Robotics I

January 9, 2015

Exercise 1

A planar 2R robot with links of length $l_1 = 0.1492$ m and $l_2 = 0.1905$ m and actuated by direct-drive motors is equipped at the two joints with incremental encoders, providing respectively 8192 and 4096 pulses per turn. When the robot is in the nominal configuration $\hat{\theta}_1 = 45°$, $\hat{\theta}_2 = -60°$, determine the maximum uncertainty (in norm) that affects the measure of the Cartesian end-effector position.

Exercise 2

Consider a 2-dof planar RP robot with the following kinematic constraints:

$$\begin{array}{rll} \text{joint ranges} & q_1 \in [0, 120°]\,, & q_2 \in [0.5, 1] \;\text{[m]}, \\ \text{joint velocity limits} & |\,\dot{q}_1| \leq 40°/\text{s}, & |\,\dot{q}_2| \leq 1.5 \;\text{[m/s]}. \end{array} \tag{1}$$

Assume that both joint velocities can switch their value instantaneously (in practice, this simplifying assumption is reasonable when the physical limits on joint accelerations are very high). Plan a straight line trajectory between two points in the Cartesian space (say, $\boldsymbol{A}$ and $\boldsymbol{B}$) such that *i)* the entire path belongs to the robot workspace, *ii)* the path has the maximum possible length, *iii)* the trajectory satisfies the velocity limits in (1), and *iv)* the transfer from $\boldsymbol{A}$ to $\boldsymbol{B}$ is realized in minimum time T (provide this value).

Exercise 3

A 3R anthropomorphic robot is characterized by the D-H parameters given in Tab. 1.

i	α_i	a_i [m]	d_i	θ_i
1	$\pi/2$	0	0	θ_1
2	0	1.5	0	θ_2
3	0	1	0	θ_3

Table 1: Denavit-Hartenberg parameters of the 3R robot

A desired trajectory $\boldsymbol{p}_d(t)$ is specified for the position $\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{\theta})$ of the robot end effector as a straight line rest-to-rest motion from point $\boldsymbol{A} = \begin{pmatrix} 0 & -2 & 0.5 \end{pmatrix}^T$ to point $\boldsymbol{B} = \begin{pmatrix} 1 & 0 & 0.5 \end{pmatrix}^T$ [m], with a trapezoidal velocity law having maximum speed $v_{\max} = 0.5$ [m/s] and maximum acceleration $a_{\max} = 5$ [m/s^2]. The initial configuration of the robot is $\boldsymbol{\theta}(0) = \begin{pmatrix} -\pi/2 & 0 & \pi/6 \end{pmatrix}^T$. Let the joint velocity $\dot{\boldsymbol{\theta}}$ be the command input. Design a controller so that the robot asymptotically tracks the desired trajectory. Furthermore, determine also the smallest feedback gains in the control law so that the norm of the Cartesian error $\boldsymbol{e} = \boldsymbol{p}_d - \boldsymbol{p}$ is brought definitely below 5% of the initial value $\|\boldsymbol{e}(0)\|$ as soon as *one fourth* of the nominal motion time of the desired trajectory has passed. Provide the expressions of all terms involved in the control law. Sketch the time evolution of the three Cartesian error components e_x, e_y and e_z. Does the robot encounter singular configurations during motion? Will all robot joints move while performing this control task?

[210 minutes; open books]

Solution

January 9, 2015

Exercise 1

The limited accuracy in the indirect measure of the end-effector position is due to the resolution of the incremental encoders, and is related to the robot Jacobian in the nominal configuration $\hat{\boldsymbol{\theta}}$. We have (with the usual shorthand notation)

$$\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{\theta}) = \begin{pmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{pmatrix} \Rightarrow \boldsymbol{J}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{f}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \begin{pmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{pmatrix}.$$

From the Taylor expansion, it is

$$\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{\theta}) \approx \boldsymbol{f}(\hat{\boldsymbol{\theta}}) + \boldsymbol{J}(\boldsymbol{\theta})\big|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} \left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}\right) = \hat{\boldsymbol{p}} + \boldsymbol{J}(\hat{\boldsymbol{\theta}})\boldsymbol{\Delta\theta}. \tag{2}$$

