Robotics I

September 10, 2009

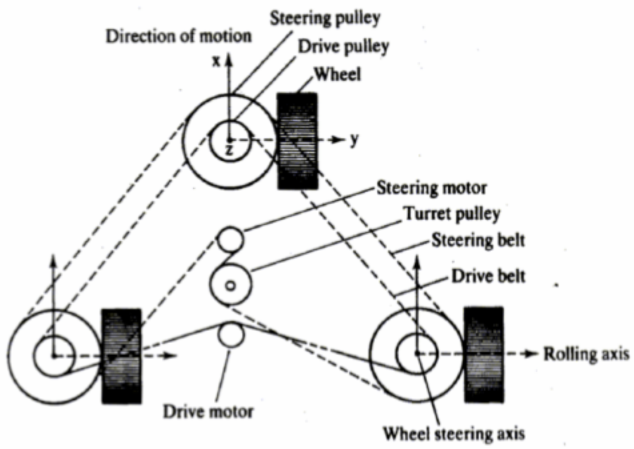

Figure 1: The mobile manipulator (left) and the *synchro-drive* of its base (right)

Consider the mobile manipulator in Figure 1, made by a nonholonomic base (Nomad) carrying a $6R$ manipulator, with shoulder and elbow off-sets (that compensate each to other) and a spherical wrist (Unimation Puma). The base has three identical steering wheels that move in coordination, driven by a *synchro-drive* actuation with one motor for driving the three wheels and one for their steering. Let v be the linear velocity of the wheels on the ground and ω the steering velocity of the wheels with respect to the base chassis. We are interested only in the position $\boldsymbol{p} \in \mathbb{R}^3$ of the center of the spherical wrist of the manipulator; the first three joints are described by the Denavit-Hartenberg coordinates $\boldsymbol{\theta} \in \mathbb{R}^3$, while the three remaining are frozen. Determine the expression of the 3×5 matrix $\boldsymbol{J}(\boldsymbol{q})$ in the relationship

$$\dot{\boldsymbol{p}} = \boldsymbol{J}(\boldsymbol{q})\boldsymbol{u},$$

where $\boldsymbol{q} \in \mathbb{R}^6$ is the vector of generalized coordinates for the mobile manipulator and

$$\boldsymbol{u} = \begin{pmatrix} v & \omega & \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{pmatrix}^T \in \mathbb{R}^5$$

is the vector of the available input velocities.

[120 minutes; open books]

Solution

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An absolute reference frame $(\boldsymbol{x}_w, \boldsymbol{y}_w, \boldsymbol{z}_w)$ is chosen, with the axis $\boldsymbol{z}_w$ being normal to the motion plane. The mobile base is described by the coordinates (x, y, θ), representing the Cartesian position of its center and the absolute orientation of the three wheels with respect to the axis $\boldsymbol{x}_w$. The (differential) kinematic model of the base is that of a unicycle

$$\begin{aligned} \dot{x} &= v \cos\theta \\ \dot{y} &= v \sin\theta \\ \dot{\theta} &= \omega. \end{aligned} \tag{1}$$

Note that the orientation of the base (and of its turret) *does not* change with the steering velocity ω (the orientation of the wheels changes w.r.t. the chassis body).

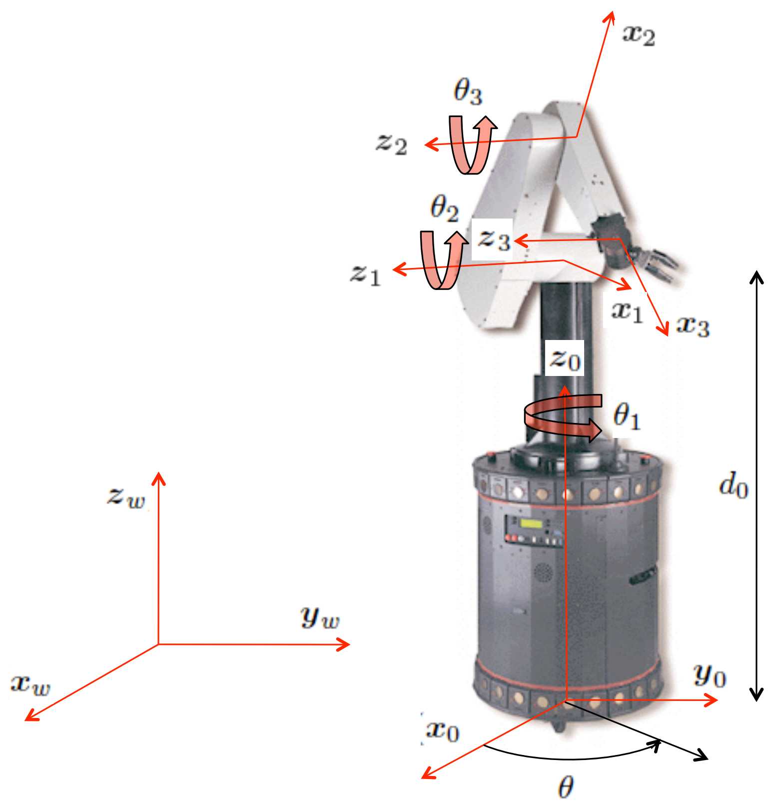

Figure 2: Reference frames for the mobile manipulator

A reference frame $(\boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{z}_0)$ is then chosen for the manipulator, located on the mobile base and aligned with the absolute frame (see Figure 2). The position of the wrist center relative to this frame is

