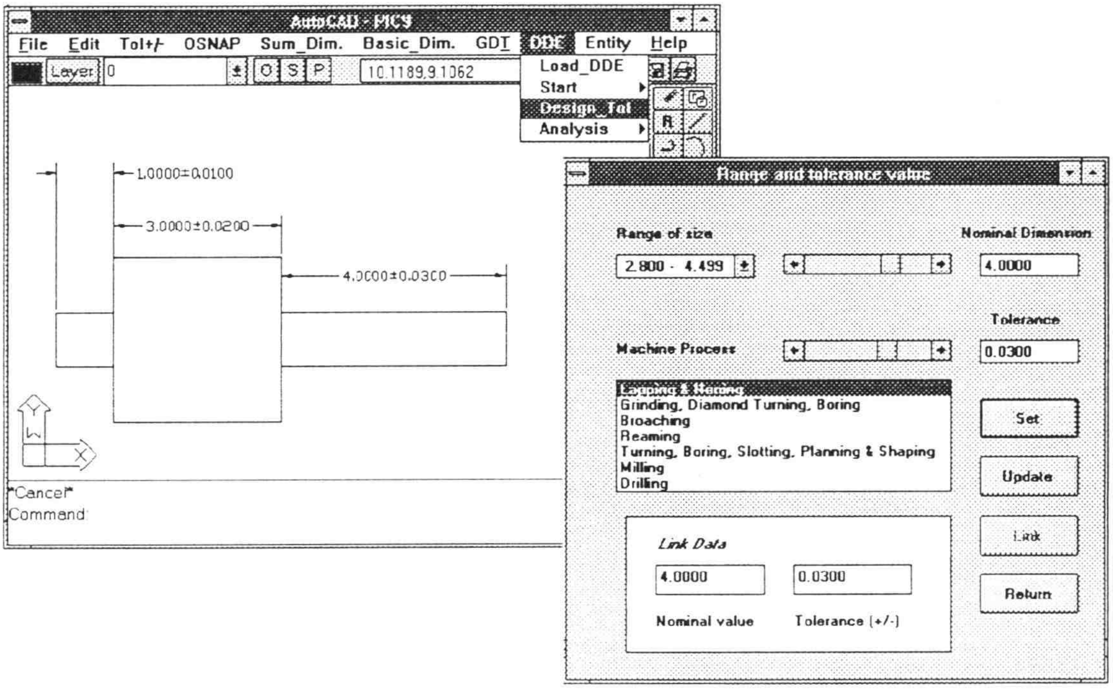

Figure A.18. Application of the Command Button "Set" in Tolerance Database Window

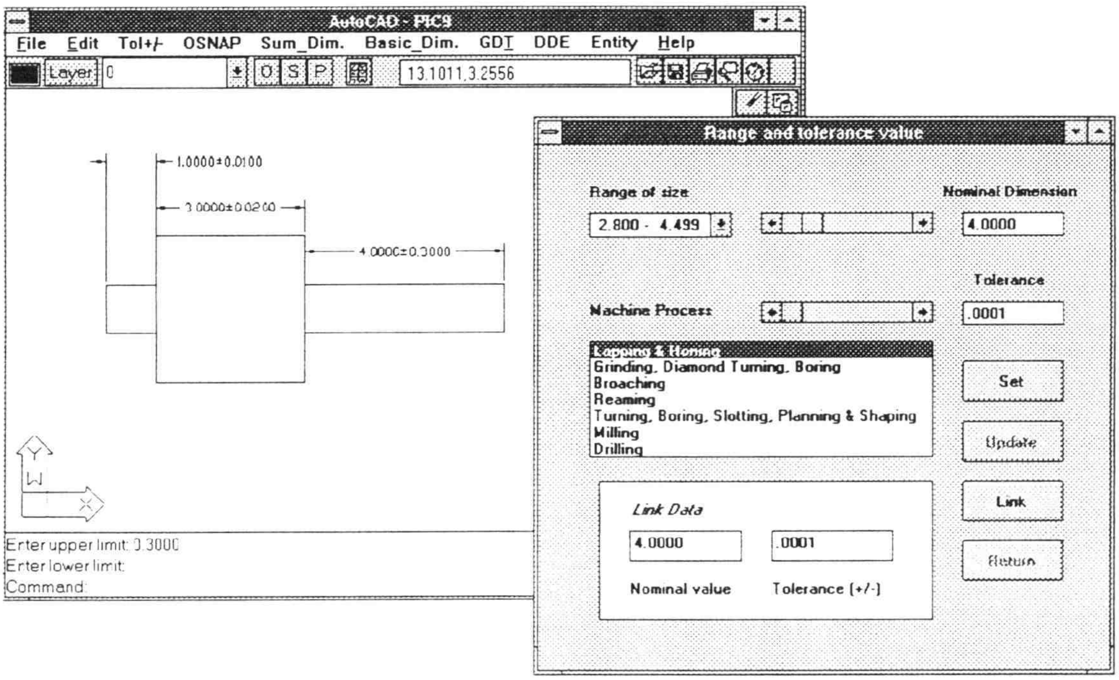

Figure A.19. Application of the Command Button "Update"