From the given data, it is

$$\hat{\boldsymbol{p}} = \begin{pmatrix} 0.2895 \\ 0.0562 \end{pmatrix} [\text{m}], \qquad \boldsymbol{\Delta\theta} = \begin{pmatrix} \pm 2\pi/8192 \\ \pm 2\pi/4096 \end{pmatrix} [\text{rad}].$$

The small joint position uncertainty due to the resolution of the encoders can be applied in two different ways to (2), depending on the choice of signs in the components of $\boldsymbol{\Delta\theta}$ —*never* use degrees here! These signs are either the same (say, positive, leading to $\boldsymbol{\Delta\theta}_1$) or opposite (say, the first negative and second negative, leading to $\boldsymbol{\Delta\theta}_2$). The two other combinations lead to values of $\boldsymbol{\Delta p} = \boldsymbol{p} - \hat{\boldsymbol{p}}$ which are the opposite of what already found, and so with same norms. We have thus

$$\boldsymbol{\Delta p}_1 = \boldsymbol{p}_1 - \hat{\boldsymbol{p}} = \boldsymbol{J}(\hat{\boldsymbol{\theta}})\boldsymbol{\Delta\theta}_1 = \begin{pmatrix} -0.0562 & 0.0493 \\ 0.2895 & 0.1840 \end{pmatrix} \begin{pmatrix} 0.0008 \\ 0.0015 \end{pmatrix} = \begin{pmatrix} 0.0325 \\ 0.5043 \end{pmatrix} [\text{mm}]$$

and

$$\boldsymbol{\Delta p}_2 = \boldsymbol{p}_2 - \hat{\boldsymbol{p}} = \boldsymbol{J}(\hat{\boldsymbol{\theta}})\boldsymbol{\Delta\theta}_2 = \begin{pmatrix} -0.0562 & 0.0493 \\ 0.2895 & 0.1840 \end{pmatrix} \begin{pmatrix} 0.0008 \\ -0.0015 \end{pmatrix} = \begin{pmatrix} -0.1187 \\ -0.0602 \end{pmatrix} [\text{mm}].$$

Therefore,

$$\max \|\boldsymbol{\Delta p}\| = \max\left\{\|\boldsymbol{\Delta p}_1\|, \|\boldsymbol{\Delta p}_2\|\right\} = \max\left\{0.5054, 0.1331\right\} = 0.5054\,[\text{mm}],$$

i.e., the maximum Cartesian uncertainty is about half a millimeter (which makes sense). Note that the given data are the actual ones for the Quanser underactuated robot (*Pendubot*) available in the Robotics Lab at DIAG.

Exercise 2

Drawing the workspace WS of the planar RP robot based on the joint ranges in (1), we obtain part of a circular sector with inner radius 0.5 m and outer radius 1 m. With reference to Fig. 1, the longest segment contained in this workspace is $\overline{AB}$ (tangent to the inner boundary of WS at point E), which connects two vertices of the admissible area. It is

$$\boldsymbol{A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \boldsymbol{B} = \begin{pmatrix} -0.5 \\ \dfrac{\sqrt{3}}{2} \end{pmatrix}, \qquad L = \|\boldsymbol{B} - \boldsymbol{A}\| = \sqrt{3} \approx 1.7321 \text{ [m]}.$$

