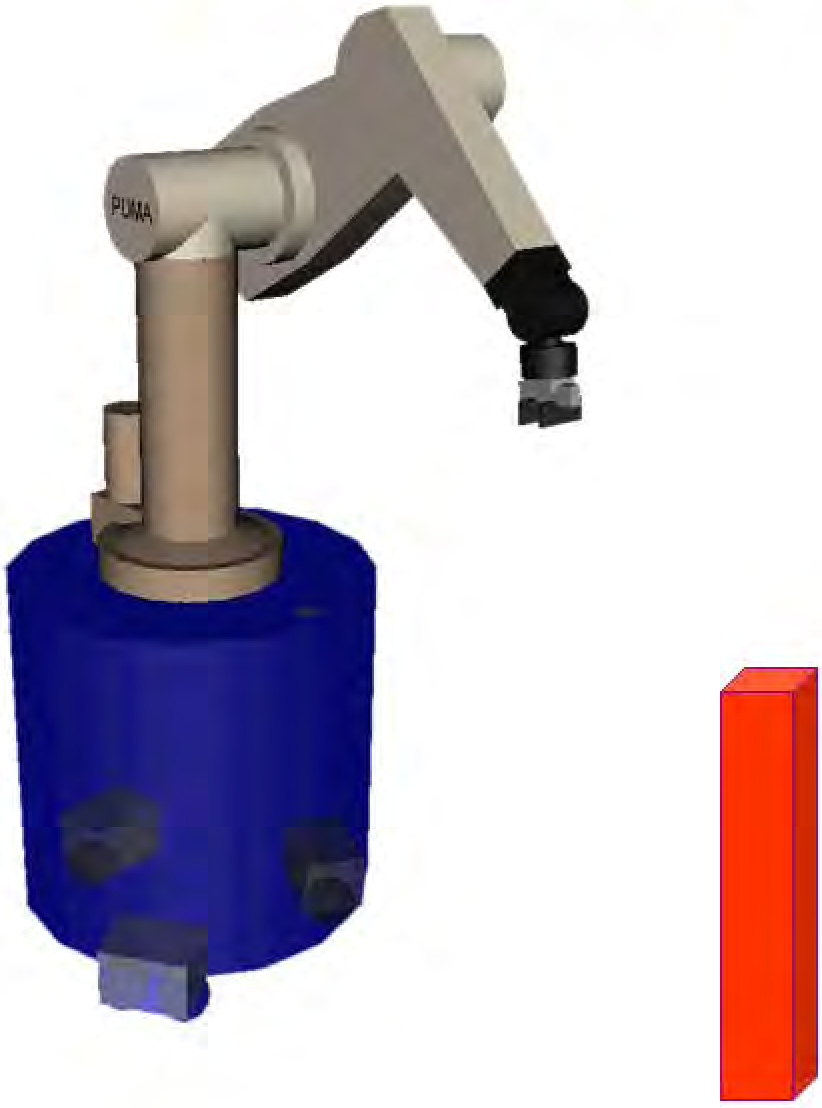
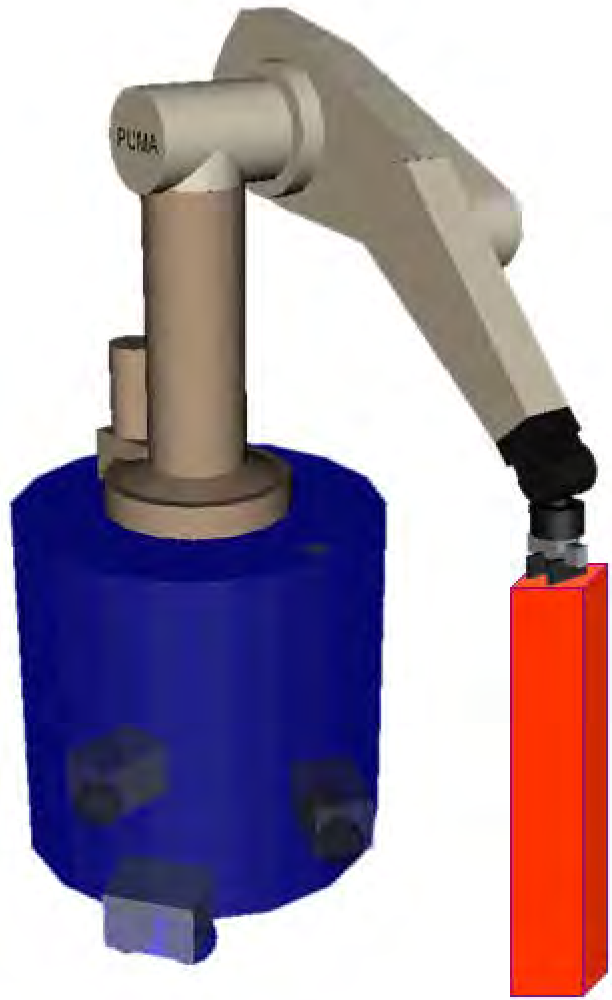
Video Segment

