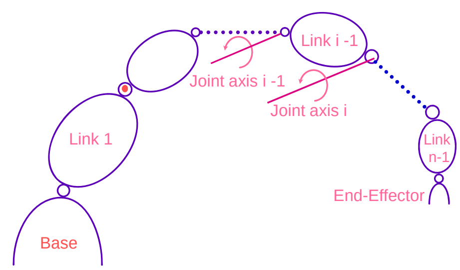
Movie Segment

HRP-4, AIST and Kwada Industries, 2010.

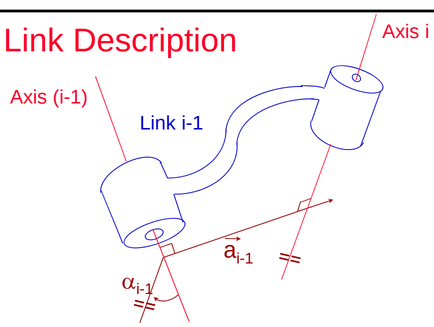
Manipulator Kinematics

- Link Description
- *Denavit-Hartenberg* Notation
- Frame Attachment
- Forward Kinematics

Manipulator



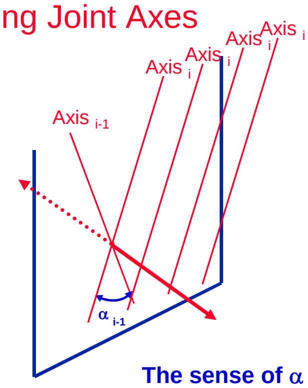
Link Description



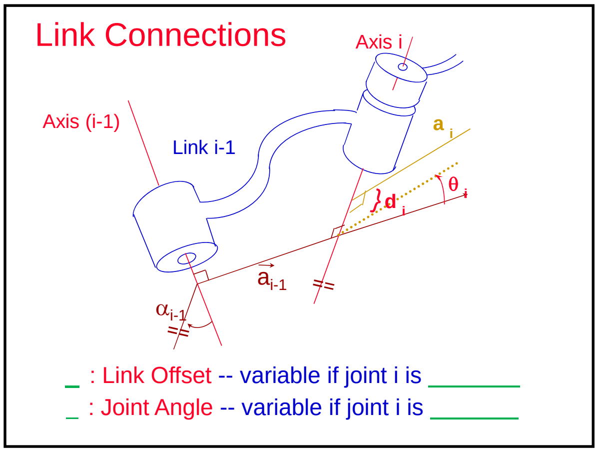
_ : Link Length - mutual perpendicular
unique except for parallel axis

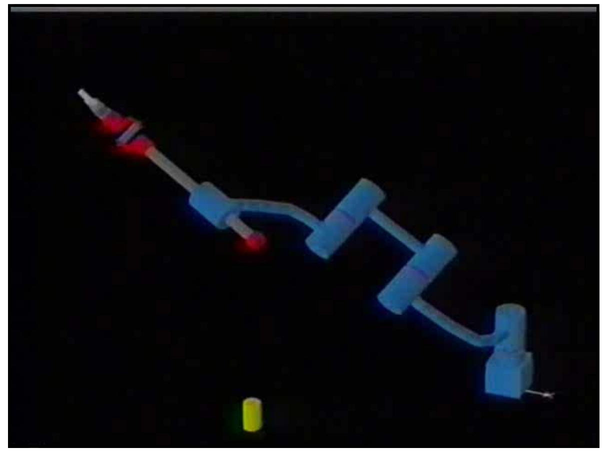
_ : Link Twist - measured in the right-hand sense about $\vec{a}_{i-1}$