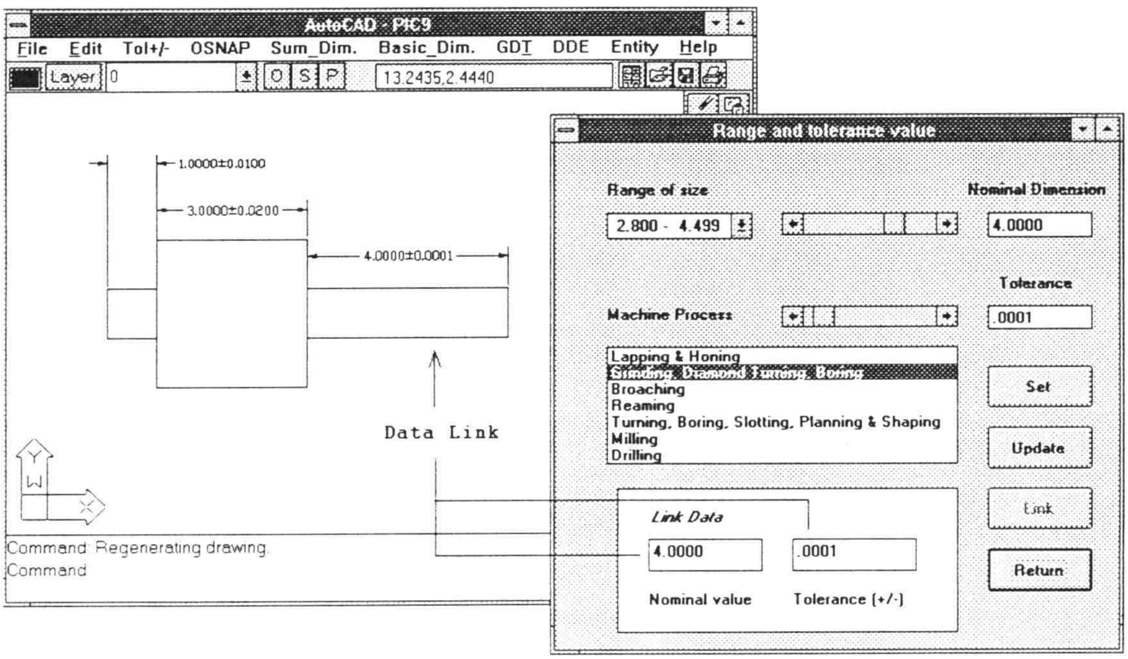

Figure A.20. Application of the Command Button "Return"

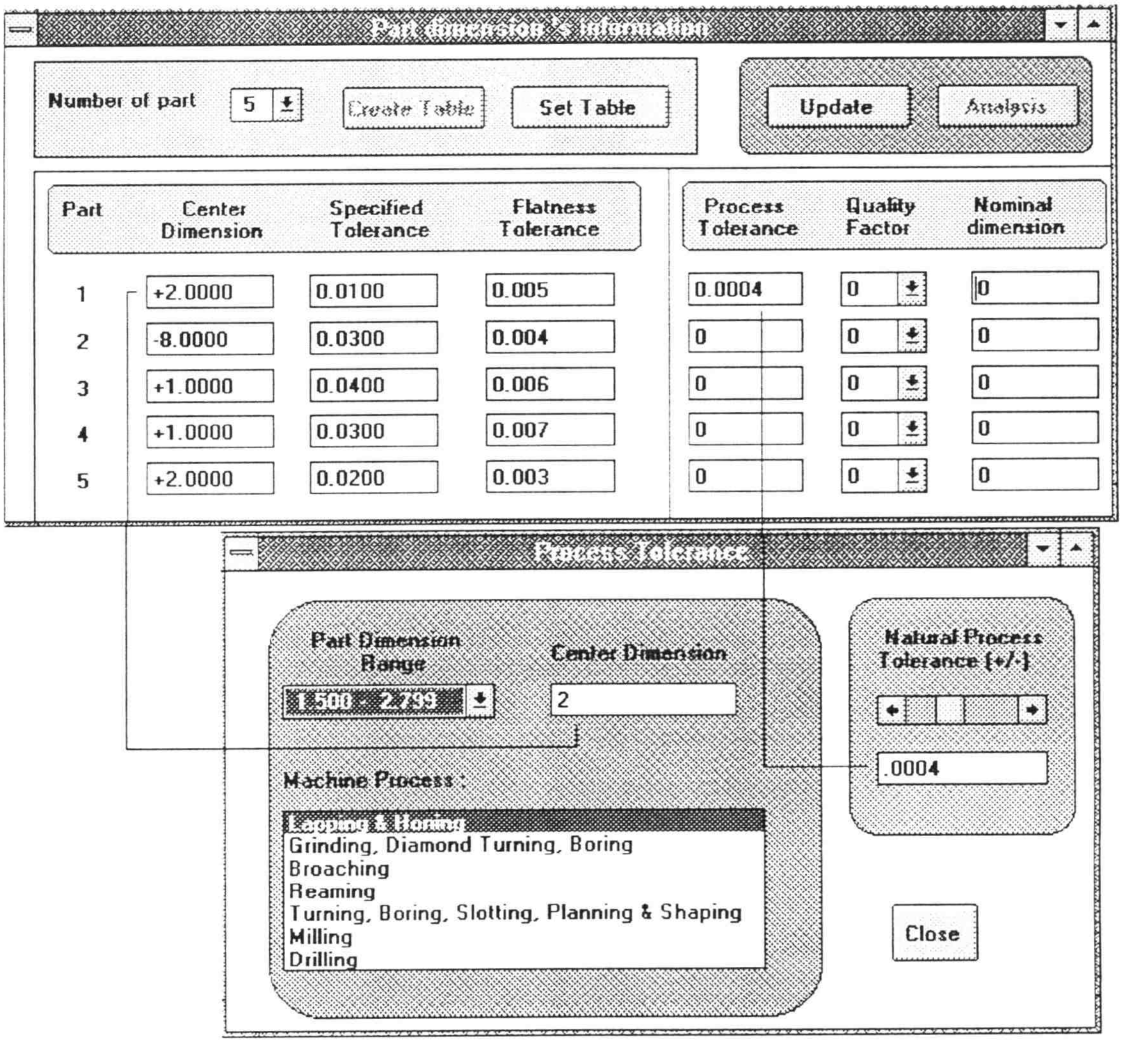

Figure A.21. Opening the Tolerance Database Window by Clicking Twice at the Selected Text Box "Process Tolerance"

2) Assembly Tolerance Analysis Process

The modified assembly tolerance analysis model runs in the user interface because it employs a large number of input variables. First, the window "Part dimension's information" must be initiated by following the steps in section "Determining a Critical Assembly Tolerance", as previously described in this Appendix. Assigning the input variable, the process tolerance (t_i), is required tolerance information, so that a user interface of tolerance database is created to support this requirement. To open the user interface, the user clicks twice at the text box of the process tolerance, as shown in Figure A.21. In the user interface, the tolerance value will change depending on part dimension range and the type of the machining process. Click "Close" command button, the user interface will close and the tolerance value will automatically be inserted in the text box of the selected process tolerance.

