Video Segment

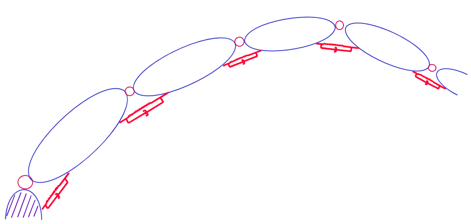
SCHAFT: DARPA Robotics Challenge 8 Tasks + Special Walking, 2013

Robot Control

Control

- Natural Systems
- PID Control
- Joint-Space Dynamic Control
- Task-Oriented Control
- Force Control

Joint-Space Control



Proportional-Derivative Control (PD)

$$m\ddot{x} \quad f \quad k_p(x \quad x_d) \quad k_v\dot{x}$$

$$m\ddot{x}+k_v\dot{x}+k_p(x-x_d)=0$$

Velocity gain (k_v) — Position gain (k_p)

$$1.\ddot{x}+\frac{k_v}{m}\dot{x}+\frac{k_p}{m}(x-x_d)=0$$

$$1.\ddot{x}+2\xi\omega\dot{x}+\omega^2(x-x_d)=0$$

$\xi = \frac{k_v}{2\sqrt{k_p m}}$ closed loop damping ratio

$\omega = \sqrt{\frac{k_p}{m}}$ closed loop frequency

Gains

$k_p \quad m \quad ^2$

$k_v \quad m(2 \quad)$

Gain Selection

set $\left(\quad \right)$ $\quad k_p \quad m \quad ^2$

$k_v \quad m(2 \quad)$

Unit mass system

$k_p \quad ^2$

$k_v \quad 2$

m - mass system

$k_p \quad m \; k_p$

$k_v \quad m \; k_v$

Control Partitioning

$m\ddot{x} \quad f \quad \Longrightarrow \quad m \; (1.\ddot{x}) = m \; f'$

$f = -k_v\dot{x} - k_p(x - x_d)$

$f = m[-k_v'\dot{x} - k_p'(x - x_d)] = m f'$

$m\ddot{x} = m f' \qquad f$

$1.\ddot{x} \quad f$ unit mass system

$1.\ddot{x} + k_v'\dot{x} + k_p'(x - x_d) = 0$

$2 \qquad\qquad ^2$

Non Linearities

$m\ddot{x} \quad b(x,\dot{x}) \quad f$

Control Partitioning

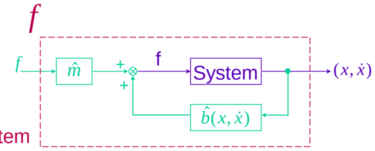
$f \quad f$

with $\hat{m}$

$\hat{b}(x,\dot{x})$

$m\ddot{x} \quad b(x,\dot{x}) \quad \hat{m}f \quad \hat{b}(x,\dot{x})$

$\longrightarrow 1.\ddot{x} \quad f$



Unit mass system

Motion Control

$m\ddot{x} \quad b(x,\dot{x}) \quad \frac{f}{f} \quad \frac{}{mf} \quad \frac{1.\ddot{x}}{b} \quad f$

Goal Position (x_d):

Control: $f \quad k_v\dot{x} \quad k_p(x \quad x_d)$

Closed-loop System: $1.\ddot{x} \quad k_v\dot{x} \quad k_p(x \quad x_d) \quad 0$

Trajectory Tracking

$x_d(t);\ \dot{x}_d(t);$ and $\ddot{x}_d(t)$

Control: $f \quad \ddot{x}_d \quad k_v(\dot{x} \quad \dot{x}_d) \quad k_p(x \quad x_d)$

Closed-loop System:

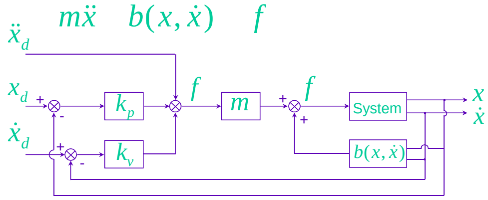
$(\ddot{x} \quad \ddot{x}_d) \quad k_v(\dot{x} \quad \dot{x}_d) \quad k_p(x \quad x_d) \quad 0$

with $e \quad x \quad x_d$

$\ddot{e} \quad k_v\dot{e} \quad k_p e \quad 0$

Disturbance Rejection

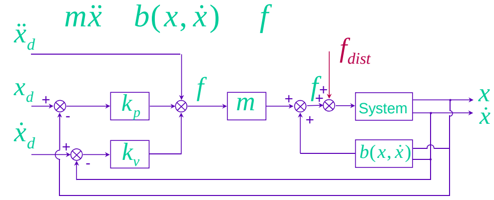
$m\ddot{x} \quad b(x,\dot{x}) \quad f$



$\ddot{e} \quad k_v\dot{e} \quad k_p e \quad 0$

Disturbance Rejection

$m\ddot{x} \quad b(x,\dot{x}) \quad f$



$m\ddot{x} \quad b(x,\dot{x}) \quad f \quad f_{dist}$ ← bounded

Control $\quad f \quad mf \quad b(x,\dot{x})$ $\qquad \{ \quad t \, |f_{dist}| \quad a\}$

