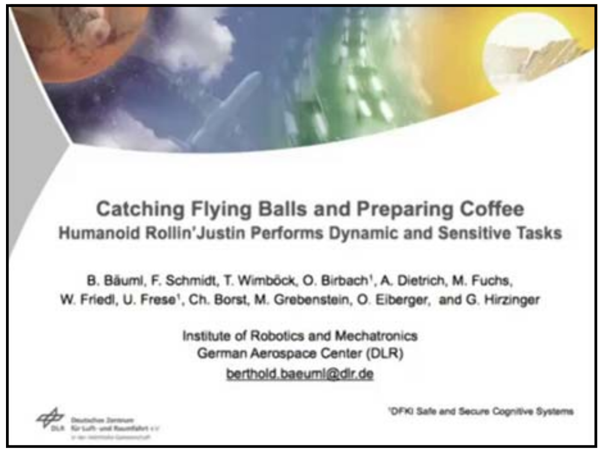
Movie Segment

Catching Flying Balls and Preparing Coffee: Humanoid Rollin'Justin Performs Dynamic and Sensitive Tasks.

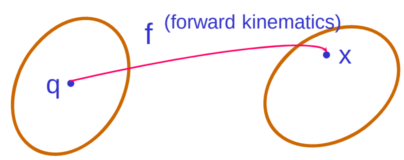
Berthold Bäuml, Florian Schmidt, Thomas Wimböck, Oliver Birbach, Alexander Dietrich, Matthias Fuchs, Werner Friedl, Udo Frese, Christoph Borst, Markus Grebenstein, Oliver Eiberger, and Gerd Hirzinger.

ICRA Video Proceedings, 2009.



Inverse Kinematics

Direct Kinematics



Joint Space (dimensions n)

Task Space (dimensions m)

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ \cdot \\ \cdot \\ \cdot \\ q_n \end{bmatrix} \qquad \mathbf{x} = f(\mathbf{q}) \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_m \end{bmatrix}$$

Joint Coordinates

Revolute Joints θ_i

Prismatic Joints d_i

$$q_i = \bar{\varepsilon}_i \theta_i + \varepsilon_i d_i$$

$$\varepsilon_i = \begin{cases} 0 & \text{revolute joint} \\ 1 & \text{prismatic joint} \end{cases}$$

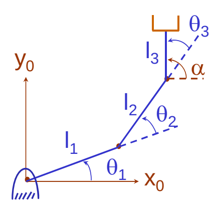
$$\bar{\varepsilon}_i \equiv 1 - \varepsilon_i$$

Direct Kinematics

Given $\mathbf{q} = (q_1 \quad q_2 \quad \dots \quad q_n)^T$

${}^0_nT = {}^0_nT(\mathbf{q})$ or $\mathbf{x} = f(\mathbf{q})$ (Geometric Model)

Inverse Kinematics

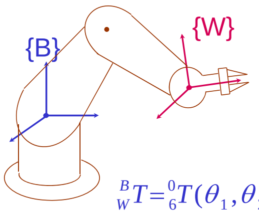


$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \alpha \end{bmatrix} = f(\mathbf{q})$$

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \alpha \end{bmatrix} = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ \theta_1 + \theta_2 + \theta_3 \end{bmatrix} \qquad \cos(\theta_1 + \theta_2) = c_{12}$$

Given **q** ⟶ a unique **x**

Inverse Kinematics



$${}^{B}_{W}T = {}^{0}_{6}T(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$$

or $X = \begin{bmatrix} X_P \\ X_R \end{bmatrix} = f(\Theta)$

Inverse Problem

Given (${}^{B}_{W}T$ or X) find Θ

Inverse Kinematics

Finding

$$\Theta = f^{-1}(X)$$

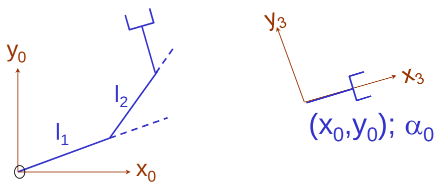
or

Solving

$${}^{0}_{6}T(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = {}^{B}_{W}T$$

(12 equations
6 unknowns)