APPENDIX B
APPLICATION OF BETA DISTRIBUTION IN PART DIMENSION

Generalized Beta Distribution

The probability density function of a generalized beta distribution in the interval [a,b] is:

$$f(x,\alpha,\beta,a,b) = \frac{1}{(b-a)\,B(\alpha,\beta)}\left(\frac{x-a}{b-a}\right)^{\alpha-1}\left(1-\frac{x-a}{b-a}\right)^{\beta-1}$$

where

$\alpha > 0$, $\beta > 0$ and $a \leq x \leq b$

$B(\alpha, \beta)$ (the gamma function) is defined by the equation:

$$B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

and

$$\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$$

where

E(X) is the expected value of X, and VAR(X) is the variance of X,

$$E(X) = a + \left[(b-a) \times \frac{\alpha}{\alpha+\beta} \right]$$

$$VAR(X) = \frac{(b-a)^2 \alpha \beta}{(\alpha+\beta+1)\,(\alpha+\beta)^2}$$

Determination of Alpha and Beta Parameters with a Symmetrical Shape in Beta Distribution

Let a be the lower limit of interval range [a,b]

let b be the upper limit of interval range [a,b]

let σ be the standard deviation

$$VAR(X) = \frac{(b-a)^2 \alpha \beta}{(\alpha+\beta+1)\,(\alpha+\beta)^2}$$

Since

$$\sigma^2 = VAR(X)$$

Therefore

$$\sigma = \sqrt{\left(\frac{(b-a)^2 \alpha \beta}{(\alpha+\beta+1)\,(\alpha+\beta)^2} \right)} \qquad \text{(b.1)}$$

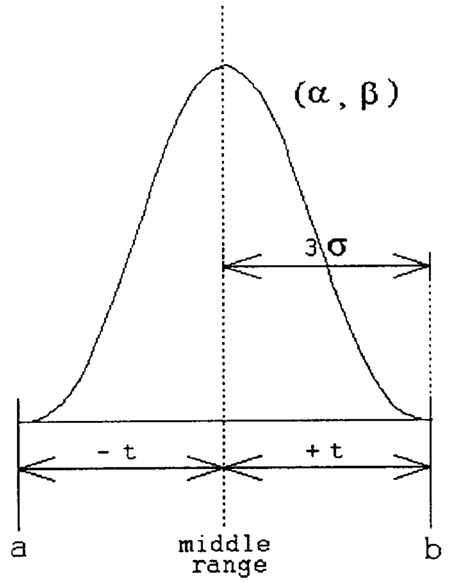

Figure B.1. The Shape of Symmetrical Beta Distribution

In Figure B.1, the process tolerance (t) is one half of the interval range [a,b].

$$2 \times t = (b-a)$$

$$t = \frac{(b-a)}{2} \qquad (b.2)$$

In manufacturing, when a part dimension is assumed to be normally distributed, the process tolerance is often assigned to be equal to 3 times the standard deviation with acceptance rate of 99.73%. To simulate the symmetrical shape in beta distribution, the alpha and beta parameters are specified by using the same rule. Unlike the normal distribution, the beta distribution has a finite range and each part dimension must be within upper and lower limits (interval range [a,b]). For this reason, all parts are assumed to be inspected before they are sent to the assembly line.

Since

$$t = 3\sigma$$

$$t = 3\sqrt{\left(\frac{(b-a)^2\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}\right)} \qquad (b.3)$$

When the shape is symmetric, the alpha and beta parameters are equal so

$$\alpha = \beta \qquad (b.4)$$

Substituting (b.4) in (b.3):

$$t = 3\sqrt{\left(\frac{(b-a)^2\alpha^2}{(2\alpha+1)(2\alpha)^2}\right)} \qquad (b.5)$$

From (b.2) and (b.5), the alpha and beta parameters can be computed as follows:

$$\frac{(b-a)}{2} = 3\sqrt{\left(\frac{(b-a)^2\alpha^2}{(2\alpha+1)\ (2\alpha)^2}\right)}$$

$$\left(\frac{(b-a)}{2}\right)^2 = 3^2\ \frac{(b-a)^2\alpha^2}{(2\alpha+1)\ (2\alpha)^2}$$

$$\frac{(b-a)^2}{4} = 9\ \frac{(b-a)^2\alpha^2}{(2\alpha+1)\ (2\alpha)^2}$$

$$\frac{1}{36} = \frac{1}{4\ (2\alpha+1)}$$

$$36 = 4\ (2\alpha+1)$$

$$9 = 2\alpha+1$$

$$\alpha = 4$$

From (b.4)

$$\beta = 4$$

Therefore, the alpha and beta parameters, which indicate the symmetrical shape in beta distribution, are assigned the same value (4,4).

APPENDIX C
NORMALIZED DIMENSION IN BETA DISTRIBUTION

Tolerances and middle point movements for normalized beta distributions

The following tables[6] give the tolerances and middle point movements for normalized beta distributions at six different confidence levels.

The parameters in the tables are as follows:

The abscissa FW - the asymmetry of a normalized dimension

The ordinate RW - range of normalized dimension

Upper number in pair TW - tolerance of a normalized dimension

Lower number in pair MW - middle point movement of a normalized dimension

Table C.1 Normalized Dimensions (Confidence Level 90.00%) [6]

	-.4	-.3	-.2	-.1	.0	.1	.2	.3	.4
5			3.22 -.72	3.27 -.38	3.28 .00	3.27 .38	3.22 .72		
6			3.25 -.98	3.29 -.51	3.30 .00	3.29 .51	3.25 .98		
7		3.19 -1.74	3.27 -1.22	3.29 -.62	3.30 .00	3.29 .62	3.27 1.22	3.19 1.74	
8		3.21 -2.10	3.28 -1.45	3.29 -.73	3.30 .00	3.29 .73	3.28 1.45	3.21 2.10	
9		3.22 -2.44	3.28 -1.67	3.29 -.84	3.30 .00	3.29 .84	3.28 1.67	3.23 2.44	
10		3.24 -2.77	3.28 -1.88	3.29 -.95	3.30 .00	3.29 .95	3.28 1.88	3.24 2.77	
11		3.25 -3.09	3.28 -2.10	3.29 -1.05	3.30 .00	3.29 1.05	3.28 2.10	3.25 3.09	
12	3.10 -4.35	3.26 -3.41	3.29 -2.31	3.29 -1.16	3.30 .00	3.29 1.16	3.29 2.31	3.65 3.41	3.10 4.35
13	3.12 -4.79	3.26 -3.73	3.29 -2.51	3.29 -1.26	3.29 .00	3.29 1.26	3.29 2.51	3.26 3.73	3.12 4.89
14	3.14 -5.22	3.27 -4.04	3.29 -2.72	3.29 -1.37	3.29 .00	3.29 1.37	3.79 2.72	3.27 4.04	3.14 5.22
15	3.16 -5.65	3.27 -4.35	3.29 -2.93	3.29 -1.47	3.29 .00	3.29 1.47	3.29 2.93	3.27 4.35	3.16 5.65
16	3.18 -6.07	3.27 -4.66	3.29 -3.13	3.29 -1.57	3.29 .00	3.29 1.57	3.29 3.13	3.27 4.66	3.18 6.07
17	3.19 -6.49	3.27 -4.97	3.29 -3.33	3.29 -1.67	3.29 .00	3.29 1.67	3.29 3.33	3.27 4.97	3.19 6.49
18	3.20 -6.91	3.28 -5.28	3.29 -3.54	3.29 -1.77	3.29 .00	3.29 1.77	3.29 3.54	3.28 5.28	3.20 6.91
19	3.21 -7.33	3.28 -5.59	3.29 -3.74	3.29 -1.87	3.29 .00	3.29 1.87	3.29 3.74	3.28 5.59	3.21 7.33
20	3.22 -7.74	3.28 -5.89	3.29 -3.95	3.29 -1.98	3.29 .00	3.29 1.98	3.29 3.95	3.28 5.89	3.22 7.74
25	3.24 -9.79	3.28 -7.41	3.29 -4.96	3.29 -2.48	3.29 .00	3.29 2.48	3.29 4.96	3.28 7.41	3.24 9.79
30	3.26 -11.83	3.28 -8.93	3.29 -5.96	3.29 -2.98	3.29 .00	3.29 2.98	3.29 5.96	3.28 8.93	3.26 11.83

