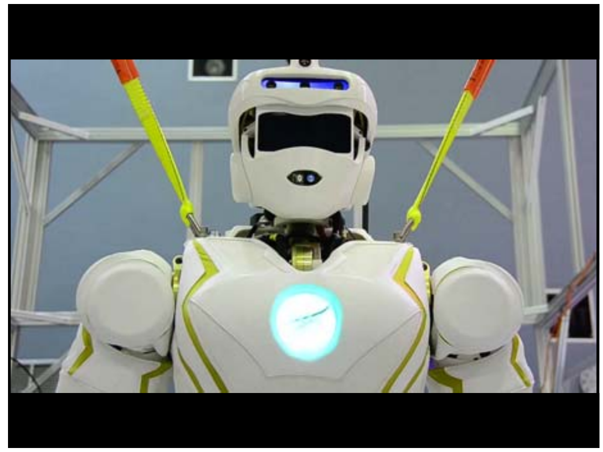
Video Segment

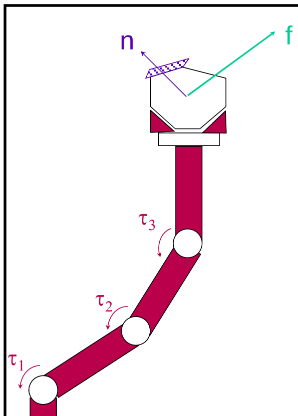
Valkyrie: NASA's Superhero Robot, IEEE Spectrum, 2013



Robot Control

Control

- Natural Systems
- PID Control
- Joint-Space Dynamic Control
- Task-Oriented Control
- Force Control

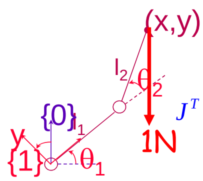


Velocity/Force Duality

$$\dot{x} = J\,\dot{\theta}$$

$$\tau = J^T F$$

Example (Static Forces)



$$J = \begin{pmatrix} -(l_1S1+l_2S12) & -l_2S12 \\ l_1C1+l_2C12 & l_2C12 \end{pmatrix}$$

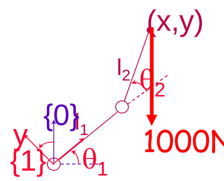
$$J^T = \begin{pmatrix} -(l_1S1+l_2S12) & l_1C1+l_2C12 \\ -l_2S12 & l_2C12 \end{pmatrix}$$

$$\tau = J^T F$$

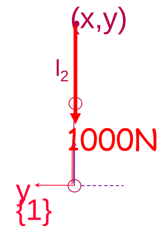
$l_1 = l_2 = 1; \quad \theta_1 = 0; \ \theta_2 = 60^\circ$

$$\tau = \begin{pmatrix} -(l_1S1+l_2S12) & l_1C1+l_2C12 \\ -l_2S12 & l_2C12 \end{pmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -\begin{bmatrix} l_1C1+l_2C12 \\ l_2C12 \end{bmatrix} = -\begin{bmatrix} 3/2 \\ 1/2 \end{bmatrix}$$

Example (Static Forces)



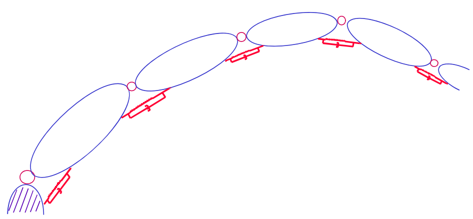
$$\tau = J^T F$$



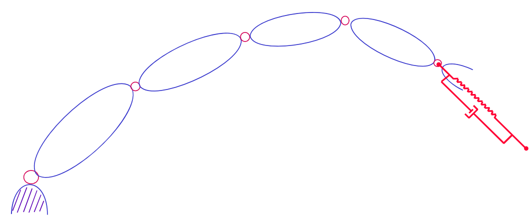
$$\tau = \begin{pmatrix} -(l_1S1+l_2S12) & l_1C1+l_2C12 \\ -l_2S12 & l_2C12 \end{pmatrix} \begin{bmatrix} 0 \\ -1K \end{bmatrix} = \begin{bmatrix} l_1C1+l_2C12 \\ l_2C12 \end{bmatrix} (-1K) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$l_1 = l_2 = 1; \quad \theta_1 = 90; \theta_2 = 0^\circ$

