Video Segment

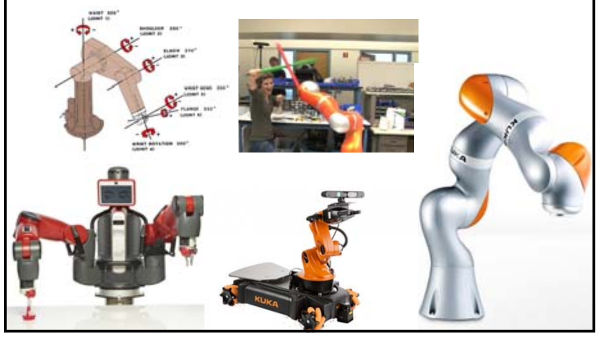
The bebionic3 Prostethic Hand, RSLSTEPPER, 2010+2013

Course Evaluation

http://axess.stanford.edu

CS225A – Experimental Robotics

Moved the Fall Quarter



Control

- Natural Systems
- PID Control
- Joint-Space Dynamic Control
- Task-Oriented Control
- Force Control

Performance

High Gains → better disturbance rejection

Gains are limited by

structural flexibilities

time delays (actuator-sensing)

sampling rate

$n \quad \frac{res}{2}$ ← lowest structural flexibility

$n \quad \frac{delay}{3}$ ← largest delay $\left(\frac{2}{delay}\right)$

$n \quad \frac{sampling \; rate}{5}$

Nonlinear Dynamic Decoupling

$M(\ )\ddot{}\quad V(\ ,\ \dot{}\ )\quad G(\ )$

$\hat{M}(\ )\_\quad \hat{V}(\ ,\ \dot{}\ )\quad \hat{G}(\ )$

$\mathbf{1.}\ \ddot{}\quad (M^{\ 1}\hat{M})\quad M^{\ 1}[(V\quad \hat{V})\quad (G\quad \hat{G})]$