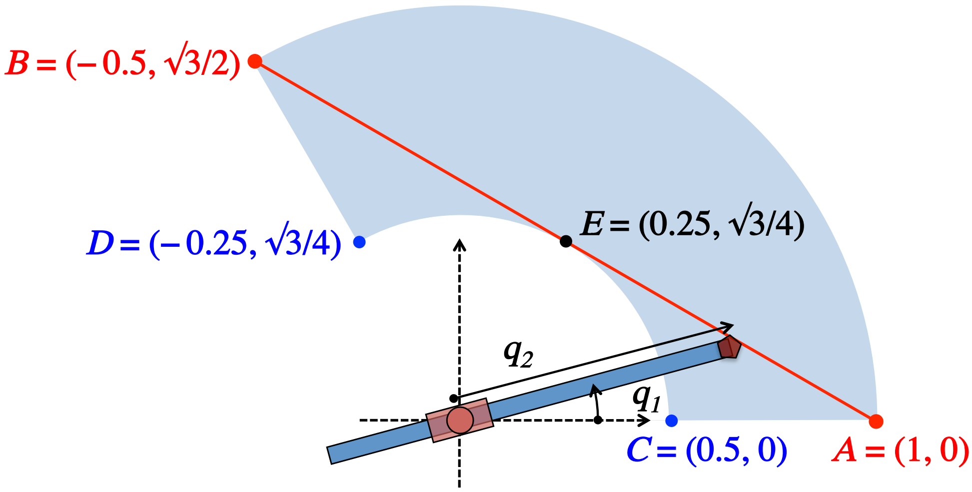

Figure 1: Workspace of the planar RP robot with the segment $\overline{AB}$ of maximum length as path

The desired Cartesian path and velocity can be parametrized as follows:

$$\boldsymbol{p}_d(s) = \boldsymbol{A} + \frac{\boldsymbol{B}-\boldsymbol{A}}{L}\, s = \begin{pmatrix} 1 - \dfrac{1.5\, s}{\sqrt{3}} \\ 0.5\, s \end{pmatrix}, \quad s \in [0, L]; \quad \dot{\boldsymbol{p}}_d(s) = \frac{\boldsymbol{B}-\boldsymbol{A}}{L}\, \dot{s} = \begin{pmatrix} -\dfrac{1.5}{\sqrt{3}} \\ 0.5 \end{pmatrix} \dot{s}. \tag{3}$$

The direct and inverse kinematics of the PR robot are[1]

$$\boldsymbol{p} = \begin{pmatrix} p_x \\ p_y \end{pmatrix} = \begin{pmatrix} q_2 \cos q_1 \\ q_2 \sin q_1 \end{pmatrix} = \boldsymbol{f}(\boldsymbol{q}) \quad \Rightarrow \quad \boldsymbol{q} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} \text{ATAN2}\{p_y, p_x\} \\ \sqrt{p_x^2 + p_y^2} \end{pmatrix} = \boldsymbol{f}^{-1}(\boldsymbol{p}), \tag{4}$$

where we have chosen only the positive solution for q_2. Corresponding to points $\boldsymbol{A}$, $\boldsymbol{E}$ (midpoint of the trajectory), and $\boldsymbol{B}$, we have thus

$$\boldsymbol{q}_A = \boldsymbol{f}^{-1}(\boldsymbol{A}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \boldsymbol{q}_E = \boldsymbol{f}^{-1}(\boldsymbol{E}) = \begin{pmatrix} 60^\circ \\ 0.5 \end{pmatrix}, \quad \boldsymbol{q}_B = \boldsymbol{f}^{-1}(\boldsymbol{B}) = \begin{pmatrix} 120^\circ \\ 1 \end{pmatrix}.$$

Finally, the differential kinematics of the PR robot is

$$\dot{\boldsymbol{p}} = \begin{pmatrix} -q_2 \sin q_1 & \cos q_1 \\ q_2 \cos q_1 & \sin_1 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = \begin{pmatrix} -p_y & \cos q_1 \\ p_x & \sin q_1 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}. \tag{5}$$

From the robot type and the shape of the path, the joint trajectories will display some symmetry in time while moving from $\boldsymbol{A}$ to $\boldsymbol{E}$ and from $\boldsymbol{E}$ to $\boldsymbol{B}$. Moreover, considering the numerical values of the velocity limits, it is clear that the revolute joint will need more time to complete its motion. Joint 1 will thus proceed at maximum positive speed, switching from rest to $V_1 = 40$ rad/s at $t = 0$ and vice versa at the (yet unknown) final time $t = T$. Simultaneously, the prismatic joint will reduce its extension during the first half of the trajectory and reverse this motion during the

[1] We have not used here the standard DH coordinate θ_1 as q_1. In that case, everything would remain the same modulo a clockwise rotation of WS and of the planned path by $\pi/2$ around the Cartesian origin.

second half, so as to *keep the robot end effector on the linear Cartesian path* between $\boldsymbol{A}$ and $\boldsymbol{B}$. In particular, the velocity of joint 2 in the segment from $\boldsymbol{A}$ to $\boldsymbol{E}$ (reached at $t = T/2$) will be negative (but neither at its maximum value nor constant, otherwise the end effector would not travel along the straight Cartesian path). The velocity profile will mirror itself for $t = (T/2, T]$ according to the rule $\dot{q}_2(t) = -\dot{q}_2((T/2) - t)$.