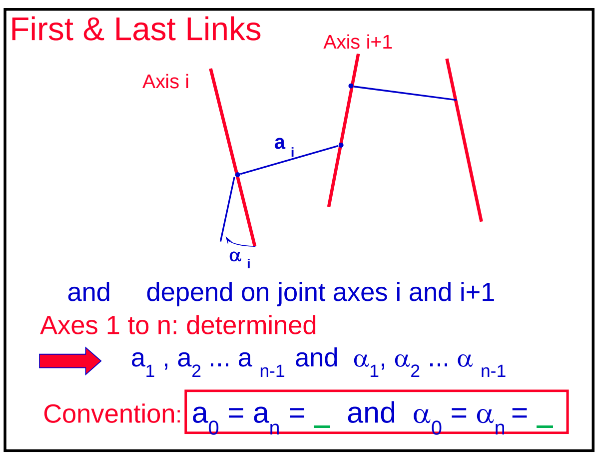
Intersecting Joint Axes

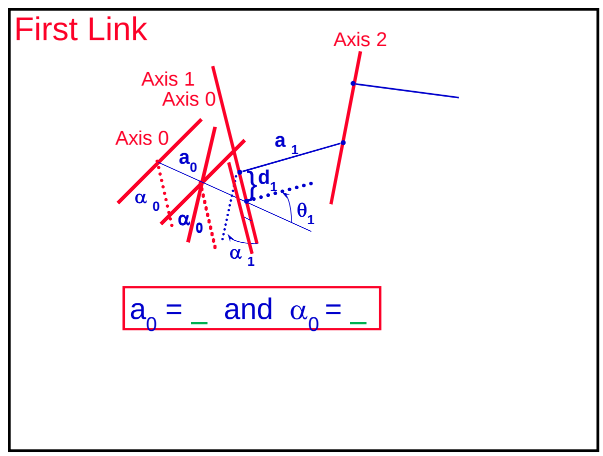


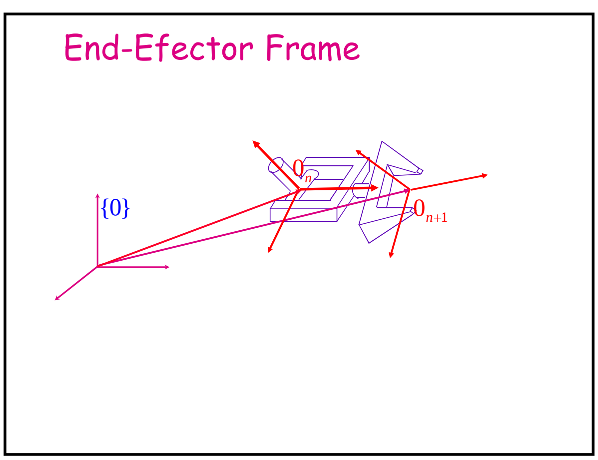
The sense of α_{i-1} is free

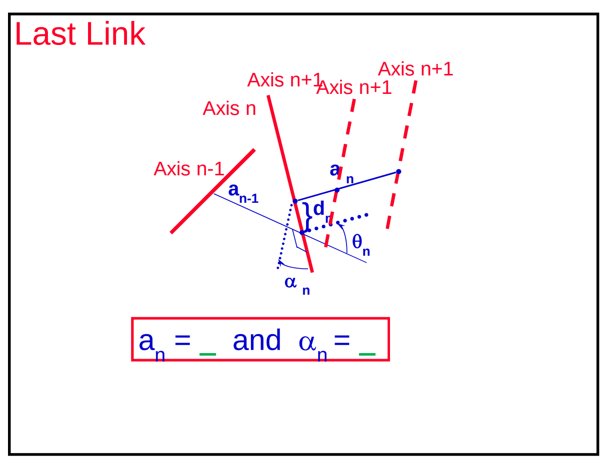

Link Connections
Axis i
Axis (i-1)
Link i-1
a i
θ i
d i
a i-1
α i-1
_ : Link Offset -- variable if joint i is _______
_ : Joint Angle -- variable if joint i is _______



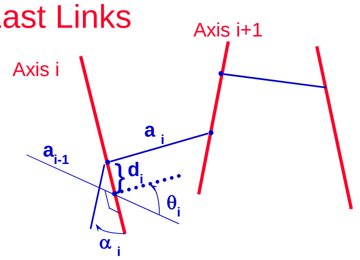

First & Last Links
Axis i+1
Axis i
a i
α i
and depend on joint axes i and i+1
Axes 1 to n: determined
a1 , a2 ... a n-1 and α1, α2 ... α n-1
Convention: a0 = an = _ and α0 = αn = _