with perfect estimates

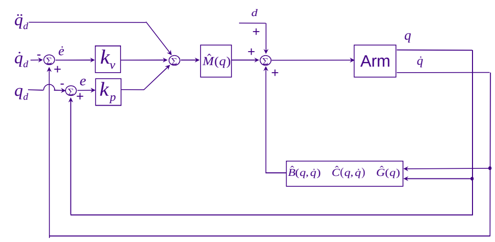
$\mathbf{1.}\ \ddot{}\qquad (t)$

: input of the unit-mass systems

$\ddot{}_{d}\quad k_v(\ \dot{}\quad \dot{}_{d})\quad k_p(\quad _{d})$

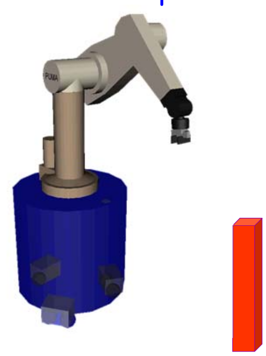
Closed-loop

$\ddot{E}\quad k_v\dot{E}\quad k_pE\quad 0\qquad (t)$

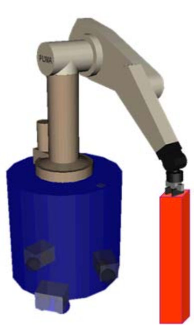


Task Oriented Control

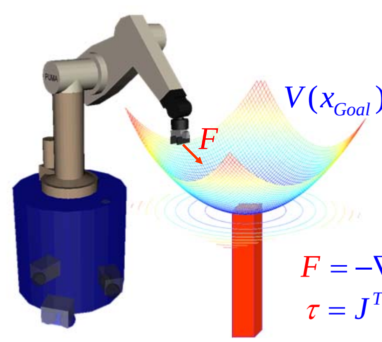
Joint Space Control



Joint Space Control



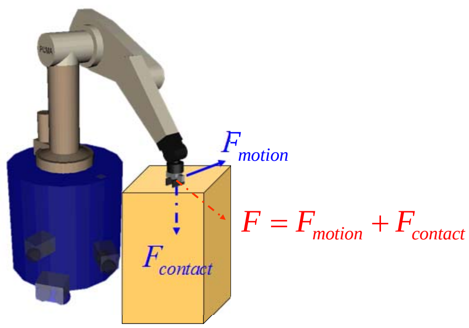
Operational Space Control



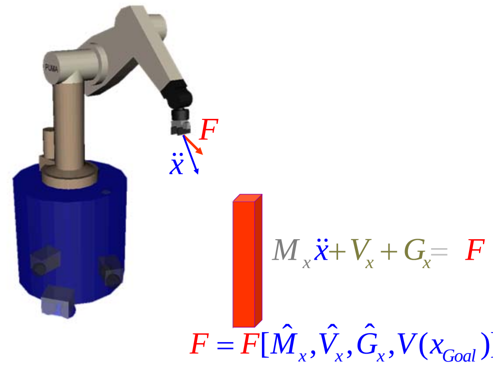
$F = -\nabla V(x_{Goal})$

$\tau = J^T F$

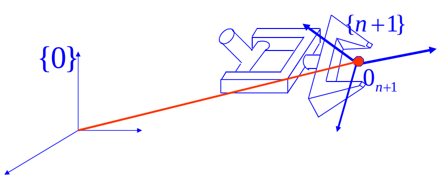
Unified Motion & Force Control



Operational Space Dynamics



Task-Oriented Equations of Motion



Non-Redundant Manipulator ; $n = m$

$$x = (x_1\ x_2 \ldots x_m)^T$$

$$q = (q_1\ q_2 \ldots q_n)^T$$

Equations of Motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F \qquad x = \begin{pmatrix} x \\ y \\ z \\ \alpha \\ \beta \\ \gamma \end{pmatrix}$$

with

$$L(x, \dot{x}) = K(x, \dot{x}) - U(x)$$

Operational Space Dynamics

$$M_x(x)\ddot{x} + V_x(x, \dot{x}) + G_x(x) = F$$

x: End-Effector Position and Orientation

$M_x(x)$: End-Effector Kinetic Energy Matrix

$V_x(x, \dot{x})$: End-Effector Centrifugal and Coriolis forces

$G_x(x)$: End-Effector Gravity forces

F: End-Effector Generalized forces

Joint Space/Task Space Relationships

Kinetic Energy

$$K_x(x, \dot{x}) \equiv K_q(q, \dot{q})$$

$$\frac{1}{2}\dot{x}^T M_x(x)\dot{x} \equiv \frac{1}{2}\dot{q}^T M(q)\dot{q}$$

Using $\quad \dot{x} = J(q)\dot{q}$

$$\frac{1}{2}\dot{q}^T (J^T M_x J)\,\dot{q} \equiv \frac{1}{2}\dot{q}^T M\,\dot{q}$$

Joint Space/Task Space Relationships

$$M_x(x) = J^{-T}(q)\,M(q)\,J^{-1}(q)$$

$$V_x(x,\dot{x}) = J^{-T}(q)\,V(q,\dot{q}) - M_x(q)\,h(q,\dot{q})$$

$$G_x(x) = J^{-T}(q)\,G(q)$$

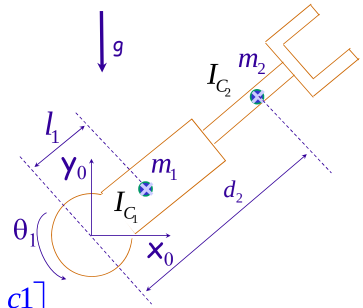
where $h(q,\dot{q}) \doteq \dot{J}(q)\dot{q}$

Example

$$q_2 = d_2$$

$$x = \begin{bmatrix} d_2 c1 \\ d_2 s1 \end{bmatrix}$$

$${}^0J = \begin{bmatrix} -d_2 s1 & c1 \\ d_2 c1 & s1 \end{bmatrix}$$

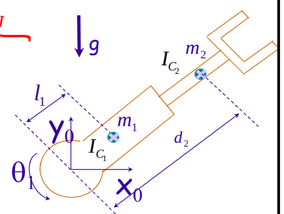


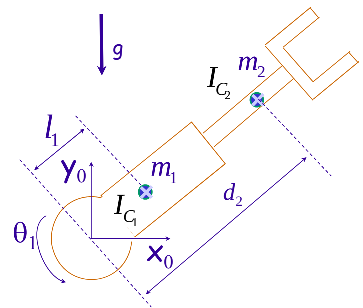
$${}^0J = \begin{bmatrix} -d_2 s1 & c1 \\ d_2 c1 & s1 \end{bmatrix}$$

$${}^0J = \begin{pmatrix} c1 & -s1 \\ s1 & c1 \end{pmatrix} \underbrace{\begin{pmatrix} 0 & 1 \\ d_2 & 0 \end{pmatrix}}_{{}^1J}$$

$${}^1J^{-1} = \begin{pmatrix} 0 & 1/d_2 \\ 1 & 0 \end{pmatrix};$$

$${}^1M_x = \begin{pmatrix} 0 & 1 \\ 1/d_2 & 0 \end{pmatrix} \begin{pmatrix} m_{11} & 0 \\ 0 & m_{22} \end{pmatrix} \begin{pmatrix} 0 & 1/d_2 \\ 1 & 0 \end{pmatrix}$$