$$ {}^0\boldsymbol{p} = \begin{pmatrix} (\ell_2 \cos\theta_2 + \ell_3 \cos(\theta_2+\theta_3)) \cos\theta_1 \\ (\ell_2 \cos\theta_2 + \ell_3 \cos(\theta_2+\theta_3)) \sin\theta_1 \\ d_0 + \ell_2 \sin\theta_2 + \ell_3 \sin(\theta_2+\theta_3) \end{pmatrix}, $$

where $(\theta_1, \theta_2, \theta_3)$ are the joint variables according to Denavit-Hartenberg and d_0 is the height from the ground of the second joint axis (which is always horizontal). The presence of shoulder and elbow offset is not relevant for the direct kinematics. It follows

$$ {}^w\boldsymbol{p} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} + {}^0\boldsymbol{p}. $$

Differentiating w.r.t. time and using (1) leads to

$$ {}^w\dot{\boldsymbol{p}} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ 0 \end{pmatrix} + {}^0\dot{\boldsymbol{p}} = \boldsymbol{J}(\boldsymbol{q})\boldsymbol{u} = $$

$$ \begin{pmatrix} c\theta & 0 & -(\ell_2 c\theta_2 + \ell_3 c(\theta_2+\theta_3))\, s\theta_1 & -(\ell_2 s\theta_2 + \ell_3 s(\theta_2+\theta_3))\, c\theta_1 & -\ell_3 s(\theta_2+\theta_3) c\theta_1 \\ s\theta & 0 & (\ell_2 c\theta_2 + \ell_3 c(\theta_2+\theta_3))\, c\theta_1 & -(\ell_2 s\theta_2 + \ell_3 s(\theta_2+\theta_3))\, s\theta_1 & -\ell_3 s(\theta_2+\theta_3) s\theta_1 \\ 0 & 0 & 0 & \ell_2 c\theta_2 + \ell_3 c(\theta_2+\theta_3) & \ell_3 c(\theta_2+\theta_3) \end{pmatrix} \begin{pmatrix} v \\ \omega \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix}, $$

where the compact notation s for sin and c for cos is used. The Jacobian $\boldsymbol{J}$ of the mobile manipulator has always a zero column, corresponding to the fact that the input ω has no effect on the wrist center velocity.

One can additionally study the singularities of $\boldsymbol{J}$ (namely, the configurations where the matrix loses its maximum rank which is 3). Note that the last three columns of this matrix are the Jacobian ${}^0\boldsymbol{J}_m = {}^w\boldsymbol{J}_m$ of the manipulator arm, when taken by itself. For the whole structure to be singular, the manipulator should necessarily be in a singularity. Since

$$ \begin{aligned} {}^0\boldsymbol{J}_m(\boldsymbol{\theta}) &= {}^0\boldsymbol{R}_1(\theta_1)\, {}^1\boldsymbol{J}_m(\boldsymbol{\theta}) \\ &= {}^0\boldsymbol{R}_1(\theta_1) \begin{pmatrix} 0 & -(\ell_2 s\theta_2 + \ell_3 s(\theta_2+\theta_3)) & -\ell_3 s(\theta_2+\theta_3) \\ \ell_2 c\theta_2 + \ell_3 c(\theta_2+\theta_3) & 0 & 0 \\ 0 & \ell_2 c\theta_2 + \ell_3 c(\theta_2+\theta_3) & \ell_3 c(\theta_2+\theta_3) \end{pmatrix}, \end{aligned} $$

it is easy to see that this happens if the wrist center is on the first joint axis

$$ \ell_2 \cos\theta_2 + \ell_3 \cos(\theta_2+\theta_3) = 0, $$

or if the third link is stretched or folded

$$ \sin\theta_3 = 0, $$

or when both situations hold together (the rank of $\boldsymbol{J}_m$ would then drop to 1). In the first kind of singularity, the manipulator wrist center cannot have a velocity along the normal to the plane of motion of the second and third link; for the base to provide mobility in this direction (through the input velocity v), the wheels must not be parallel to the plane containing the second and third link, i.e., $\theta \neq \theta_1 + k\pi$, $k = 0, 1$. In the second kind of singularity, the wrist center cannot have a velocity along the direction of alignment of the second and third link; for the base to provide mobility in this direction, the wheels must not be oriented normally to this direction, i.e., $\theta \neq \theta_1 \pm \pi/2$. When the manipulator is in a double singularity, the mobile base cannot recover the lack of mobility so as to give full row rank to $\boldsymbol{J}$.

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