First Link
Axis 2
Axis 1
Axis 0
Axis 0
a 1
a0
d1
α 0
α0
θ1
α 1
a0 = _ and α0 = _


End-Efector Frame
{0}


Last Link
Axis n+1
Axis n+1
Axis n+1
Axis n
Axis n-1
an-1
an
dn
θn
α n
an = _ and αn = _

First & Last Links



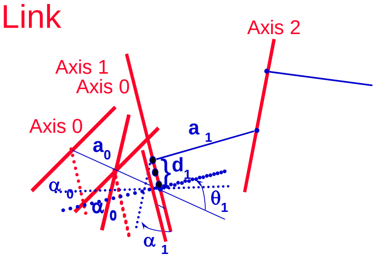
θ_i and d_i depend on links i-1 and i

➡ $\theta_2, \theta_3, \ldots, \theta_{n-1}$ and $d_2, d_3, \ldots, d_{n-1}$

Convention: set the constant parameters to zero

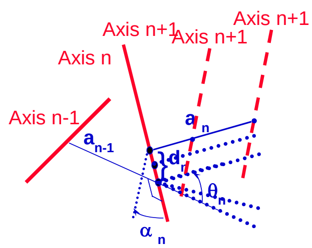
Following joint type: d_1 or $\theta_1 = 0$ and d_n or $\theta_n = 0$

First Link



d_1 or θ_1 = _

Last Link



d_n or θ_n = _

Denavit-Hartenberg Parameters

4 D-H parameters $(\alpha_i,\ a_i,\ d_i,\ \theta_i)$

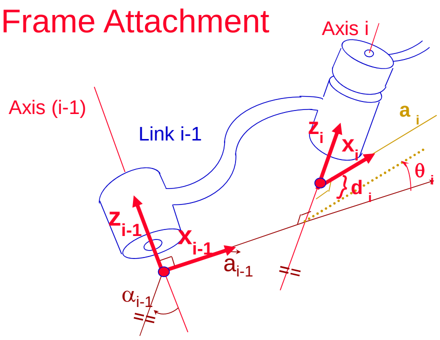
3 fixed link parameters

1 joint variable { θ_i revolute joint; d_i prismatic joint }

α_i and a_i : describe the Link i

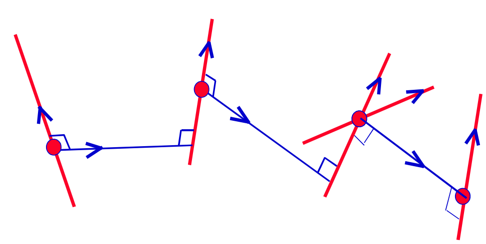
d_i and θ_i : describe the Link's connection

Frame Attachment



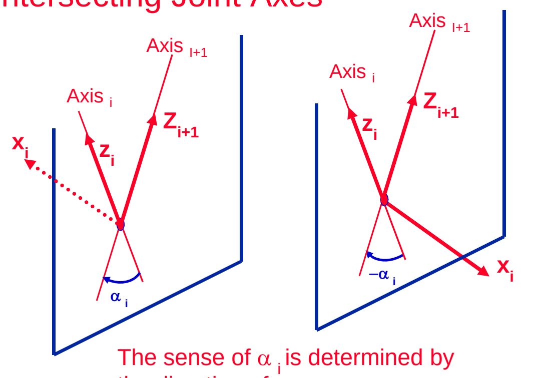
y-vectors: complete right-hand frames

Summary – Frame Attachment



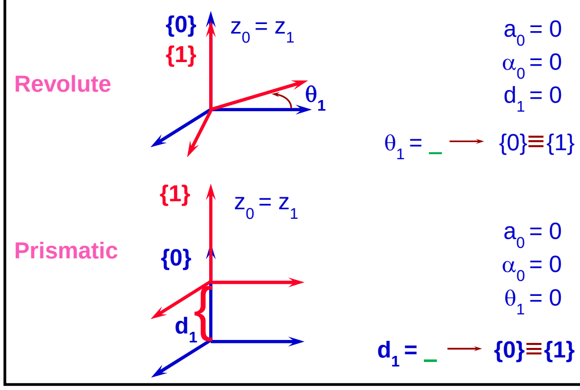
1. Normals
2. Origins
3. Z-axes
4. X-axes