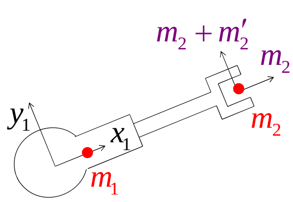




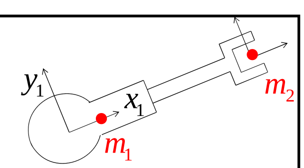
$$M \begin{bmatrix} m_1 l_1^2 \quad I_{zz1} \quad m_2 d_2^2 \quad I_{zz2} & 0 \\ 0 & m_2 \end{bmatrix}$$

$$m_2' = \frac{I_{zz1} + I_{zz2} + m_1 l_1^2}{d_2^{\,2}}$$

$${}^1M_x = \begin{pmatrix} m_2 & 0 \\ 0 & m_2 + m_2' \end{pmatrix}$$



$$M \begin{bmatrix} m_1 l_1^2 \quad I_{zz1} \quad m_2 d_2^2 \quad I_{zz2} & 0 \\ 0 & m_2 \end{bmatrix}$$



$${}^0M_x = \begin{pmatrix} c1 & -s1 \\ s1 & c1 \end{pmatrix} \begin{pmatrix} m_2 & 0 \\ 0 & m_2 + m_2' \end{pmatrix} \begin{pmatrix} c1 & s1 \\ -s1 & c1 \end{pmatrix}$$

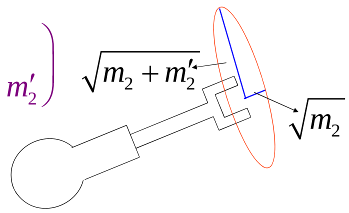
Task Space

$${}^0M_x = \begin{pmatrix} m_2 + m_2' s1^2 & -m_2' s\,c1 \\ -m_2' s\,c1 & m_2 + m_2' c1^2 \end{pmatrix}$$

Joint Space

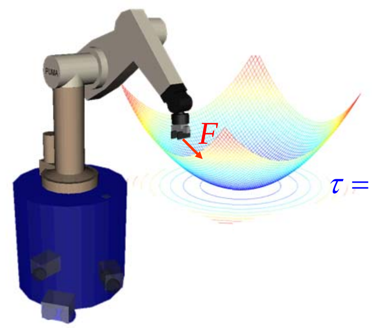
$$M \begin{bmatrix} m_1 l_1^2 \quad I_{zz1} \quad m_2 d_2^2 \quad I_{zz2} & 0 \\ 0 & m_2 \end{bmatrix}$$

$$ {}^{1}M_x = \begin{pmatrix} m_2 & 0 \\ 0 & m_2 + m_2' \end{pmatrix} $$



$$ {}^{0}\Lambda = \begin{pmatrix} m_2 + m_2' s1^2 & -m_2' s\, c1 \\ -m_2' s\, c1 & m_2 + m_2' c1^2 \end{pmatrix} $$

End-Effector Control



$$ \tau = J^T(q) F $$

Passive Systems (Stability)

$$ V_{goal} = \frac{1}{2} k_p \left(x - x_g \right)^T \left(x - x_g \right) $$

System $$ \frac{d}{dt}\left(\frac{\partial (K - V)}{\partial \dot{x}} \right) - \frac{\partial (K - V)}{\partial x} = F $$

$$ \Downarrow \; F = -\frac{\partial}{\partial X}\left(V_{goal} - \hat{V} \right) $$

Conservative Forces

$$ \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}} \right) - \frac{\partial \left(K - V_{goal} \right)}{\partial x} = 0 $$