Table C.2 Normalized Dimensions(Confidence Level 95.00%)

	-.4	-.3	-.2	-.1	.0	.1	.2	.3	.4
5			3.63 -.58	3.69 -.32	3.71 .00	3.69 .32	3.63 .58		
6			3.74 -.86	3.78 -.45	3.79 .00	3.78 .45	3.74 .86		
7		3.71 -1.55	3.79 -1.22	3.82 -.58	3.63 .00	3.92 .58	3.79 1.22	3.71 1.55	
8		3.76 -1.92	3.83 -1.36	3.85 -.69	3.86 .00	3.85 .69	3.83 1.36	3.76 1.92	
9		3.79 -2.28	3.85 -1.59	3.87 -.81	3.87 .00	3.87 .81	3.85 1.59	3.79 2.28	
10		3.82 -2.63	3.86 -1.81	3.88 -.92	3.88 .00	3.88 .92	3.86 1.81	3.82 2.63	
11		3.83 -2.96	3.87 -2.03	3.89 -1.03	3.89 .00	3.89 1.03	3.87 2.03	3.83 3.96	
12	3.71 -4.09	3.85 -3.29	3.88 -2.24	3.89 -1.13	3.89 .00	3.89 1.13	3.88 2.24	3.85 3.29	3.71 4.09
13	3.73 -4.54	3.86 -3.62	3.89 -2.46	3.90 -1.24	3.90 .00	3.90 1.24	3.89 2.46	3.86 3.62	3.73 4.54
14	3.75 -4.99	3.87 -3.94	3.89 -2.67	3.90 -1.37	3.90 .00	3.90 1.34	3.89 2.67	3.87 3.94	3.75 4.99
15	3.77 -5.43	3.87 -4.26	3.90 -2.88	3.90 -1.45	3.90 .00	3.90 1.45	3.90 2.88	3.87 4.26	3.77 5.43
16	3.78 -5.87	3.88 -4.57	3.90 -3.09	3.90 -1.55	3.91 .00	3.90 1.55	3.90 3.09	3.88 4.57	3.78 5.87
17	3.80 -6.30	3.88 -4.89	3.90 -3.29	3.91 -1.65	3.91 .00	3.91 1.65	3.90 3.39	3.88 4.89	3.80 6.30
18	3.81 -6.73	3.89 -5.28	3.91 -3.50	3.91 -1.76	3.91 .00	3.91 1.76	3.90 3.50	3.89 5.28	3.81 6.73
19	3.82 -7.15	3.89 -5.51	3.91 -3.70	3.91 -1.86	3.91 .00	3.91 1.86	3.91 3.70	3.89 5.51	3.82 7.15
20	3.83 -7.57	3.89 -5.82	3.91 -3.91	3.91 -1.96	3.91 .00	3.91 1.96	3.91 3.91	3.89 5.82	3.83 7.57
25	3.86 -9.66	3.90 -7.36	3.91 -4.93	3.91 -2.47	3.91 .00	3.91 2.47	3.91 4.93	3.90 7.36	3.86 9.66
30	3.88 -11.72	3.91 -8.83	3.91 -5.94	3.92 -2.97	3.92 .00	3.92 2.97	3.91 5.94	3.91 8.83	3.88 11.72

Table C.3 Normalized Dimensions(Confidence Level 99.00%)[6]

	-.4	-.3	-.2	-.1	.0	.1	.2	.3	.4
5			4.24 -.35	4.30 -.35	4.32 .00	4.30 .20	4.24 .35		
6			4.51 -.63	4.57 -.34	4.59 .00	4.57 .34	4.51 .63		
7		4.61 -1.16	4.68 -.90	4.73 -.48	4.74 .00	4.73 .48	4.68 .90	4.61 1.16	
8		4.72 -1.56	4.79 -1.16	4.83 -.60	4.84 .00	4.83 .60	4.79 1.16	4.72 1.56	
9		4.80 -1.94	4.87 -1.40	4.90 -.73	4.91 .00	4.90 .73	4.87 1.40	4.80 1.94	
10		4.87 -2.31	4.92 -1.64	4.95 -.84	4.96 .00	4.95 .84	4.92 1.64	4.87 2.31	
11		4.91 -2.67	4.96 -1.88	4.99 -.96	4.99 .00	4.99 .96	4.96 1.88	4.91 2.67	
12	4.93 -3.52	4.95 -3.02	4.99 -2.10	5.01 -1.07	5.02 .00	5.01 1.07	4.99 2.10	4.95 3.02	4.93 3.52
13	4.94 -4.00	4.98 -3.36	5.02 -2.32	5.03 -1.18	5.04 .00	5.03 1.18	5.02 2.32	4.98 3.36	4.94 4.00
14	4.96 -4.47	5.00 -3.70	5.04 -2.54	5.05 -1.29	5.05 .00	5.05 1.29	5.04 2.54	5.00 3.70	4.96 4.47
15	4.97 -4.94	5.02 -4.03	5.05 -2.76	5.06 -1.40	5.07 .00	5.06 1.40	5.05 2.76	5.02 4.03	3.97 4.94
16	4.98 -5.40	5.04 -4.36	5.06 -2.98	5.07 -1.50	5.08 .00	5.07 1.50	5.06 2.98	5.04 4.36	4.98 5.40
17	5.00 -5.85	5.05 -4.69	5.07 -3.19	5.08 -1.61	5.09 .00	5.08 1.61	5.07 3.19	5.05 4.69	5.00 5.85
18	5.01 -6.30	5.06 -5.01	5.08 -3.40	5.09 -1.71	5.09 .00	5.09 1.71	5.08 3.40	5.06 5.01	5.01 6.30
19	5.02 -6.74	5.07 -5.33	5.09 -3.61	5.10 -1.82	5.10 .00	5.10 1.82	5.09 3.61	5.07 5.33	5.82 6.74
20	5.83 -7.18	5.06 -5.65	5.09 -3.82	5.10 -1.92	5.10 .00	5.10 1.92	5.09 3.82	5.08 5.65	5.03 7.18
25	5.07 -9.34	5.10 -7.22	5.12 -4.86	5.12 -2.44	5.12 .00	5.12 2.44	5.12 4.86	5.10 7.22	5.07 9.34
30	5.09 -11.45	5.12 -8.77	5.13 -5.88	5.13 -2.95	5.13 .00	5.13 2.95	5.12 5.88	5.12 8.77	5.09 11.45