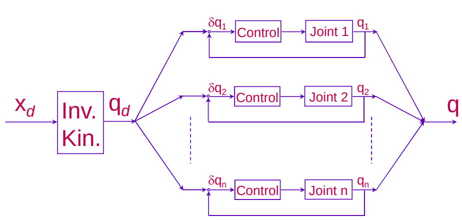
Joint-Space Control



Task-Oriented Control



Joint Space Control



Resolved Motion Rate Control (Whitney 72)

$$x \qquad J(\ )$$

Outside singularities

$$J^{1}(\ )\ x$$

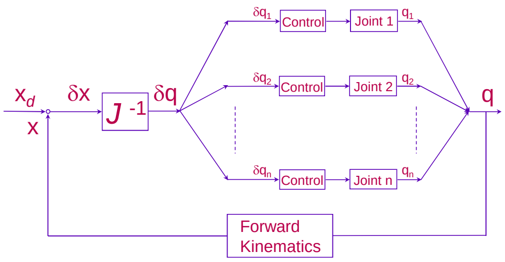
Arm at Configuration

$$x \qquad f(\ )$$

$$x \qquad x_d \qquad x$$

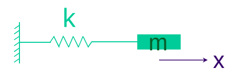
$$J^{1}\ x$$

Resolved Motion Rate Control



Natural Systems

Conservative Systems



$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial \dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=0 \qquad K=\frac{1}{2}m\dot{x}^2$$

Natural Systems

Conservative Forces

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right)-\frac{\partial K}{\partial x}=-\frac{\partial V}{\partial x}$$

$$m\ddot{x} = F = -kx$$

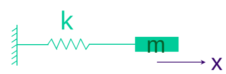
$$-\frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right)$$

Potential Energy of a spring

$$V = Work = \int_x^0 (-kx)\delta x = \frac{1}{2}kx^2$$

Natural Systems

Conservative Forces



$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right)-\frac{\partial K}{\partial x}=-\frac{\partial V}{\partial x}$$

Potential Energy of a spring

$$V = Work = \int_x^0 (-kx)\delta x = \frac{1}{2}kx^2$$

$$m\ddot{x} = F = -kx$$

$$-\frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right)$$

Natural Systems

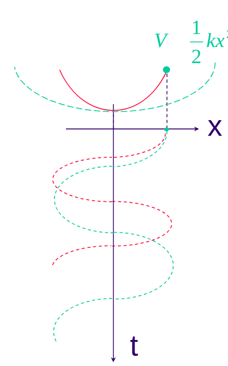
Conservative Systems

$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial \dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=0 \qquad K=\frac{1}{2}m\dot{x}^2$$

$$m\ddot{x} + kx = 0 \qquad V=\frac{1}{2}kx^2$$

Natural Systems

Conservative Systems



$$m\ddot{x} + kx = 0$$

Frequency increases
with stiffness
and inverse mass

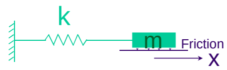
Natural Frequency $\quad {}_n \quad \sqrt{\frac{k}{m}}$

$$\ddot{x} \quad {}_n^2 x \quad 0$$

$$x(t) \quad c\ \cos(\quad {}_n t \quad)$$

Natural Systems

Dissipative Systems

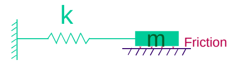


$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial \dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=f_{friction}$$

Viscous friction: $f_{friction} \quad b\dot{x}$

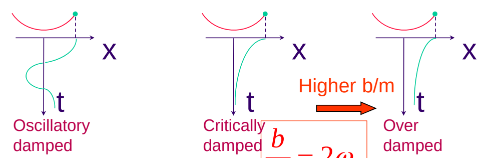
$$m\ddot{x} \quad b\dot{x} \quad kx \quad 0$$

Dissipative Systems



$$m\ddot{x} \quad b\dot{x} \quad kx \quad 0$$

$$\ddot{x} \quad \frac{b}{m}\dot{x} \quad \frac{k}{m}x \quad 0$$



$$\frac{b}{m}=2\omega_n$$

2d order systems

$$m\ddot{x} \quad b\dot{x} \quad kx \quad 0$$

$$\ddot{x} \quad \frac{b}{m}\dot{x} \quad \frac{k}{m}x \quad 0$$

b/m — $2 \quad {}_n$; ${}^2_n$

$2 \quad {}_n$

Natural damping ratio

Critically damped when $b/m=2\omega_n$

$${}_n \quad \frac{b}{2 \quad {}_n m} \quad \frac{b}{2\sqrt{km}}$$

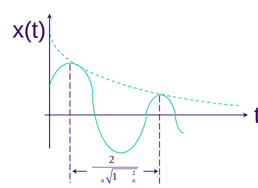
Critically damped system: ${}_n \quad 1 \quad (b \quad 2\sqrt{km})$

