Robotics I

June 5, 2015

Exercise 1

Consider a helix path whose parametrization is given by

$$\boldsymbol{p}(s) = \begin{pmatrix} x(s) \\ y(s) \\ z(s) \end{pmatrix} = \begin{pmatrix} r\,(\cos s - 1) + x_0 \\ r \sin s + y_0 \\ ks + z_0 \end{pmatrix}, \qquad s \in \mathbb{R}, \tag{1}$$

and let two Cartesian points $\boldsymbol{P}_A = \begin{pmatrix} p_{Ax} & p_{Ay} & p_{Az} \end{pmatrix}^T$ and $\boldsymbol{P}_B = \begin{pmatrix} p_{Bx} & p_{By} & p_{Bz} \end{pmatrix}^T$ be assigned. Define an interval $s \in [0, s_{\max}]$ and scalar values r, k, x_0, y_0, and z_0 in (1) such that $\boldsymbol{p}(0) = \boldsymbol{P}_A$ and $\boldsymbol{p}(s_{\max}) = \boldsymbol{P}_B$. Moreover, associate to this path a rest-to-rest timing law given by a cubic polynomial $s = s(t)$, $t \in [0, T]$, where T is the total motion time.

- Does the trajectory interpolation problem always have a solution? Is the solution unique?
- Determine a path (1) that solves the above problem for the numerical data $\boldsymbol{P}_A = \begin{pmatrix} 0 & 2 & -10 \end{pmatrix}^T$ and $\boldsymbol{P}_B = \begin{pmatrix} -2 & 0 & 10 \end{pmatrix}^T$. Compute the expression of the curvature $\kappa(s)$ of this path.
- For the chosen timing law, provide the expressions of $\dot{\boldsymbol{p}}(t)$ and $\ddot{\boldsymbol{p}}(t)$, and determine the minimum time T that realizes the interpolation under the constraint $\|\dot{\boldsymbol{p}}(t)\| \leq V_{\max}$.

Exercise 2

Consider a 3R elbow-type robot having its base mounted on the plane $z = 0$. The shoulder joint is at a height $\ell_1 = 5$. The links 2 and 3 have equal lengths $\ell_2 = \ell_3 = 10$.

- Place the robot base at a point (x_b, y_b) on the plane $z = 0$ so that the end-effector is capable of executing the solution path of Exercise 1.
- Find a robot configuration $\boldsymbol{q} = \boldsymbol{q}^*$ at which the end-effector is placed in the (single) point of path (1) where the norm of the Cartesian velocity $\dot{\boldsymbol{p}}$ in the minimum time trajectory of Exercise 1 has its maximum value.
- Compute at $\boldsymbol{q}^*$ the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^3$ of the robot that realizes the desired velocity $\dot{\boldsymbol{p}}$ of the above minimum time trajectory.

[150 minutes; open books]