For this intuitively described trajectory to be also the desired *time optimal* solution, we just need to compute the resulting velocity of joint 2 and check its feasibility against the limit $V_2 = 0.5$ m/s during the entire motion interval $[0, T]$. The time profile of the first joint is

$$q_{d1}(t) = q_{d1}(0) + V_1 t, \quad t \in [0, T], \quad \text{with } q_{d1}(0) = 0 \qquad \Rightarrow \qquad T = \frac{\Delta q_1}{V_1} = \frac{120°}{40°/\text{s}} = 3 \text{ s}. \tag{6}$$

A closed-form solution for the time profile $q_{d2}(t)$ of joint 2 and for the timing law $s(t)$ along the Cartesian path are obtained with the following method, which provides also $\dot{q}_{d2}(t)$ and $\dot{s}(t)$:

1. For each instant t (sampling uniformly the interval $[0, T]$, say every $T_c = 1$ ms), equate the desired path position $\boldsymbol{p}_d(s)$, expressed from the task side by (3) as a function of s, with the direct kinematics of the end effector, as given by (4) from the robot side:

$$\begin{pmatrix} 1 - \dfrac{1.5\,s}{\sqrt{3}} \\ 0.5\,s \end{pmatrix} = \begin{pmatrix} q_2 \cos q_{d1} \\ q_2 \sin q_{d1} \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \cos q_{d1} & \dfrac{1.5}{\sqrt{3}} \\ \sin q_{d1} & -0.5 \end{pmatrix} \begin{pmatrix} q_2 \\ s \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{7}$$

2. Solve the linear system (7) for $q_2 = q_{d2}(t)$ and $s = s(t)$, and substitute therein $q_1 = q_{d1} = V_1 t$:

$$\begin{pmatrix} q_{d2}(t) \\ s(t) \end{pmatrix} = \frac{1}{0.5 \cos V_1 t + \frac{1.5}{\sqrt{3}} \sin V_1 t} \begin{pmatrix} 0.5 \\ \sin V_1 t \end{pmatrix}. \tag{8}$$

3. Similarly, equate at the differential level the desired Cartesian velocity on the path, expressed from the task side by the second relation in (3), with the velocity of the end effector, as given by (5) from the robot side, substituting therein $\boldsymbol{p} = \boldsymbol{p}_d(s)$, with $s = s(t)$, and $\dot{q}_{d1} = V_1$:

$$\begin{pmatrix} -\dfrac{1.5}{\sqrt{3}} \\ 0.5 \end{pmatrix} \dot{s} = \begin{pmatrix} -p_{dy}(s) \\ p_{dx}(s) \end{pmatrix} \dot{q}_{d1} + \begin{pmatrix} \cos q_{d1} \\ \sin q_{d1} \end{pmatrix} \dot{q}_2 = \begin{pmatrix} -0.5\,s(t) \\ 1 - \dfrac{1.5\,s(t)}{\sqrt{3}} \end{pmatrix} V_1 + \begin{pmatrix} \cos V_1 t \\ \sin V_1 t \end{pmatrix} \dot{q}_2$$

or

$$\begin{pmatrix} \cos V_1 t & \dfrac{1.5}{\sqrt{3}} \\ \sin V_1 t & -0.5 \end{pmatrix} \begin{pmatrix} \dot{q}_2 \\ \dot{s} \end{pmatrix} = \begin{pmatrix} 0.5 V_1\, s(t) \\ -V_1 \left(1 - \dfrac{1.5\,s(t)}{\sqrt{3}}\right) \end{pmatrix}. \tag{9}$$