Closed loop $\quad \ddot{e} \quad k_v\dot{e} \quad k_p e \quad \frac{f_{dist}}{m}$

Steady-State Error

$$\ddot{e} \quad k_v \dot{e} \quad k_p e \quad \frac{f_{dist}}{m}$$

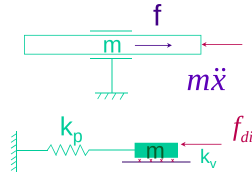
The steady-state $\left(\dot{e} = \ddot{e} = 0\right)$:

$$k_p e \quad \frac{f_{dist}}{m}$$

$$e \quad \frac{f_{dist}}{mk_p} \quad \frac{f_{dist}}{k_p}$$

Closed loop position gain (stiffness)

Steady-State Error - Example



$$m\ddot{x} \quad k_v \dot{x} \quad k_p(x \quad x_d) \quad 0$$

$$k_p(x \quad x_d) \quad f_{dist}$$

$$x \quad x_d \quad \frac{f_{dist}}{k_p}$$

$$x$$

$$f_{dist} \quad k_p \quad x$$

$$x \quad \frac{f_{dist}}{k_p}$$

Closed Loop Stiffness

PID (adding Integral action)

System $\quad m\ddot{x} \quad b(x,\dot{x}) \quad f \quad f_{dist}$

Control $\quad f \quad mf \quad b(x,\dot{x})$

$$f \quad \ddot{x}_d \quad k_v(\dot{x} \quad \dot{x}_d) \quad k_p(x \quad x_d) \quad k_i\int(x \quad x_d)dt$$

Closed-loop System

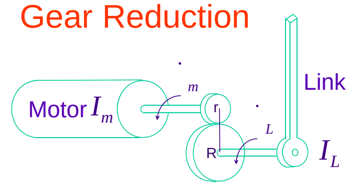
$$\ddot{e} \quad k_v \dot{e} \quad k_p e \quad k_i \int edt \quad \frac{f_{dist}}{m}$$

$$\dddot{e} \quad k_v \ddot{e} \quad k_p \dot{e} \quad k_i e \quad 0$$

constant

Steady-state Error $\quad e \quad 0$

Gear Reduction



Gear ratio $\quad \frac{R}{r}$

$$L \quad (\frac{1}{}) \quad m$$

$$L \quad m$$

$$m \quad I_m \ddot{}_m \quad \frac{1}{}(I_L \ddot{}_L) \quad b_m \dot{}_m \quad \frac{1}{} b_L \dot{}_L$$

$$\ddot{}_L \quad \frac{1}{}\ddot{}_m$$

$$m \quad (I_m \quad \frac{I_L}{^2})\ddot{}_m \quad (b_m \quad \frac{b_L}{^2})\dot{}_m$$