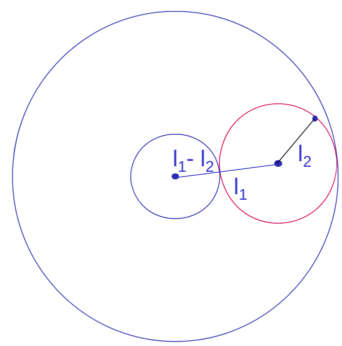
Existence of Solutions



$${}^{0}_{3}T = \begin{pmatrix} c_{123} & -s_{123} & 0 & l_1c_1 + l_2c_{12} \\ s_{123} & c_{123} & 0 & l_1s_1 + l_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c\alpha_0 & -s\alpha_0 & 0 & x_0 \\ s\alpha_0 & c\alpha_0 & 0 & y_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

solution if

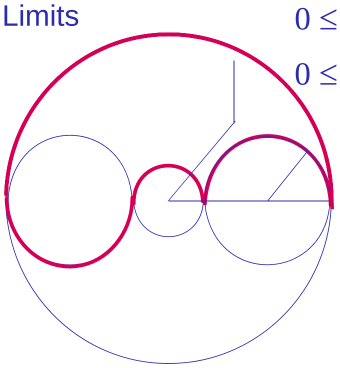
$$(l_1 - l_2)^2 \le x_0^2 + y_0^2 \le (l_1 + l_2)^2$$



Joint Limits

$$0 \le \theta_1 \le 180^o$$

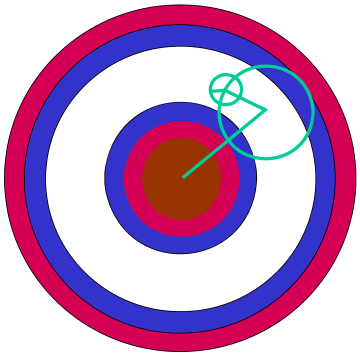
$$0 \le \theta_2 \le 180^o$$



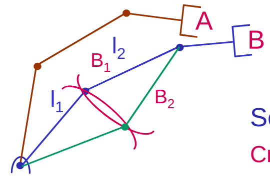
Workspace

- Reachable Workspace
- Dextrous Workspace

Dextrous Workspace



Multiplicity of Solutions



Selection of a solution

Criterion: Joint distance

$$C_1 = \left\| \Theta_{(B1)} - \Theta_{(A)} \right\|$$

$$C_2 = \left\| \Theta_{(B2)} - \Theta_{(A)} \right\|$$

Weighted Joint distance
moving smaller joints

Number of Solutions

It depends on

- Number of Joints
- Link Parameters

  e.g. 6-revolute-joint manipulator

  if all $a_i \neq 0$ Number solutions $\leq$ 16

  if $a_1 = a_3 = a_5 = 0$ Number solutions $\leq$ 4
- Range of Motion

General Mechanism with 6 d.o.f.

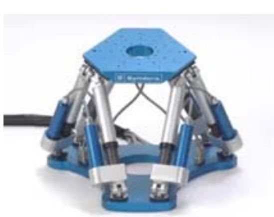
Number of solutions $\leq$ 16

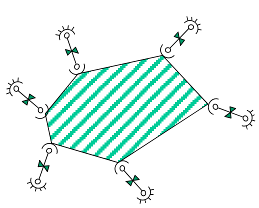
Main Results

General 6R open-chain 16 solutions
General 5RP open-chain 16 solutions
General 4R2P open-chain 8 solutions
General 3R3P open-chain 2 solutions

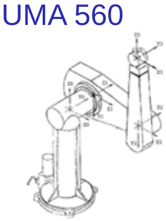
Special conditions in the structure [such as intersecting or parallel axes] cause the general number of solutions to reduce.
There exist open-chain manipulators with 16, 14, 12, 10, 8, 6, 4, 2 solutions.

For a given set of 6 lengths of the legs
General in-parallel structure has
40 configurations
By specializing structure the number of configurations can be reduced





PUMA 560



__Solutions

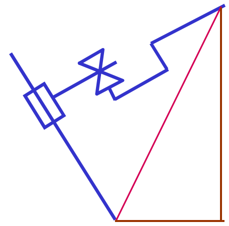
$\theta_4 \longrightarrow \theta_4 + 180^o$

$\theta_5 \longrightarrow -\theta_5$

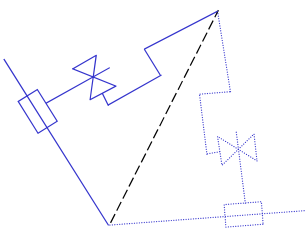
$\theta_6 \longrightarrow \theta_6 + 180^o$

Stanford Scheinman Arm

Stanford Scheinman Arm



Stanford Scheinman Arm



Solvability

A manipulator is solvable if ALL the sets of solutions can be determined.

6 d.o.f. open-chain mechanisms are "now" solvable.

(the general solution is a numerical one)

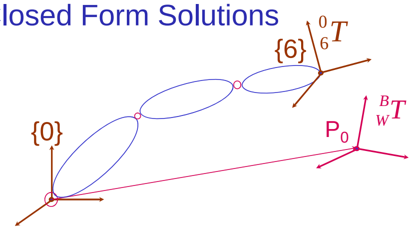
Closed Form Solutions

Analytical Solutions - Exist for a large class of mechanisms.