4. Solve the linear system (9) for $\dot{q}_2 = \dot{q}_{d2}(t)$ and $\dot{s} = \dot{s}(t)$:

$$\begin{pmatrix} \dot{q}_{d2}(t) \\ \dot{s}(t) \end{pmatrix} = \frac{V_1}{0.5 \cos V_1 t + \frac{1.5}{\sqrt{3}} \sin V_1 t} \begin{pmatrix} \left(0.25 + \left(\frac{1.5}{\sqrt{3}}\right)^2\right) s(t) - \frac{1.5}{\sqrt{3}} \\ \cos V_1 t + \left(0.5 \sin V_1 t - \frac{1.5}{\sqrt{3}} \cos V_1 t\right) s(t) \end{pmatrix}, \tag{10}$$

where the expression of $s(t)$ from (8) should be be used.

Note that the above steps 3 and 4 can be replaced (approximately) by a numerical derivative of the expressions (8), e.g., by finite differences at the sampling rate $1/T_c$. The final check is indeed

$$|\dot{q}_{d2}(t)| \leq V_2 = 1.5 \text{ [m/s]}, \qquad \forall t \in [0, T]. \tag{11}$$

The following simple Matlab code implements the above method:

```
V1=40*pi/180; T=3;
Tc=0.001; t=[0:Tc:T];
% solution for desired q2 and s
dets=0.5*cos(V1*t)+(1.5/sqrt(3))*sin(V1*t);
qd2=0.5./dets;
sd=sin(V1*t)./dets;
% solution for desired velocity of q2 and s
dotqd2=V1*(0.25*sd+(1.5/sqrt(3))^2*sd-(1.5/sqrt(3)))./dets;
dotsd=V1*((0.5*sin(V1*t)-(1.5/sqrt(3))*cos(V1*t)).*sd+cos(V1*t))./dets;
```

With the obtained values, we can verify that the constraint (11) is indeed always satisfied. Therefore, the optimal solution is given by the joint trajectory $\boldsymbol{q}_d(t)$ already found in (6) and (8). Figure 2 shows the actual Cartesian path that has been planned, while Figs. 3–4 report the time evolution of all the relevant variables. Note in particular that the speed $\dot{s}(t)$ on the linear path is *not* constant.