Table C.4 Normalized Dimensions(Confidence Level 99.50%)[6]

	-.4	-.3	-.2	-.1	.0	.1	.2	.3	.4
5			4.41 -.28	4.46 -.17	4.48 .00	4.46 .17	4.41 .28		
6			4.75 -.55	4.81 -.30	4.83 .00	4.81 .30	4.75 .55		
7		4.91 -1.03	4.97 -1.82	5.02 -.44	5.04 .00	5.02 .44	4.97 0.82	4.91 1.03	
8		5.05 -1.42	5.12 -1.08	5.16 -.57	5.18 .00	5.16 .57	5.12 1.08	5.05 1.42	
9		5.16 -1.81	5.22 -1.33	5.26 -.69	5.27 .00	5.26 .69	5.22 1.33	5.16 1.81	
10		5.23 -2.18	5.29 -1.57	5.33 -.81	5.34 .00	5.33 .81	5.29 1.57	5.23 2.18	
11		5.30 -2.55	5.25 -1.81	5.38 -.93	5.38 .00	5.38 .93	5.35 1.81	5.30 2.55	
12	5.41 -3.29	5.34 -2.90	5.39 -2.04	5.41 -1.04	5.42 .00	5.41 1.04	5.39 2.04	5.34 2.90	5.41 3.29
13	5.40 -3.78	5.38 -3.25	5.42 -2.27	5.44 -1.15	5.45 .00	5.44 1.15	5.42 2.27	5.38 3.25	5.40 3.78
14	5.42 -4.26	5.41 -3.60	5.45 -2.49	5.47 -1.26	5.47 .00	5.47 1.26	5.47 2.49	5.41 3.60	5.42 4.26
15	5.42 -4.74	5.44 -3.94	5.47 -2.71	5.49 -1.37	5.49 .00	5.49 1.37	5.47 2.71	5.44 3.94	5.42 4.74
16	5.43 -5.20	5.46 -4.27	5.49 -2.93	5.50 -1.48	5.51 .00	5.50 1.48	5.49 2.93	5.46 4.27	5.43 5.20
17	5.45 -5.66	5.48 -4.60	5.50 -3.14	5.52 -1.59	5.52 .00	5.52 1.59	5.50 3.14	5.48 4.60	5.45 5.66
18	5.46 -6.12	5.49 -4.93	5.51 -3.36	5.53 -1.69	5.53 .00	5.53 1.69	5.52 3.36	5.49 4.93	5.46 6.12
19	5.47 -6.57	5.50 -5.25	5.53 -3.57	5.53 -1.80	5.54 .00	5.53 1.80	5.53 3.57	5.50 5.25	5.47 6.57
20	5.48 -7.02	5.51 -5.57	5.53 -3.78	5.54 -1.90	5.55 .00	5.54 1.90	5.53 3.78	5.51 5.57	5.48 7.02
25	5.52 -9.20	5.55 -7.16	5.56 -4.83	5.57 -2.42	5.57 .00	5.57 2.42	5.56 4.83	5.55 7.16	5.52 9.20
30	5.54 -11.33	5.57 -8.71	5.58 -5.86	5.58 -2.94	5.58 .00	5.58 2.94	5.58 5.86	5.57 8.71	5.55 11.33

Table C.5 Normalized Dimensions (Confidence Level 99.73%) [6]

	-.4	-.3	-.2	-.1	.0	.1	.2	.3	.4
5			4.52 -.23	4.57 -.14	4.59 .00	4.57 .14	4.52 .23		
6			4.92 -.48	4.98 -.27	5.00 .00	4.98 .27	4.92 .48		
7		5.14 -.92	5.19 -.75	5.24 -.41	5.26 .00	5.24 .41	5.19 .75	5.14 .92	
8		5.31 -1.31	5.37 -1.01	5.42 -.54	5.44 .00	5.42 .54	5.37 1.01	5.31 1.31	
9		5.44 -1.69	5.50 -1.27	5.54 -.66	5.55 .00	5.54 .66	5.50 1.27	5.44 1.69	
10		5.53 -2.07	5.59 -1.51	5.63 -.78	5.64 .00	5.63 .78	5.59 1.51	5.53 2.07	
11		5.60 -2.44	5.66 -1.75	5.69 -.90	5.70 .00	5.69 .90	5.66 1.75	5.60 2.44	
12	5.79 -3.10	5.66 -2.61	5.71 -1.99	5.74 -1.02	5.75 .00	5.74 1.02	5.71 1.99	5.66 2.61	5.79 3.10
13	5.81 -3.58	5.71 -3.16	5.76 -2.22	5.78 -1.13	5.79 .00	5.78 1.13	5.76 2.22	5.71 3.16	5.81 3.58
14	5.81 -4.07	5.75 -3.51	5.79 -2.44	5.81 -1.24	5.82 .00	5.81 1.24	5.79 2.44	5.75 3.51	5.81 4.07
15	5.80 -4.56	5.78 -3.85	5.82 -2.67	[illegible] -1.35	5.84 .00	5.83 1.35	5.82 2.67	5.78 3.85	5.80 4.56
16	5.81 -5.03	5.80 -4.19	5.84 -2.89	5.85 -1.46	5.86 .00	5.85 1.46	5.84 2.89	5.80 4.19	5.81 5.03
17	5.82 -5.50	5.83 -4.52	5.86 -3.10	5.87 -1.57	5.88 .00	5.87 1.57	5.86 3.10	5.83 4.52	5.82 5.50
18	5.83 -5.96	5.84 -4.86	5.87 -3.32	5.88 -1.68	5.89 .00	5.88 1.68	5.87 3.32	5.84 4.86	5.83 5.96
19	5.84 -6.42	5.86 -5.18	5.89 -3.53	5.90 -1.78	5.90 .00	5.90 1.78	5.90 3.53	5.86 5.18	5.84 6.42
20	5.86 -6.87	5.87 -5.51	5.89 -3.75	5.91 -1.89	5.91 .00	5.91 1.89	5.91 3.75	5.87 5.51	5.85 6.87
25	5.90 -9.08	5.92 -7.10	5.93 -4.80	5.94 -2.41	5.94 .00	5.94 2.41	5.93 4.80	5.92 7.10	5.90 9.08
30	5.92 -11.23	5.94 -8.67	5.95 -5.83	5.96 -2.93	5.96 .00	5.96 2.93	5.95 5.83	5.94 8.67	5.92 11.23