$$L \quad (I_L \quad ^2 I_m)\ddot{}_L \quad (b_L \quad ^2 b_m)\dot{}_L$$

Effective Inertia $\qquad$ Effective Damping

Effective Inertia

$$I_{eff} \quad I_L \quad ^2 I_m$$

for a manipulator $\quad I_L \quad I_L(q)$

1

Direct Drive

Gain Selection

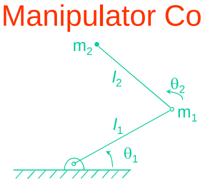
$$k_p \quad (I_L \quad ^2 I_m)k_p$$

$$k_v \quad (I_L \quad ^2 I_m)k_v$$

Time Optimal Selection

$$\hat{I}_L \quad \frac{1}{4}(\sqrt{I_{L_{min}}} \quad \sqrt{I_{L_{max}}})^2$$

Manipulator Control

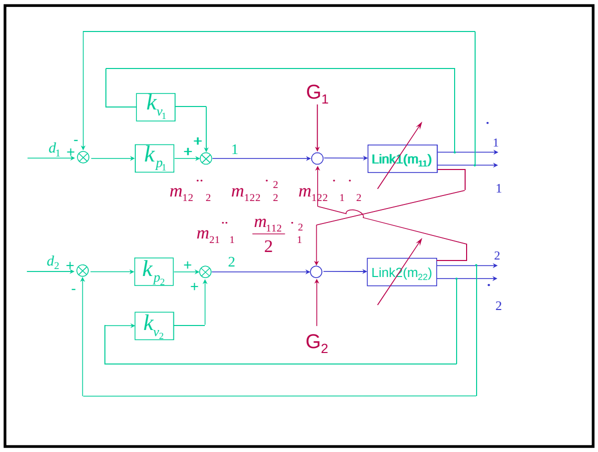


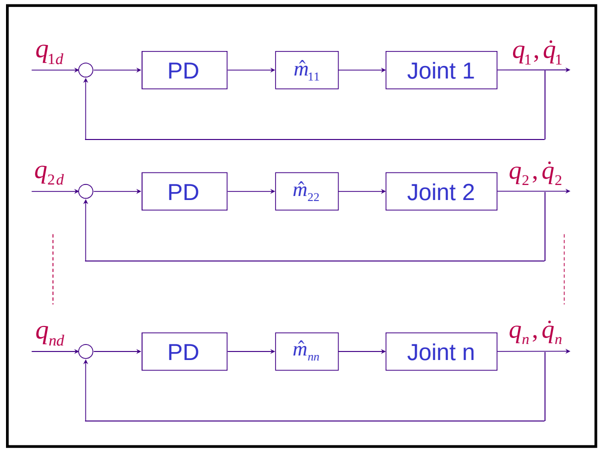
$$M(\;)\ddot{} \quad V(\;,\;\dot{}\;) \quad G(\;)$$

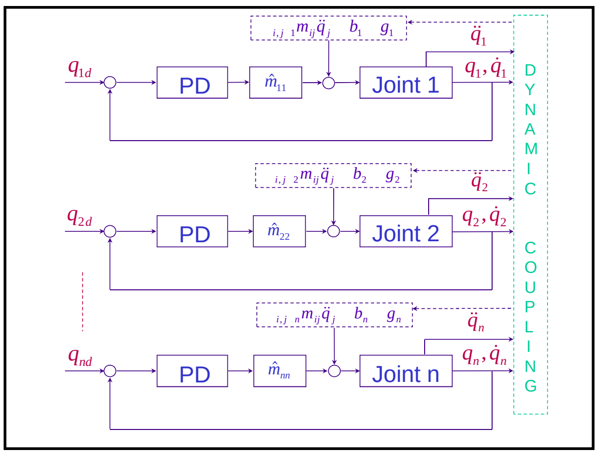
$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}\begin{pmatrix} \ddot{}_1 \\ \ddot{}_2 \end{pmatrix} \begin{pmatrix} m_{112} \\ 0 \end{pmatrix}(\dot{}_1 \quad \dot{}_2) \begin{pmatrix} 0 & m_{122} \\ \frac{m_{112}}{2} & 0 \end{pmatrix}\begin{pmatrix} \dot{}_1^2 \\ \dot{}_2^2 \end{pmatrix} \begin{pmatrix} G_1 \\ G_2 \end{pmatrix} \begin{pmatrix} _1 \\ _2 \end{pmatrix}$$

$$\underline{m_{11}\ddot{}_1} \quad m_{12}\ddot{}_2 \quad m_{112}\dot{}_1\dot{}_2 \quad m_{122}\dot{}_2^2 \quad G_1 \quad \underline{\;_1}$$

$$\underline{m_{22}\ddot{}_2} \quad m_{21}\ddot{}_1 \quad \frac{m_{112}}{2}\dot{}_1^2 \quad G_2 \quad \underline{\;_2}$$







PD Control Stability

$$M(q)\ddot{q}+B(q)[\dot{q}\dot{q}]+C(q)[\dot{q}^2]+G(\theta)=\tau$$

$$\tau=-k_p(q-q_d)-k_v\dot{q}$$

$$V_d=1/2k_p(q-q_d)^2$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right)-\frac{\partial K}{\partial q}+\frac{\partial V_s}{\partial q}=\tau\frac{\partial V_d}{\partial q}-k_v\dot{q}$$

PD Control Stability

$$M(q)\ddot{q}+B(q)[\dot{q}\dot{q}]+C(q)[\dot{q}^2]+G(\theta)=\tau$$

$$\tau=-k_p(q-q_d)-k_v\dot{q}$$

$$V_d=1/2k_p(q-q_d)^T(q-q_d)$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right)-\frac{\partial K}{\partial q}+\frac{\partial (V_s-V_d)}{\partial q}=\tau_s$$

$\tau_s=-k_v\dot{q}$ with $\tau_s^T\dot{q}<0$ for $\dot{q}\neq 0;\ k_v>0$

Performance

High Gains → better disturbance rejection

Gains are limited by
- structural flexibilities
- time delays (actuator-sensing)
- sampling rate

$n \quad \frac{res}{2}$ ← lowest structural flexibility

$n \quad \frac{delay}{3}$ ← largest delay $\left(\frac{2}{delay}\right)$

$n \quad \frac{sampling\ rate}{5}$

Nonlinear Dynamic Decoupling

$$M(\ )\ddot{\ }\quad V(\ ,\dot{\ })\quad G(\ )$$

$$\hat{M}(\ )\_\quad \hat{V}(\ ,\dot{\ })\quad \hat{G}(\ )$$

1. $\ddot{\ }\quad (M^{1}\hat{M})\quad M^{1}[(V\quad \hat{V})\quad (G\quad \hat{G})]$

with perfect estimates

1. $\ddot{\ }\quad (t)$

: input of the unit-mass systems

$$\ddot{\ }_d\quad k_v(\dot{\ }\quad \dot{\ }_d)\quad k_p(\quad _d)$$

Closed-loop

$$\ddot{E}\quad k_v\dot{E}\quad k_pE\quad 0\quad (t)$$