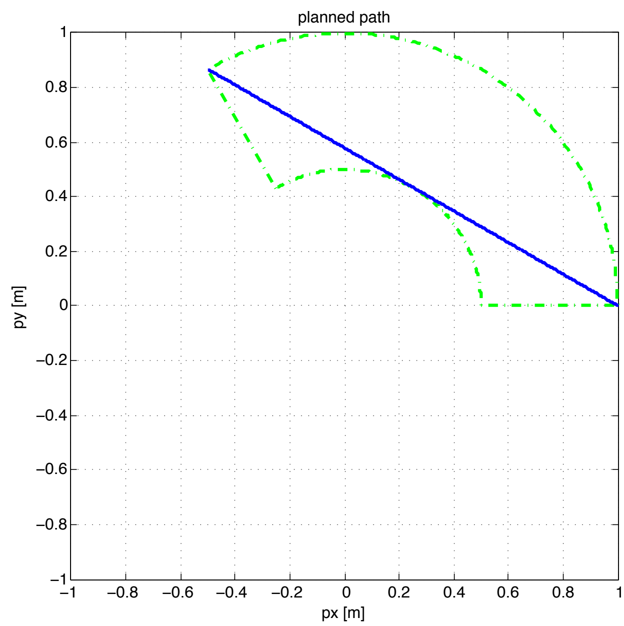

Figure 2: Actual Cartesian path obtained with the planned joint trajectories

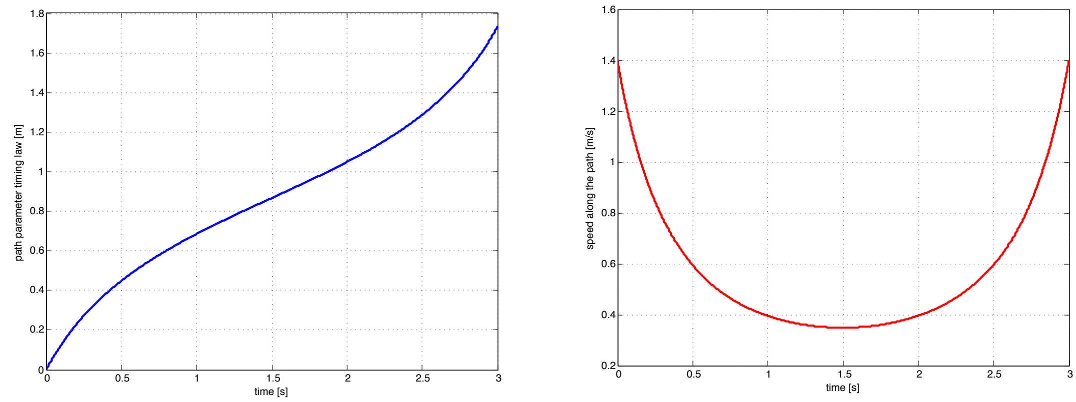

Figure 3: Timing law $s(t)$ for the path parameter in (3) and its speed $\dot{s}(t)$, as computed from (8) and (10). The minimum speed is at point $\boldsymbol{E}$, where the motion of joint 2 is orthogonal to the path and only joint 1 contributes with $\|\boldsymbol{E}\| \cdot V_1 \approx 0.35$ m/s