Table C.6 Normalized Dimensions(Confidence Level 99.90%)[6]

	-.4	-.3	-.2	-.1	.0	.1	.2	.3	.4
5			4.66 -.17	4.70 -.10	4.72 .00	4.76 .10	4.66 .17		
6			5.15 -.39	5.21 -.23	5.23 .00	5.21 .23	5.15 .39		
7		5.46 -.76	5.49 -.65	5.55 -.36	5.57 .00	5.55 .36	5.49 0.65	5.46 .76	
8		5.67 -1.14	5.72 -.91	5.78 -.49	5.80 .00	5.78 .49	5.72 .91	5.67 1.14	
9		5.83 -1.52	5.89 -1.17	5.94 -.62	5.96 .00	5.94 .62	5.89 1.17	5.83 1.52	
10		5.96 -1.91	6.02 -1.42	6.06 -.74	6.07 .00	6.06 .74	6.02 1.42	5.96 1.91	
11		6.05 -2.28	6.11 -1.66	6.15 -.86	6.16 .00	6.15 .86	6.11 1.66	6.05 2.28	
12	6.32 -2.84	6.13 -2.65	6.18 -1.90	6.22 -.98	6.23 .00	6.22 .98	6.18 1.90	6.13 2.65	6.32 2.84
13	6.41 -3.29	6.19 -3.01	6.24 -2.14	6.27 -1.10	6.28 .00	6.27 1.10	6.24 2.14	6.19 3.01	6.41 3.29
14	6.37 -3.60	6.24 -3.37	6.29 -2.37	6.31 -1.21	6.32 .00	6.31 1.21	6.29 2.37	6.24 3.37	6.37 3.60
15	6.37 -4.29	6.28 -3.72	6.32 -2.60	6.35 -1.32	6.35 .00	6.35 1.32	6.32 2.60	6.28 3.72	6.37 4.29
16	6.39 -4.77	6.32 -4.06	6.35 -2.82	6.37 -1.43	6.38 .00	6.37 1.43	6.35 2.82	6.32 4.06	6.39 4.77
17	6.39 -5.25	6.38 -4.40	6.38 -3.04	6.40 -1.54	6.40 .00	6.41 1.54	6.38 3.04	6.35 4.40	6.39 5.25
18	6.40 -5.72	6.37 -4.74	6.40 -3.26	6.42 -1.65	6.42 .00	6.42 1.65	6.40 3.26	6.37 4.74	6.40 5.72
19	6.41 -6.18	6.39 -5.07	6.42 -3.48	6.43 -1.76	6.44 .00	6.43 1.76	6.42 3.48	6.39 5.07	6.41 6.18
20	6.42 -6.64	6.41 -5.40	6.43 -3.69	6.45 -1.87	6.45 .00	6.45 1.87	6.43 3.69	6.41 5.40	6.42 6.64
25	6.46 -8.89	6.47 -7.02	6.49 -4.75	6.50 -2.39	6.50 .00	6.50 2.39	6.49 4.75	6.47 7.02	6.46 8.88
30	6.49 -11.06	6.50 -8.59	6.52 -5.79	6.52 -2.91	6.52 .00	6.52 2.91	6.51 5.80	6.50 8.59	6.49 11.06

APPENDIX D
DETERMINATION OF ALPHA AND BETA PARAMETERS OF THE SUM DIMENSION

When the beta distribution is assumed for each part dimension, four parameters: the alpha parameter, the beta parameter, the process tolerance, and the middle range of process tolerance must be assigned.

let

$X_{n,i}$ be the nominal dimension of part i

T_i be the flatness tolerance zone of part i

t_i be the process tolerance of part i

f_i be the mean shift factor of part i

MX_i be the middle range of part i

C_i be the inverse process capability index of part i

Referring to Figure 408, the nominal dimension ($X_{n,i}$) is effected by the mean shift factor (f_i). The middle range of part i (MX_i) is determined as follows:

$$MX_i = X_{n,i} + f_i(1-C_i)T_i$$

where

$$C_i = \frac{t_i}{T_i/2}.$$

The mean and the variance of the beta distribution can be determined as follows:

Let

a_i be the minimum dimension of part i

b_i be the maximum dimension of part i

$E(X_i)$ be the expected value of part i

$VAR(X_i)$ be the variance of part i

The middle range of part i is equal to the middle point of interval a and b, and the process tolerance is equal to one half of the range of interval [a,b]:

$$MX_i = \frac{a_i+b_i}{2}$$

$$t_i = \frac{(b_i-a_i)}{2}$$

The expected value and the variance of part i are modified in terms of part dimension as follows:

From Appendix B,

$$E(X_i) = a_i + \left[(b_i - a_i) \times \frac{\alpha_i}{\alpha_i + \beta_i} \right]$$

$$= \frac{a_i + b_i}{2} - \frac{b_i - a_i}{2} + \left[(b_i - a_i) \times \frac{\alpha_i}{\alpha_i + \beta_i} \right]$$

since

$$MX_i = \frac{a_i + b_i}{2} \quad \text{and} \quad t_i = \frac{(b_i - a_i)}{2}$$