Intersecting Joint Axes

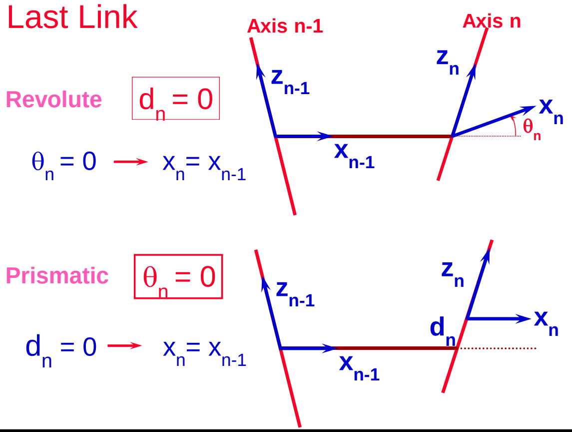


The sense of α_i is determined by the direction of x

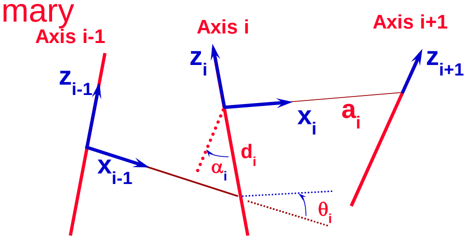
First Link



Last Link



Summary



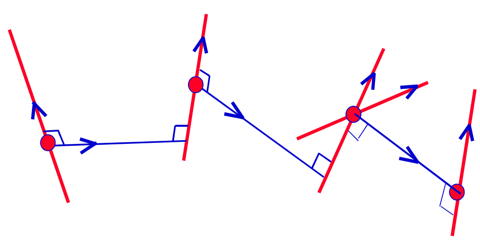
a_i : distance (z_i , z_{i+1}) along x_i

α_i : angle (z_i , z_{i+1}) about x_i

d_i : distance (x_{i-1} , x_i) along z_i

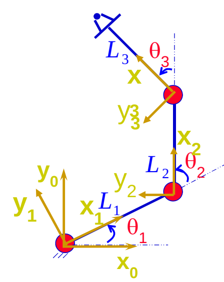
θ_i : angle (x_{i-1} , x_i) about z_i

Summary – Frame Attachment



1. Normals
2. Origins
3. Z-axes
4. X-axes

Example – RRR Arm

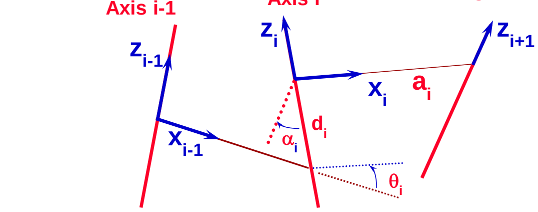


i	α_{i-1}	a_{i-1}	d_i	θ_i
1	_	_	_	_
2	_	_	_	_
3	_	_	_	_

Movie Segment

BigDog, Boston Dynamics, 2010

Summary



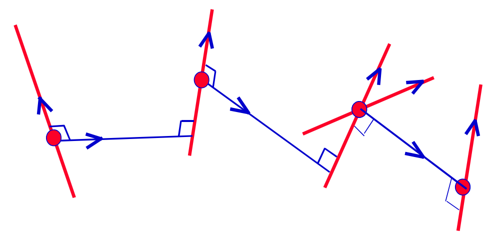
a_i : distance (z_i , z_{i+1}) along x_i