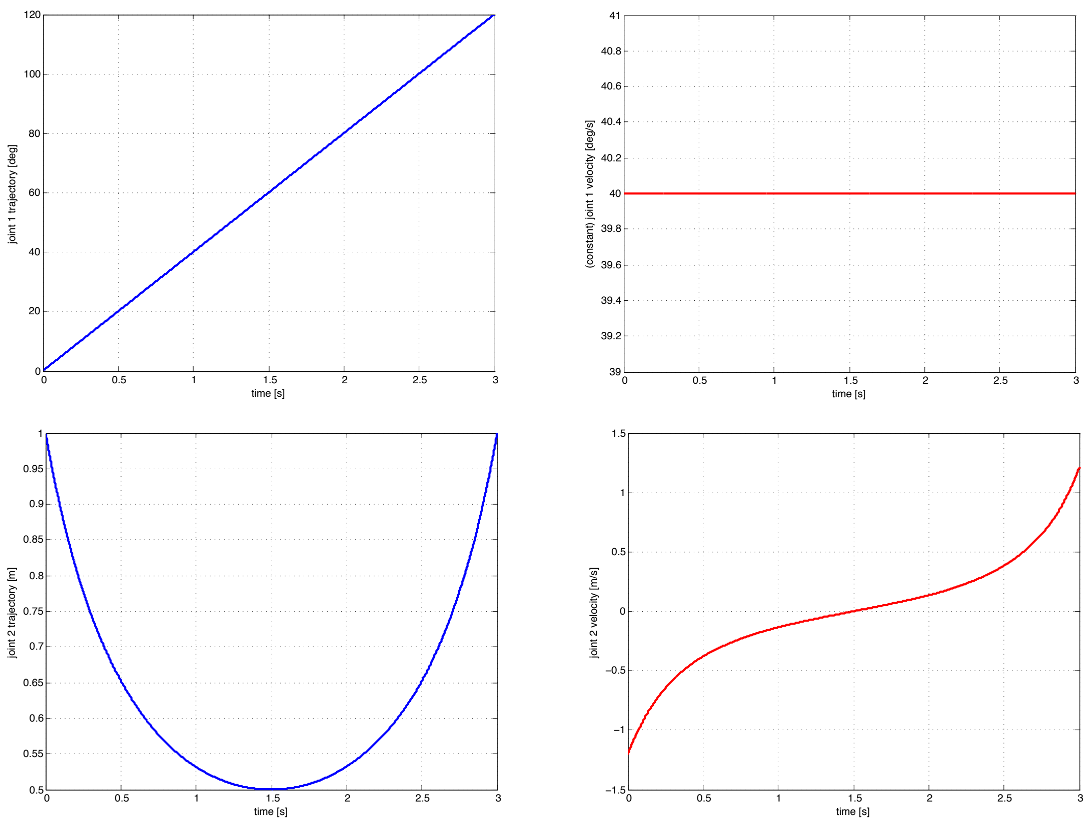

Figure 4: Planned trajectories in position (blue) and velocity (red) for the revolute (top) and prismatic (bottom) joints, as computed from (6), (8), and (10). As anticipated, motion of joint 2 is symmetric vs. the path midpoint $\boldsymbol{E}$, and its velocity is maximum at the initial and final points

Exercise 3

The length of the desired path is $L = \|\boldsymbol{B} - \boldsymbol{A}\| = \sqrt{5} \approx 2.2361$ m. Since

$$L = \sqrt{5} > 0.05 = \frac{v_{\max}^2}{a_{\max}},$$

the existence of a coast phase at constant speed is verified, and the nominal motion time to trace the path with a trapezoidal velocity profile can be computed as

$$T = \frac{L\, a_{\max} + v_{\max}^2}{v_{\max}\, a_{\max}} = 4.5721\,\text{s}.$$

The desired trajectory is written in parametrized form as

$$\boldsymbol{p}_d(t) = \boldsymbol{A} + \frac{\boldsymbol{B} - \boldsymbol{A}}{L}\, s(t), \quad \text{for } t \in [0, T] \rightarrow s(t) \in [0, L], \quad \dot{\boldsymbol{p}}_d(t) = \frac{\boldsymbol{B} - \boldsymbol{A}}{L}\, \dot{s}(t) = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \dot{s}(t),$$

with

$$s(t) = \begin{cases} 2.5\, t^2, & t \in [0, 0.1] \\ 0.5\,(t - 0.05), & t \in [0.1, T - 0.1] \\ -2.5\,(t - T)^2 + 0.5\,(T - 0.1), & t \in [T - 0.1, T] \end{cases}$$

and

$$\dot{s}(t) = \begin{cases} 5\, t, & t \in [0, 0.1] \\ 0.5, & t \in [0.1, T - 0.1] \\ -5\,(t - T), & t \in [T - 0.1, T]. \end{cases}$$

The nominal path is internal to the primary workspace and never crosses the axis of joint 1 (the minimum distance to $\boldsymbol{z}_0$ is about 0.89 m) nor reaches the external boundary (where the links 2 and 3 would be stretched). Thus, if the end effector were always on this desired path, the robot would not encounter any kinematic singularity.

Using the values in Tab. 1, we have for the direct kinematics of the robot end-effector position

$$\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{\theta}) = \begin{pmatrix} \cos\theta_1\,(a_2 \cos\theta_2 + a_3 \cos(\theta_2 + \theta_3)) \\ \sin\theta_1\,(a_2 \cos\theta_2 + a_3 \cos(\theta_2 + \theta_3)) \\ a_2 \sin\theta_2 + a_3 \sin(\theta_2 + \theta_3) \end{pmatrix}, \quad \text{with } a_2 = 1.5,\ a_3 = 1 \text{ [m]}.$$

The associated Jacobian is

$$\boldsymbol{J}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{f}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \begin{pmatrix} -s_1\,(a_2 c_2 + a_3 c_{23}) & -c_1\,(a_2 s_2 + a_3 s_{23}) & -a_3 c_1 s_{23} \\ c_1\,(a_2 c_2 + a_3 c_{23}) & -s_1\,(a_2 s_2 + a_3 s_{23}) & -a_3 s_1 s_{23} \\ 0 & a_2 c_2 + a_3 c_{23} & a_3 c_{23} \end{pmatrix}.$$

In the initial configuration $\boldsymbol{\theta}(0) = \begin{pmatrix} -\pi/2 & 0 & \pi/6 \end{pmatrix}^T$, we have

$$\boldsymbol{p}(0) = \boldsymbol{f}(\boldsymbol{\theta}(0)) = \begin{pmatrix} 0 \\ -2.3660 \\ 0.5 \end{pmatrix} \quad \Rightarrow \quad \boldsymbol{e}(0) = \boldsymbol{p}_d(0) - \boldsymbol{p}(0) = \boldsymbol{A} - \boldsymbol{p}(0) = \begin{pmatrix} 0 \\ 0.3660 \\ 0 \end{pmatrix},$$

so that only the $e_y(0)$ component is different from zero, while $e_x(0) = e_z(0) = 0$.