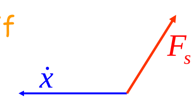
Stable

Asymptotic Stability

a system $$ \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}} \right) - \frac{\partial \left(K - V_{goal} \right)}{\partial x} = 0 \; F_s $$

is asymptotically stable if

$$ F_s^T \dot{x} < 0 \quad ; \; for \; \dot{x} \neq 0 $$



$$ F_s = -k_v \dot{x} \quad \rightarrow \quad k_v > 0 $$

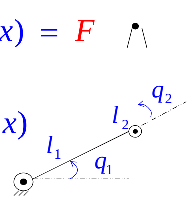
Control

$$ F = -k_p \left(x - x_{goal} \right) + \hat{G}_x - k_v \dot{x} $$

Example 2-d.o.f arm: Non-Dynamic Control

$$ M_x(x)\ddot{x} + V_x(x, \dot{x}) + G_x(x) = F $$

$$ F = -k_p \left(x - x_g \right) - k_v \dot{x} + \hat{G}(x) $$



$$ \left(m_1^* c^2 12 + m_2 \right) \ddot{x} + m_1^* \ddot{y} + V_{x1} = -k_p \left(x - x_g \right) - k_v \dot{x} $$

$$ \left(m_1^* c^2 12 + m_2 \right) \ddot{y} + m_1^* \ddot{x} + V_{x2} = -k_p \left(y - y_g \right) - k_v \dot{y} $$

Closed loop behavior

$$ m_{11}(q)\ddot{x} + k_v \dot{x} + k_p \left(x - x_g \right) = -\left(m_1^* \ddot{y} + V_{x1} \right) $$

$$ m_{22}(q)\ddot{y} + k_v \dot{y} + k_p \left(y - y_g \right) = -\left(m_1^* \ddot{x} + V_{x2} \right) $$

Nonlinear Dynamic Decoupling

Model

$$M_x(x)\ddot{x} + V_x(x,\dot{x}) + G_x(x) = F$$

Control Structure

$$F = \hat{M}(x)F' + \hat{V}_x(x,\dot{x}) + \hat{G}_x(x)$$

Decoupled System

$$I\,\ddot{x} = F'$$

with $\tau = J^T F$

Perfect Estimates

$$I\,\ddot{x} = F'$$

F' input of decoupled end-effector

Goal Position Control

$$F' = -k_v'\,\dot{x} - k_p'\left(x - x_g\right)$$

Closed Loop

$$I\,\ddot{x} + k_v'\dot{x} + k_p'(x - x_g) = 0$$

Trajectory Tracking

Trajectory: x_d, $\dot{x}_d$, $\ddot{x}_d$

$$F' = I\,\ddot{x}_d - k_v'(\dot{x} - \dot{x}_d) - k_p'(x - x_d)$$

$$(\ddot{x} - \ddot{x}_d) + k_v'(\dot{x} - \dot{x}_d) + k_p'(x - x_d) = 0$$

or $\ddot{\varepsilon}_x + k_v'\dot{\varepsilon}_x + k_p'\varepsilon_x = 0$

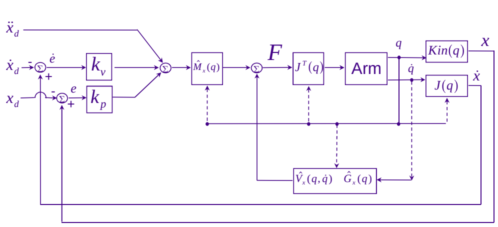
with $\varepsilon_x = x - x_d$

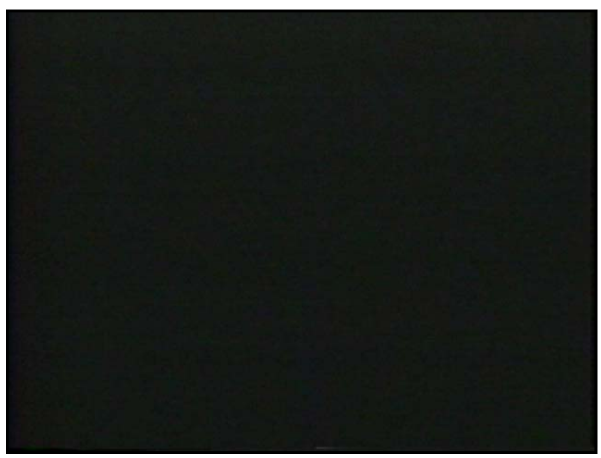
In joint space

$$\ddot{\varepsilon}_q + k_v'\dot{\varepsilon}_q + k_p'\varepsilon_q = 0$$

with $\varepsilon_q = q - q_d$

Task-Oriented Control



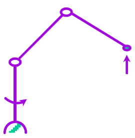


Compliance

$$I\ \ddot{x} = F'$$

$$F' = -\begin{pmatrix} k'_{p_x} & 0 & 0 \\ 0 & k'_{p_y} & 0 \\ 0 & 0 & k'_{p_z} \end{pmatrix}(x - x_d) - k'_v \dot{x}$$