α_i : angle (z_i , z_{i+1}) about x_i

d_i : distance (x_{i-1} , x_i) along z_i

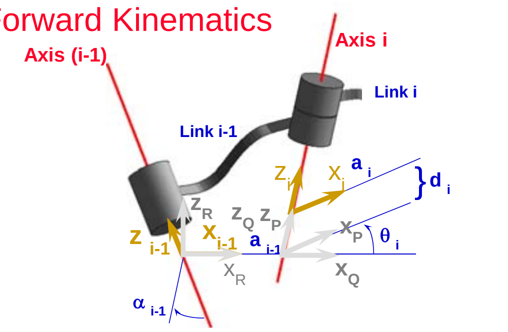
θ_i : angle (x_{i-1} , x_i) about z_i

Summary – Frame Attachment



1. Normals
2. Origins
3. Z-axes
4. X-axes

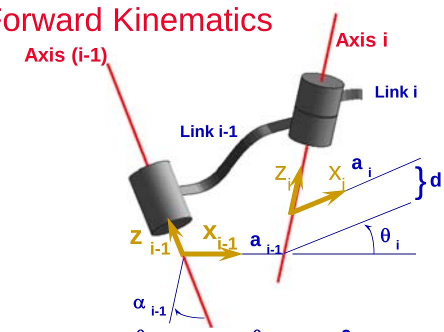
Forward Kinematics



$${}^{i-1}_{i}T = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T$$

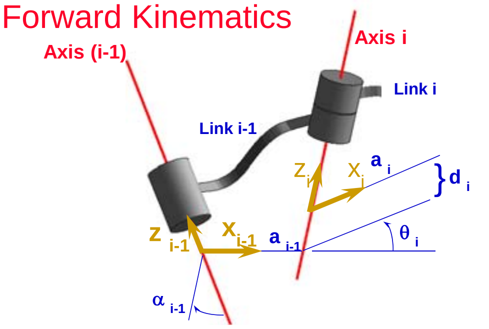
$${}^{i-1}_{i}T_{(\alpha_{i-1}, a_{i-1}, \theta_i, d_i)} = R_x(\alpha_{i-1}) \, D_x(a_{i-1}) \, R_z(\theta_i) \, D_z(d_i)$$

Forward Kinematics



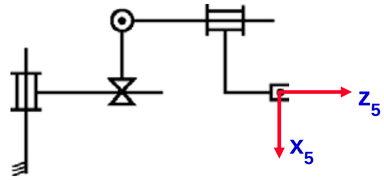
$${}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i \, c\alpha_{i-1} & c\theta_i \, c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i \, s\alpha_{i-1} & c\theta_i \, s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics

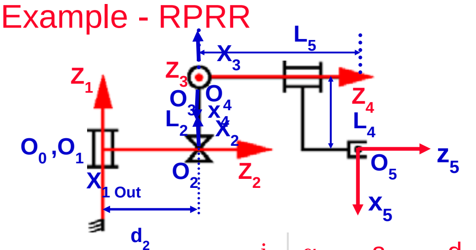


Forward Kinematics: $^{0}_{N}T = {}^{0}_{1}T\ {}^{1}_{2}T \dots {}^{N-1}_{N}T$

Example - RPRR



Example - RPRR



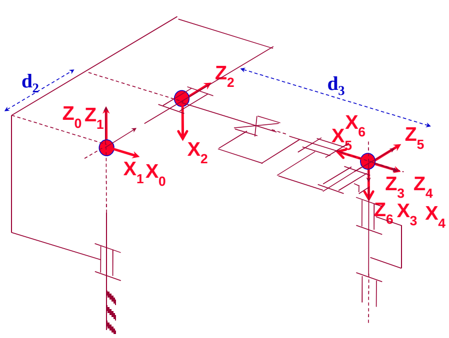
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	–	–	–	–
2	–	–	–	–
3	–	–	–	–
4	–	–	–	–
5	–	–	–	–

a_i : distance (z_i , z_{i+1}) along x_i
α_i : angle (z_i , z_{i+1}) about x_i
d_i : distance (x_{i-1} , x_i) along z_i
θ_i : angle (x_{i-1} , x_i) about z_i

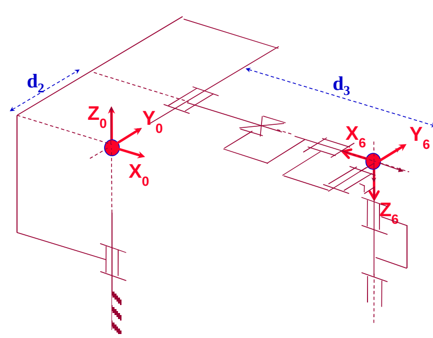
Stanford Scheinman Arm

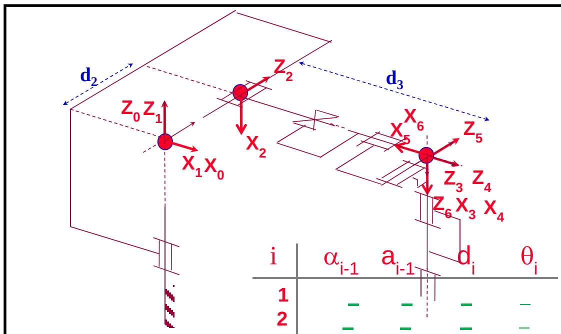
Victor Scheinman (around 1968)