The kinematic control law that allows to obtain the desired characteristics has to be designed on the *Cartesian error*, and with a Cartesian velocity feedforward, as

$$\dot{\boldsymbol{\theta}} = \boldsymbol{J}^{-1}(\boldsymbol{\theta}) \left(\dot{\boldsymbol{p}}_d + \boldsymbol{K} \left(\boldsymbol{p}_d - \boldsymbol{f}(\boldsymbol{\theta})\right)\right), \qquad \text{with } \boldsymbol{K} = \text{diag}\{k_x, k_y.k_z\} > 0, \tag{12}$$

where the expressions of the required terms $\boldsymbol{f}(\boldsymbol{\theta})$, $\boldsymbol{J}(\boldsymbol{\theta})$, $\boldsymbol{p}_d(t)$, and $\dot{\boldsymbol{p}}_d(t)$ have already been given. In fact, the law (12) guarantees that the Cartesian tracking error $\boldsymbol{e}(t) = \boldsymbol{p}_d(t) - \boldsymbol{p}(t)$ behaves as

$$\dot{\boldsymbol{e}} = -\boldsymbol{K}\boldsymbol{e} \qquad \Rightarrow \qquad e_i(t) = e_i(0) \exp(-k_i t) \to 0 \qquad \text{for } t \geq 0, \quad i = x, y, z.$$

Any choice of strictly positive values for k_x, k_y, and k_z will work. In this case, being the initial errors on two Cartesian components already zero, it will be $e_x(t) = e_z(t) = 0$ for all times —this is a consequence of the Cartesian decoupling achieved by the control law (12). Note also that $\|\boldsymbol{e}(t)\| = |e_y(t)|$ holds for all $t \geq 0$. For the gain k_y, the requested minimum value is found by imposing at $t = T/4 = 1.1430$ s

$$e_y(T/4) = e_y(0) \exp(-k_y T/4) = 0.05\, e_y(0) \qquad \Rightarrow \qquad k_y = -\frac{4}{T} \ln 0.05 = 2.6209.$$

Figure 5 shows the evolution of the norm of the Cartesian tracking error with this choice, and confirms the satisfaction of the error reduction as soon as $t \geq T/4$.

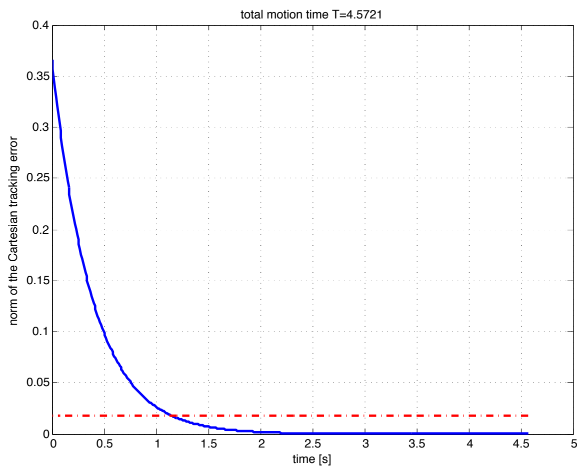

Figure 5: Evolution of the norm of the Cartesian tracking error with $k_y = 2.6209$

Moreover, the y-component of the Cartesian trajectory followed by the robot end effector will not overshoot its initial value and will always be *larger* than that of the nominal trajectory, practically coinciding with the desired one after five times the time constant τ of the exponential trajectory (i.e., for $t \geq 5\tau = 5 \cdot (1/k_y) \approx 1.9$ s). As a consequence, also the *actual* path executed by the robot will never encounter kinematic singularities. Finally, all joints will be simultaneously in motion during the execution of the controlled task.

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