Sufficient Condition

3 intersecting neighboring axes (most industrial robots)

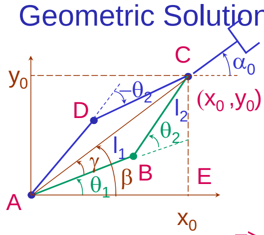
Closed Form Solutions



$$^0_6T(\theta_1, \theta_2, \ldots, \theta_6) = {}^B_WT$$

Solutions:
- Algebraic
- Geometric

Geometric Solutions



First θ_2:

$$l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta_2 = x_0^2 + y_0^2$$

$$\cos\theta_2 = \frac{(x_0^2 + y_0^2) - (l_1^2 + l_2^2)}{2 l_1 l_2}$$

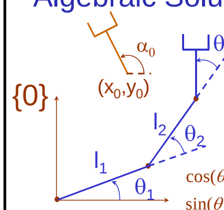
=> θ_2 and $-\theta_2$

θ_1: $l_2^2 = l_1^2 + (x_0^2 + y_0^2) - 2 l_1 \sqrt{x_0^2 + y_0^2}\cos\gamma$

$$\cos\gamma = \frac{x_0^2 + y_0^2 + l_1^2 - l_2^2}{2 l_1 \sqrt{x_0^2 + y_0^2}} \quad \text{and} \quad \tan\beta = \frac{y_0}{x_0}$$

θ_3: $\theta_1 = \beta \pm \gamma$ $\qquad \theta_3 = \alpha_0 - (\theta_1 + \theta_2)$

Algebraic Solutions



$${}^0_3T \equiv {}^B_WT$$

$$\begin{pmatrix} c_{123} & -s_{123} & 0 & l_1c_1 + l_2c_{12} \\ s_{123} & c_{123} & 0 & l_1s_1 + l_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c\alpha_0 & -s\alpha_0 & 0 & x_0 \\ s\alpha_0 & c\alpha_0 & 0 & y_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\left.\begin{matrix} \cos(\theta_1 + \theta_2 + \theta_3) = \cos\alpha_0 \\ \sin(\theta_1 + \theta_2 + \theta_3) = \sin\alpha_0 \end{matrix}\right\} \Rightarrow \theta_1 + \theta_2 + \theta_3 = \alpha_0$$

For θ_1 and θ_2: $l_1c_1 + l_2c_{12} = x_0$

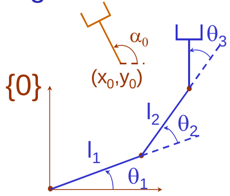
$l_1s_1 + l_2s_{12} = y_0$

Solution if (x_0, y_0) is in the workspace

$$-1 \le \cos\theta_2 = \frac{(x_0^2 + y_0^2) - (l_1^2 + l_2^2)}{2 l_1 l_2} \le 1$$

=> $\theta_2 = A\tan2(\pm\sqrt{1 - \cos^2\theta_2}, \cos\theta_2)$

Algebraic Solutions



$l_1c_1 + l_2c_{12} = x_0$

$l_1s_1 + l_2s_{12} = y_0$

For θ_1:

$$\left.\begin{matrix} (l_1 + l_2c_2)c_1 - (l_2s_2)s_1 = x_0 \\ (l_1 + l_2c_2)s_1 + (l_2s_2)c_1 = y_0 \end{matrix}\right\}$$

$$\Rightarrow \left.\begin{matrix} k_1c_1 - k_2s_1 = x_0 \\ k_1s_1 + k_2c_1 = y_0 \end{matrix}\right\}$$

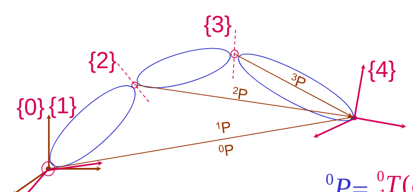
$$(k_1, k_2) \xrightarrow[\tan\gamma = k_2/k_1]{r = \sqrt{k_1^2 + k_2^2}} \begin{pmatrix} k_1 = r.\cos\gamma \\ k_2 = r.\sin\gamma \end{pmatrix}$$