set to zero



$$\ddot{x} + k'_v \dot{x} + k'_{px}(x - x_d) = 0$$

$$\ddot{y} + k'_v \dot{y} + k'_{py}(y - y_d) = 0$$

$$\ddot{z} + k'_v \dot{z} = 0$$

Compliance along Z

Stiffness

$$\ddot{z} + k'_v \dot{z} + k'_{p_z}(z - z_d) = 0$$

determines stiffness along z

Closed-Loop Stiffness: $\hat{M}_x k'_p = k_p$

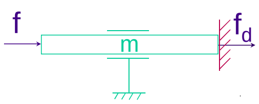
$$F = K_x(x - x_d)$$

$$J^T F \quad J^T K_x \quad x \quad (J^T K_x J) \qquad K$$

$$K \qquad J^T(\ \ )K_x J(\ \ )$$

Force Control

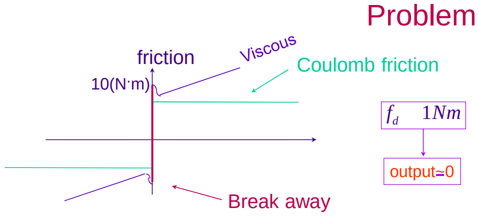
1-d.o.f.



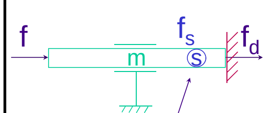
$m\ddot{x} \quad f$

set $f \quad f_d$

Problem



Force Sensing



Ⓢ —ww— ; $f_s \quad k_s x$

$m\ddot{x} \quad \underline{k_s x} \quad f$

f_s

At static Equilibrium

$f_s \quad f_d \qquad f \quad f_d$

Dynamics

$m\ddot{x} \quad k_s x \quad f_d \quad f_{Dynamic}$

Dynamics

$m\ddot{x} \quad \underline{k_s x} \quad f \qquad f_s \quad k_s x$

$\downarrow f_s \qquad\qquad \dot{f}_s \quad k_s \dot{x}$

$\frac{m}{k_s}\ddot{f}_s \quad f_s \quad f \qquad \ddot{f}_s \quad k_s \ddot{x}$

Control

$f_d \quad \frac{m}{k_s}(\ k_{p_f}(f_s \quad f_d) \quad k_{v_f}\dot{f}_s)$

Closed Loop

$$\frac{m}{k_s}[\ddot{f}_s \quad k_{v_f}\dot{f}_s \quad k_{p_f}(f_s \quad f_d)] \quad f_s \quad f_d$$

Steady-State error

$$\frac{m}{k_s}(\ddot{f}_s \quad k_{v_f}\dot{f}_s \quad k_{p_f}(f_s \quad f_d)) \quad (f_s \quad f_d) \quad 0$$

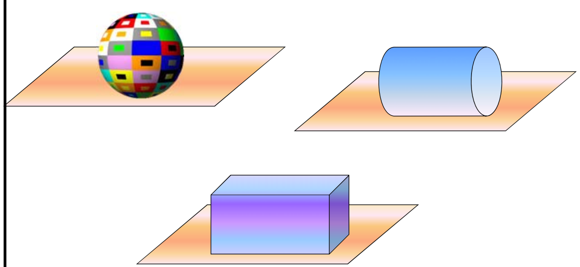
f_{dist}

$$\ddot{f}_s \quad \dot{f}_s \quad 0$$

$$(\frac{mk_{p_f}}{k_s} \quad 1)e_f \quad f_{dist}$$

$$e_f \quad \frac{f_{dist}}{1 \quad \frac{mk_{p_f}}{k_s}}$$

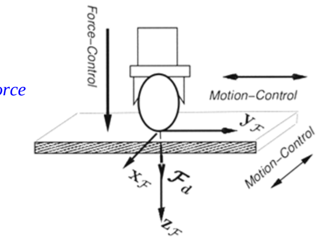
Task Description



Task Specification

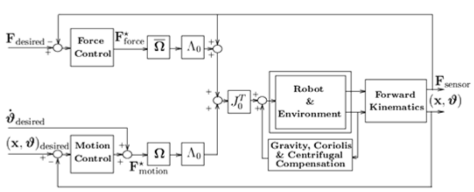
$$F = \Omega F_{motion} + \bar{\Omega} F_{force}$$

Selection matrix



$$\Omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \bar{\Omega} = I - \Omega$$

Unified Motion & Force Control



Two decoupled Subsystems

$$\Omega \dot{\vartheta} = \Omega F^{*}_{motion}$$

$$\bar{\Omega} \dot{\vartheta} = \bar{\Omega} F^{*}_{force}$$