Stanford Scheinman Arm



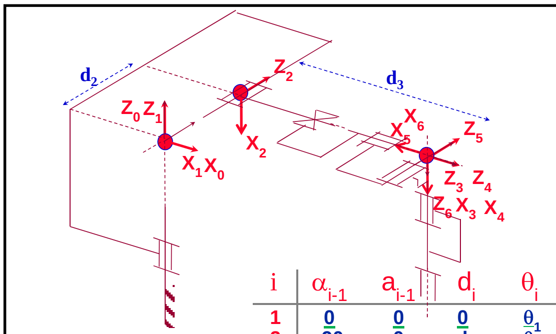
Stanford Scheinman Arm





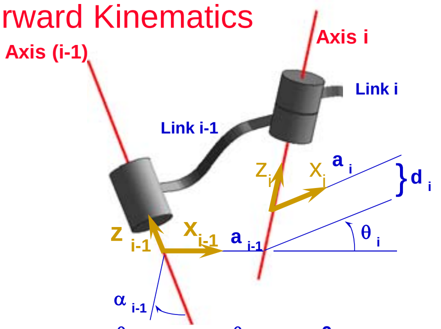
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	–	–	–	–
2	–	–	–	–
3	–	–	–	–
4	–	–	–	–
5	–	–	–	–
6	–	–	–	–

a_i : distance (z_i, z_{i+1}) along x_i
α_i : angle (z_i, z_{i+1}) about x_i
d_i : distance (x_{i-1}, x_i) along z_i
θ_i : angle (x_{i-1}, x_i) about z_i



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90	0	d_2	θ_2
3	90	0	d_3	0
4	0	0	0	θ_4
5	-90	0	0	θ_5
6	90	0	0	θ_6

Forward Kinematics



$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i\, c\alpha_{i-1} & c\theta_i\, c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}\, d_i \\ s\theta_i\, s\alpha_{i-1} & c\theta_i\, s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}\, d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Stanford Scheinman Arm

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	d_2	θ_2
3	90	0	d_3	0
4	0	0	0	θ_4
5	-90	0	0	θ_5
6	90	0	0	θ_6

$$ \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i\, c\alpha_{i-1} & c\theta_i\, c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}\, d_i \\ s\theta_i\, s\alpha_{i-1} & c\theta_i\, s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}\, d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}T \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

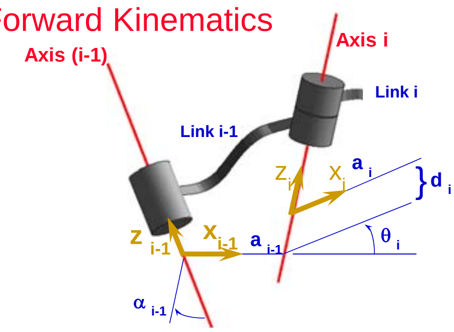
$$ {}^{2}_{3}T \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{4}T \begin{bmatrix} c_4 & s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{4}_{5}T \begin{bmatrix} c_5 & s_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{5}_{6}T \begin{bmatrix} c_6 & s_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Forward Kinematics



Forward Kinematics: $ {}^{0}_{N}T = {}^{0}_{1}T \; {}^{1}_{2}T \; \ldots \; {}^{N-1}_{N}T $

$$
{}^{0}_{1}T \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{2}T \begin{bmatrix} c_1 c_2 & c_1 s_2 & s_1 & s_1 d_2 \\ s_1 c_2 & s_1 s_2 & c_1 & c_1 d_2 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