Time Response

$$\ddot{x} \quad 2 \quad {}_n \quad {}_n\dot{x} \quad {}^2_n x \quad 0$$

Natural frequency ${}_n \quad \sqrt{\frac{k}{m}}$; ${}_n \quad \frac{b}{2\sqrt{km}}$ Natural damping ratio

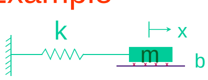
$$x(t) \quad ce \quad {}^{n \; nt} \cos(\quad {}_n\sqrt{1 \quad {}^2_n}\,t \quad)$$



damped Natural frequency

$$\omega=\omega_n\sqrt{1-\xi_n^2}$$

Example



$$m\ddot{x} \quad b\dot{x} \quad kx \quad 0$$

$m \quad 2.0$

$b \quad 4.8$

$k \quad 8.0$

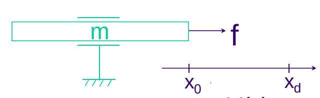
what is the "damped Natural frequency"

$${}_n\sqrt{1 \quad {}^2_n}$$

$${}_n \quad \sqrt{\frac{k}{m}} \quad 2 \; ; \quad {}_n \quad \frac{b}{2\sqrt{km}} \quad 0.6$$

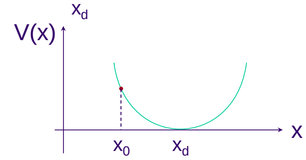
$$2\sqrt{1 \quad 0.36} \quad 1.6$$

1-dof Robot Control



$$m\ddot{x} \quad f$$

Potential Field



$V(x) \quad 0, x \quad x_d$

$V(x) \quad 0, x \quad x_d$

$$V(x) \quad \frac{1}{2}k_p(x \quad x_d)^2 \; ; \; f \quad V(x) \quad \frac{V}{x}$$

$$m\ddot{x} \quad \frac{}{x}\left[\frac{1}{2}k_p(x \quad x_d)^2\right] \; ; \; m\ddot{x} \quad k_p(x \quad x_d) \quad 0$$

Position gain

Passive Systems (Stability)

$$V_{goal} = \frac{1}{2} k_p \left(x - x_g\right)^T \left(x - x_g\right)$$

System $\quad \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial K}{\partial x} = f$

$$\Downarrow \quad f = -\frac{\partial V_{goal}}{\partial X}$$

Conservative Forces

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial \left(K - V_{goal}\right)}{\partial x} = 0$$

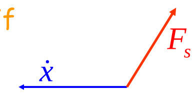
Stable

Asymptotic Stability

a system $\quad \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial \left(K - V_{goal}\right)}{\partial x} = 0 F_s$

is asymptotically stable if

$$F_s^T \dot{x} < 0 \quad ; \; for \; \dot{x} \neq 0$$



$$F_s = -k_v \dot{x} \;\rightarrow\; k_v > 0$$

Control

$$F = -k_p \left(x - x_{goal}\right) - k_v \dot{x}$$

Proportional-Derivative Control (PD)

$$m\ddot{x} \quad f \qquad k_p(x \quad x_d) \quad k_v \dot{x}$$

$$m\ddot{x} + k_v \dot{x} + k_p (x - x_d) = 0$$

Velocity gain $\quad$ Position gain

$$1.\ddot{x} + \frac{k_v}{m}\dot{x} + \frac{k_p}{m}(x - x_d) = 0$$

$$1.\ddot{x} + 2\xi\omega\dot{x} + \omega^2 (x - x_d) = 0$$

$\frac{k_v}{2\sqrt{k_p m}}$ closed loop damping ratio

$\sqrt{\frac{k_p}{m}}$ closed loop frequency

Gains

$k_p \quad m \quad ^2$

$k_v \quad m(2 \quad)$

Gain Selection

set $\left(\quad \right)$ $\quad k_p \quad m \quad ^2$

$k_v \quad m(2 \quad)$

Unit mass system

$k_p \quad ^2$

$k_v \quad 2$

m - mass system

$k_p \quad m \; k_p$

$k_v \quad m \; k_v$

Control Partitioning

$$m\ddot{x} \quad f \;\Longrightarrow\; m \;(1.\ddot{x}) = m \;\; f'$$

$$f = -k_v \dot{x} - k_p (x - x_d)$$

$$f = m[-k_v' \dot{x} - k_p' (x - x_d)] = m f'$$

$$m\ddot{x} = m f' \qquad f$$

$1.\ddot{x} \quad f$ unit mass system

$$1.\ddot{x} + k_v' \dot{x} + k_p' (x - x_d) = 0$$

$2 \qquad\qquad ^2$