Therefore

$$E(X_i) = MX_i - \frac{b_i - a_i}{2} + \left[(b_i - a_i) \times \frac{\alpha_i}{\alpha_i + \beta_i} \right]$$

$$= MX_i + \left[\frac{\alpha_i}{\alpha_i + \beta_i} - \frac{1}{2} \right] (b_i - a_i)$$

$$E(X_i) = MX_i + \left[\frac{\alpha_i}{\alpha_i + \beta_i} - \frac{1}{2} \right] 2\, t_i \qquad (d.1)$$

The varince:

$$VAR(X_i) = \frac{4\,t_i^2\,\alpha_i\,\beta_i}{(\alpha_i+\beta_i+1)\;(\alpha_i+\beta_i)^2}$$

$$VAR(X_i) = \frac{(b_i-a_i)^2\,\alpha_i\,\beta_i}{(\alpha_i+\beta_i+1)\;(\alpha_i+\beta_i)^2} \qquad \text{(d.2)}$$

To investigate the behavior of the assembly sum (critical) dimension, four parameters are determined.

1. The range of the sum dimension.
2. The middle range of the sum dimension.
3. The alpha parameter of the sum dimension.
4. The beta parameter of the sum dimension.

1. <u>The range of the sum dimension</u>

The range of the sum dimension is equal to the sum of the range dimensions, or process tolerance limits of each part component.

let $R\Delta X_\Sigma$ be the range of the sum dimension:

$$R\Delta X_\Sigma = 2\sum_{i=1}^{n} t_i$$

where n is the number of part dimensions.

2. The middle range of the sum dimension

The middle range of the sum dimension is equal to the sum of the middle range of part dimension.

Let $MX_{\Sigma R}$ be the middle range of the sum dimension.

$$MX_{\Sigma R} = \sum_{i=1}^{n} MX_i$$

since

$$MX_i = X_{n,i} + f_i(1-C_i)\,T_i$$

then

$$MX_{\Sigma R} = \sum_{i=1}^{n} X_{n,i} + \sum_{i=1}^{n} f_i(1-C_i)\,T_i$$

3. and 4. The alpha and the beta parameters of the sum dimension

If X_1 and X_2 are random variables with joint pdf $f(x_1, x_2)$ then

$$E(X_1 + X_2) = E(X_1) + E(X_2)$$

If a_1, a_2, a_3... a_k are constants and X_1, X_2.... X_k are jointly distributed random variables, then

$$E\left(\sum_{i=1}^{k} a_i X_i\right) = \sum_{i=1}^{k} a_i E(X_i) \tag{d.3}$$

If X_1 and X_2 are random variables with joint pdf $f(x_1,x_2)$ then:

$$VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2\,Cov(X_1, X_2)$$

and

$$VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2)$$

whenever X_1 and X_2 are independent.

It can also be verified that if $X_1,..,X_k$ are random variables and a_1, a_2, a_3... a_k are constants, then

$$VAR\left(\sum_{i=1}^{k} a_i X_i\right) = \sum_{i=1}^{k} a_i^2 VAR(X_i) + 2\sum\sum_{i<j} a_i a_j Cov(X_i, X_j)$$

and if $X_1,..,X_k$ are independent, then

$$VAR\left(\sum_{i=1}^{k} a_i X_i\right) = \sum_{i=1}^{k} a_i^2 VAR(X_i) \tag{d.4}$$

Assuming that each part dimension is independent and the assembly is linear, the mean and the variance of the sum dimension can be determined by using equations (d.3) and (d.4). First, the alpha and the beta parameters of the sum dimension are computed as follows:

let

α_Σ	be the alpha parameter of the sum dimension
β_Σ	be the beta parameter of the sum dimension
$E(X_\Sigma)$	be the mean of the sum dimension
$VAR\Delta X_\Sigma$	be the variance of the sum dimension

From Appendix B, the mean and the variance can be found as follows:

$$E(X_\Sigma) = MX_{\Sigma R} + \left[\frac{\alpha_\Sigma}{\alpha_\Sigma + \beta_\Sigma} - \frac{1}{2} \right] R\Delta X_\Sigma \tag{d.5}$$

$$VAR\Delta X_\Sigma = \frac{(R\Delta X_\Sigma)^2 \alpha_\Sigma \beta_\Sigma}{(\alpha_\Sigma + \beta_\Sigma + 1)(\alpha_\Sigma + \beta_\Sigma)^2} \tag{d.6}$$

Substituting equations (d.1) and (d.5) in equation (d.3):

$$E(X_\Sigma) = \sum_{i=1}^{n} E(X_i) \tag{d.7}$$

Substituting equations (d.2) and (d.6) in equation

$$VAR\Delta X_{\Sigma} = \sum_{i=1}^{n} VAR(X_i) \qquad (d.8)$$

where

n is the number of part dimensions

Finally, the two unknown variables, α_{Σ} and β_{Σ}, are mathematically found (equations d.1, d.3, d.5 and d.7) as follows:

$$MX_{\Sigma R} + \left[\frac{\alpha_{\Sigma}}{\alpha_{\Sigma}+\beta_{\Sigma}} - \frac{1}{2}\right] R\Delta X_{\Sigma} = \sum_{i=1}^{n}\left(MX_i + \left[\frac{\alpha_i}{\alpha_i+\beta_i} - \frac{1}{2}\right] 2\, t_i\right)$$

$$MX_{\Sigma R} + \left[\frac{\alpha_{\Sigma}}{\alpha_{\Sigma}+\beta_{\Sigma}} - \frac{1}{2}\right] R\Delta X_{\Sigma} = \sum_{i=1}^{n} MX_i + \sum_{i=1}^{n}\left[\frac{\alpha_i}{\alpha_i+\beta_i} - \frac{1}{2}\right] 2\, t_i$$

$$MX_{\Sigma R} = \sum_{i=1}^{n} MX_i \quad \text{and} \quad R\Delta X_{\Sigma} = \sum_{i=1}^{n} 2\, t_i$$

then,

$$\left[\frac{\alpha_\Sigma}{\alpha_\Sigma+\beta_\Sigma} - \frac{1}{2}\right] R\Delta X_\Sigma = \sum_{i=1}^{n}\left[\frac{\alpha_i}{\alpha_i+\beta_i}-\frac{1}{2}\right] 2\,t_i$$

$$\frac{\alpha_\Sigma}{\alpha_\Sigma+\beta_\Sigma}\sum_{i=1}^{n} 2\,t_i - \sum_{i=1}^{n} t_i = \sum_{i=1}^{n}\frac{\alpha_i}{\alpha_i+\beta_i} 2\,t_i - \sum_{i=1}^{n} t_i$$

$$\frac{\alpha_\Sigma}{\alpha_\Sigma+\beta_\Sigma}\sum_{i=1}^{n} t_i = \sum_{i=1}^{n}\frac{\alpha_i}{\alpha_i+\beta_i}\,t_i \qquad \text{(d.9)}$$

As discussed in Appendix B, β may be kept constant (equal to 4) and α is assumed to be an integer not less than 4. Equation d.9 results in:

$$\frac{\alpha_\Sigma}{\alpha_\Sigma+\beta_\Sigma}\sum_{i=1}^{n} t_i = \sum_{i=1}^{n}\frac{\alpha_i}{\alpha_i+4}\,t_i \qquad \text{(d.10)}$$

where

α is an integer and not less than 4.