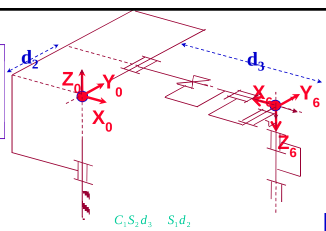
$$
{}^{0}_{3}T \begin{bmatrix} c_1 c_2 & s_1 & c_1 s_2 & c_1 d_3 s_2 & s_1 d_2 \\ s_1 c_2 & c_1 & s_1 s_2 & s_1 d_3 s_2 & c_1 d_2 \\ s_2 & 0 & c_2 & & d_3 c_2 \\ 0 & 0 & 0 & & 1 \end{bmatrix}
$$

$$
{}^{0}_{4}T = \begin{bmatrix} c_1 c_2 c_4 - s_1 s_4 & -c_1 c_2 s_4 - s_1 c_4 & c_1 s_2 & c_1 d_3 s_2 - s_1 d_2 \\ s_1 c_2 c_4 + c_1 s_4 & -s_1 c_2 s_4 + c_1 c_4 & s_1 s_2 & s_1 d_3 s_2 + c_1 d_2 \\ -s_2 c_4 & s_2 s_4 & c_2 & d_3 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{5}T \begin{bmatrix} X & X & c_1 c_2 s_4 & s_1 c_4 & c_1 d_3 s_2 & s_1 d_2 \\ X & X & s_1 c_2 s_4 & c_1 c_4 & s_1 d_3 s_2 & c_1 d_2 \\ X & X & s_2 s_4 & & d_3 c_2 & \\ 0 & 0 & 0 & & 1 & \end{bmatrix}
$$

$$
{}^{0}_{6}T \begin{bmatrix} X & X & c_1 c_2 c_4 s_5 & s_1 s_4 s_5 & c_1 s_2 s_5 & c_1 d_3 s_2 & s_1 d_2 \\ X & X & s_1 c_2 c_4 s_5 & c_1 s_4 s_5 & s_1 s_2 c_5 & s_1 d_3 s_2 & c_1 d_2 \\ X & X & s_2 c_4 s_5 & c_5 c_2 & & d_3 c_2 & \\ 0 & 0 & 0 & & & 1 & \end{bmatrix}
$$

$$
{}^{0}_{6}T \begin{bmatrix} X & X & c_1 c_2 c_4 s_5 & s_1 s_4 s_5 & c_1 s_2 s_5 & c_1 d_3 s_2 & s_1 d_2 \\ X & X & s_1 c_2 c_4 s_5 & c_1 s_4 s_5 & s_1 s_2 c_5 & s_1 d_3 s_2 & c_1 d_2 \\ X & X & s_2 c_4 s_5 & c_5 c_2 & & d_3 c_2 & \\ 0 & 0 & 0 & & & 1 & \end{bmatrix}
$$



$$
x = \begin{pmatrix} x_P \\ r_1 \\ r_2 \\ r_3 \end{pmatrix} = \left| \begin{array}{c} C_1 S_2 d_3 \quad S_1 d_2 \\ S_1 S_2 d_3 \quad C_1 d_2 \\ C_2 d_3 \\ C_1[C_2(C_4 C_5 C_6 \quad S_4 S_6) \quad S_2 S_5 C_6] \quad S_1(S_4 C_5 C_6 \quad C_4 S_6) \\ S_1[C_2(C_4 C_5 C_6 \quad S_4 S_6) \quad S_2 S_5 C_6] \quad C_1(S_4 C_5 C_6 \quad C_4 S_6) \\ S_2(C_4 C_5 C_6 \quad S_4 S_6) \quad C_2 S_5 C_6 \\ C_1[\quad C_2(C_4 C_5 S_6 \quad S_4 C_6) \quad S_2 S_5 S_6] \quad S_1(\quad S_4 C_5 S_6 \quad C_4 C_6) \\ S_1[\quad C_2(C_4 C_5 S_6 \quad S_4 C_6) \quad S_2 S_5 S_6] \quad C_1(\quad S_4 C_5 S_6 \quad C_4 C_6) \\ S_2(C_4 C_5 S_6 \quad S_4 C_6) \quad C_2 S_5 S_6 \\ C_1(C_2 C_4 S_5 \quad S_2 C_5) \quad S_1 S_4 S_5 \\ S_1(C_2 C_4 S_5 \quad S_2 C_5) \quad C_1 S_4 S_5 \\ S_2 C_4 S_5 \quad C_2 C_5 \end{array} \right|
$$