=> $x_0 = r.\cos(\theta_1 + \gamma)$

$y_0 = r.\sin(\theta_1 + \gamma)$

=> $\theta_1 = A\tan2(y_0, x_0) - A\tan2(k_2, k_1)$

Pieper's Solution

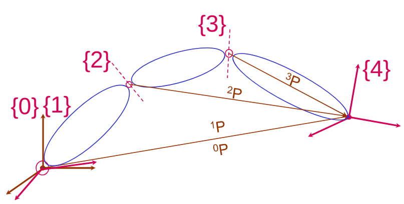


$${}^1P = {}^1_2T(\theta_2)\,.{}^2P$$

$${}^0P = {}^0_1T(\theta_1)\,.{}^1P$$

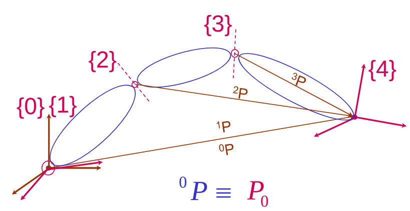
$${}^1P = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ 1 \end{bmatrix}; g_i = g_i(c_2, s_2, f_i) \qquad {}^0P = \begin{bmatrix} c_1g_1 - s_1g_2 \\ s_1g_1 + c_1g_2 \\ g_3 \\ 1 \end{bmatrix}$$

Pieper's Solution



$${}^3P = \begin{bmatrix} a_3 \\ -s\alpha_3.d_4 \\ c\alpha_3.d_4 \\ 1 \end{bmatrix} \qquad {}^2P = {}^2_3T(\theta_3).{}^3P \qquad {}^2P = \begin{bmatrix} f_1(\theta_3) \\ f_2(\theta_3) \\ f_3(\theta_3) \\ 1 \end{bmatrix}$$

Pieper's Solution



$${}^0P \equiv P_0$$

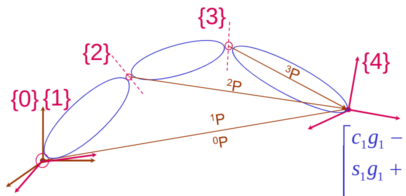
$$\begin{bmatrix} c_1g_1 - s_1g_2 \\ s_1g_1 + c_1g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

For θ_1: $\left.\begin{matrix} c_1g_1 - s_1g_2 = x_0 \\ s_1g_1 + c_1g_2 = y_0 \end{matrix}\right\} \theta_1$

if g_1 and g_2 are known

$$\theta_1 = \text{Atan2}(y_0, x_0) - \text{Atan2}(g_2, g_1)$$

Pieper's Solution



$$\begin{bmatrix} c_1 g_1 - s_1 g_2 \\ s_1 g_1 + c_1 g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

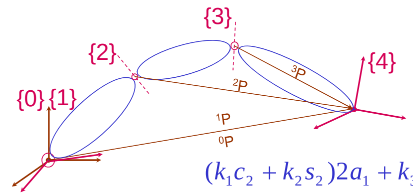
For θ_2: $\begin{cases} g_1^2 + g_2^2 + g_3^2 = x_0^2 + y_0^2 + z_0^2 = r_0^2 \\ g_3 = z_0 \end{cases}$

$g_i = g_i(c_2, s_2, f_1, f_2, f_3)$

$$(k_1 c_2 + k_2 s_2) 2 a_1 + k_3 = r_0^2$$
$$(k_1 s_2 - k_2 c_2) s\alpha_1 + k_4 = z_0$$

$k_i = k_i(f_1, f_2, f_3) \rightarrow \theta_2$ if k_i are known

Pieper's Solution



$$(k_1 c_2 + k_2 s_2) 2 a_1 + k_3 = r_0^2$$
$$(k_1 s_2 - k_2 c_2) s\alpha_1 + k_4 = z_0$$

For θ_3:

$$(r_0^2 - k_3)^2 . s^2\alpha_1 + (Z_0 - k_4)^2 . 4. a_1^2 = 4. a_1^2 . s^2\alpha_1 (k_1^2 + k_2^2)$$

$$k_i = k_i(f_i(c_3, s_3))$$

Transcendental Equations
Reduction to Polynomial

$$u = \tan\frac{\theta}{2} \Rightarrow \begin{cases} \cos\theta = \dfrac{1-u^2}{1+u^2} \\ \sin\theta = \dfrac{2u}{1+u^2} \end{cases}$$

For θ_3: $k_i = k_i(u, u^2)$

$$A.u^4 + B.u^3 + C.u^2 + D.u + E = 0$$

with $u = \tan\dfrac{\theta_3}{2}$

For θ_4, θ_5, and θ_6

$$^0_6R(\Theta) \equiv R_0$$

$$^0_6R(\Theta) = {}^0_1R(\theta_1).{}^1_2R(\theta_2).{}^2_3R(\theta_3).{}^3_4R(\theta_4).{}^4_5R(\theta_5).{}^5_6R(\theta_6)$$

$$^3_4R(\theta_4) = {}^3_4R|_{\theta_4=0} . R_Z(\theta_4)$$

$$[{}^0_4R|_{\theta_4=0}(\theta_1, \theta_2, \theta_3)].[R_Z(\theta_4).{}^4_6R(\theta_5, \theta_6)] = R_0$$

is known — R

$$R(\theta_4, \theta_5, \theta_6) = R'_0$$

Euler Angle Solution