From equations (d.2), (d.4), (d.6), and (d.8):

$$\frac{(R\Delta X_\Sigma)^2\,\alpha_\Sigma\,\beta_\Sigma}{(\alpha_\Sigma+\beta_\Sigma+1)\,(\alpha_\Sigma+\beta_\Sigma)^2} = \sum_{i=1}^{n}\frac{4\,t_i^2\,\alpha_i\,\beta_i}{(\alpha_i+\beta_i+1)\,(\alpha_i+\beta_i)^2}$$

Since $\beta = 4$ then

$$\frac{\left(\sum_{i=1}^{n} 2\, t_i\right)^2 \alpha_\Sigma \beta_\Sigma}{(\alpha_\Sigma+\beta_\Sigma+1)\,(\alpha_\Sigma+\beta_\Sigma)^2} = 16 \sum_{i=1}^{n} \frac{t_i^2 \alpha_i}{(\alpha_i+5)\,(\alpha_i+4)^2} \qquad \text{(d.11)}$$

To solve both (d.10) and (d.11), the following procedure is followed:

let

$$q_i = \alpha_i + 4$$

$$J = \sum_{i=1}^{n} \frac{t_i}{q_i}$$

$$M = \sum_{i=1}^{n} t_i$$

$$A = \sum_{i=1}^{n} \frac{t_i^2}{q_i}$$

$$B = \sum_{i=1}^{n} \left(\frac{t_i}{q_i}\right)^2$$

$$C = \sum_{i=1}^{n} \frac{t_i^2}{q_i+1}$$

q_i, J, and M are substituted in equation d.10

$$\frac{\alpha_\Sigma}{\alpha_\Sigma+\beta_\Sigma}\sum_{i=1}^{n} t_i = \sum_{i=1}^{n}\frac{\alpha_i}{\alpha_i+4}\,t_i$$

$$\frac{\alpha_\Sigma}{\alpha_\Sigma+\beta_\Sigma}M = \sum_{i=1}^{n}\frac{q_i-4}{q_i}\,t_i$$

$$\frac{\alpha_\Sigma}{\alpha_\Sigma+\beta_\Sigma}M = \sum_{i=1}^{n} t_i - 4\sum_{i=1}^{n}\frac{t_i}{q_i}$$

$$\frac{\alpha_\Sigma}{\alpha_\Sigma+\beta_\Sigma}M = M - 4J$$

$$\alpha_\Sigma M = (M-4J)(\alpha_\Sigma+\beta_\Sigma)$$

$$\beta_\Sigma = \frac{4J}{M-4J}\alpha_\Sigma \qquad \text{(d.12)}$$

Substituting q_i, A, B, C, and M in equation (d.11):

$$\frac{\left(\sum_{i=1}^{n} 2\,t_i\right)^2 \alpha_\Sigma\beta_\Sigma}{(\alpha_\Sigma+\beta_\Sigma+1)\,(\alpha_\Sigma+\beta_\Sigma)^2} = 16\sum_{i=1}^{n}\frac{t_i^2\,\alpha_i}{(\alpha_i+5)\,(\alpha_i+4)^2}$$

$$\frac{M^2 \alpha_\Sigma \beta_\Sigma}{(\alpha_\Sigma+\beta_\Sigma+1)\,(\alpha_\Sigma+\beta_\Sigma)^2} = 4 \sum_{i=1}^{n} \frac{(q_i-4)}{(q_i+1)\,(q_i)^2}\, t_i^2$$

$$\frac{M^2 \alpha_\Sigma \beta_\Sigma}{(\alpha_\Sigma+\beta_\Sigma+1)\,(\alpha_\Sigma+\beta_\Sigma)^2} = 4 \sum_{i=1}^{n} \left(\frac{5q_i-4}{q_i^2} - \frac{5}{(q_i+1)} \right) t_i^2$$

$$\frac{M^2 \alpha_\Sigma \beta_\Sigma}{(\alpha_\Sigma+\beta_\Sigma+1)\,(\alpha_\Sigma+\beta_\Sigma)^2} = 4 \left(\sum_{i=1}^{n} \frac{5}{q_i} t_i^2 - \sum_{i=1}^{n} \frac{4}{q_i^2} t_i^2 - \sum_{i=1}^{n} \frac{5}{(q_i+1)} t_i^2 \right)$$

$$\frac{M^2 \alpha_\Sigma \beta_\Sigma}{(\alpha_\Sigma+\beta_\Sigma+1)\,(\alpha_\Sigma+\beta_\Sigma)^2} = 4\,[5A-4B-5C]$$

Using equation (d.12), α and β parameters are solved:

$$\frac{M^2 \alpha_\Sigma \frac{4J}{M-4J} \alpha_\Sigma}{\left(\alpha_\Sigma + \frac{4J}{M-4J}\alpha_\Sigma + 1\right)\left(\alpha_\Sigma + \frac{4J}{M-4J}\alpha_\Sigma\right)^2} = 4\,[5A-4B-5C]$$

$$\frac{(M-4J)^2 J}{\alpha_\Sigma M+M-4J} = 5A-4B-5C$$

let

$$K = 5A - 4B - 5C$$

then

$$(M-4J)^2 J = K\alpha_\Sigma M + KM - 4JK$$

Finally,

$$\alpha_\Sigma = \frac{(M-4J)^2 J-(M-4J)K}{KM} \quad \text{(d.13)}$$

and

$$\beta_\Sigma = 4\left(\frac{(M-4J)J^2-JK}{KM}\right) \quad \text{(d.14)}$$