The bebionic3 Prostethic Hand,
RSLSTEPPER, 2010+2013

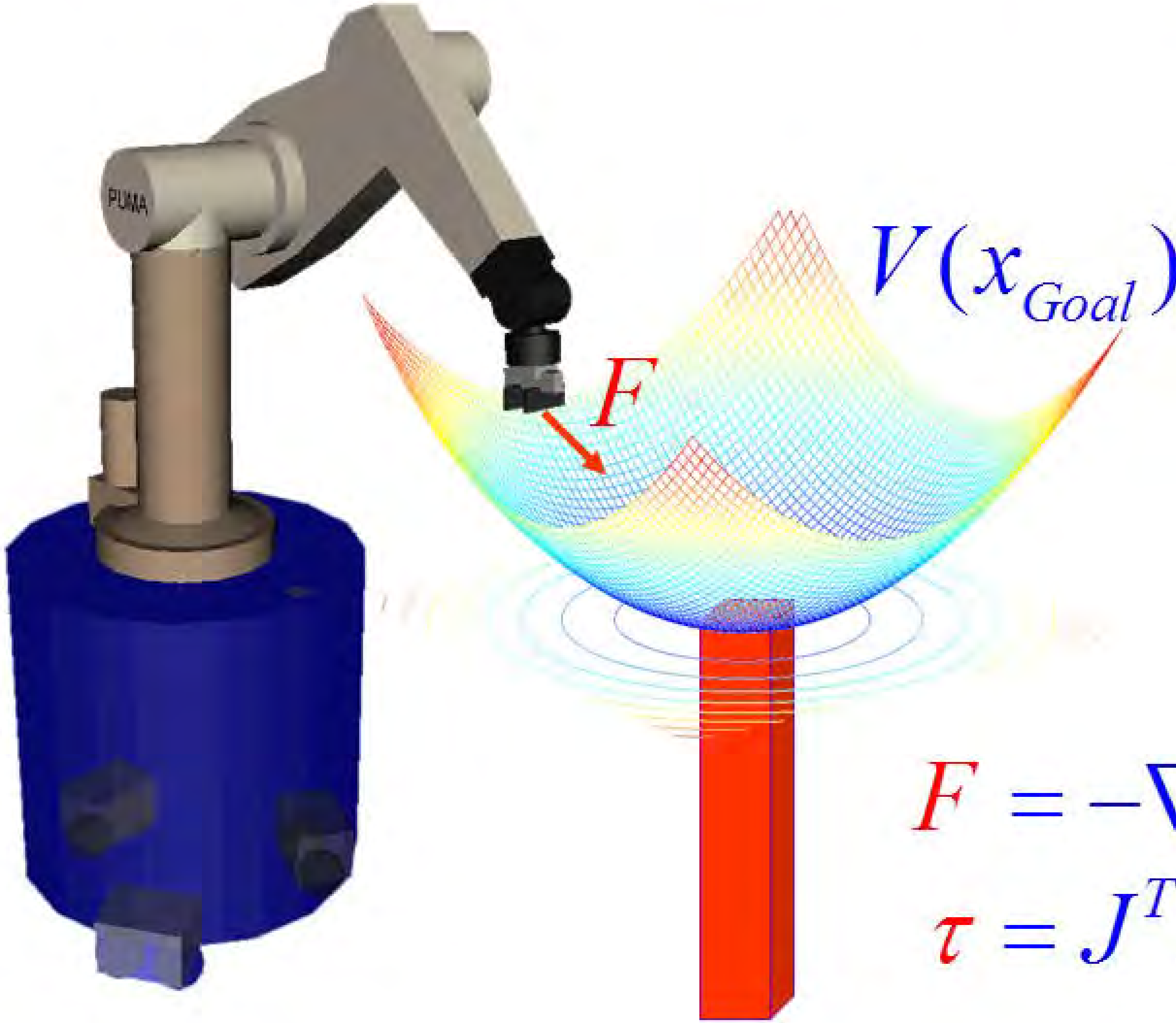
Joint Space Control



Joint Space Control



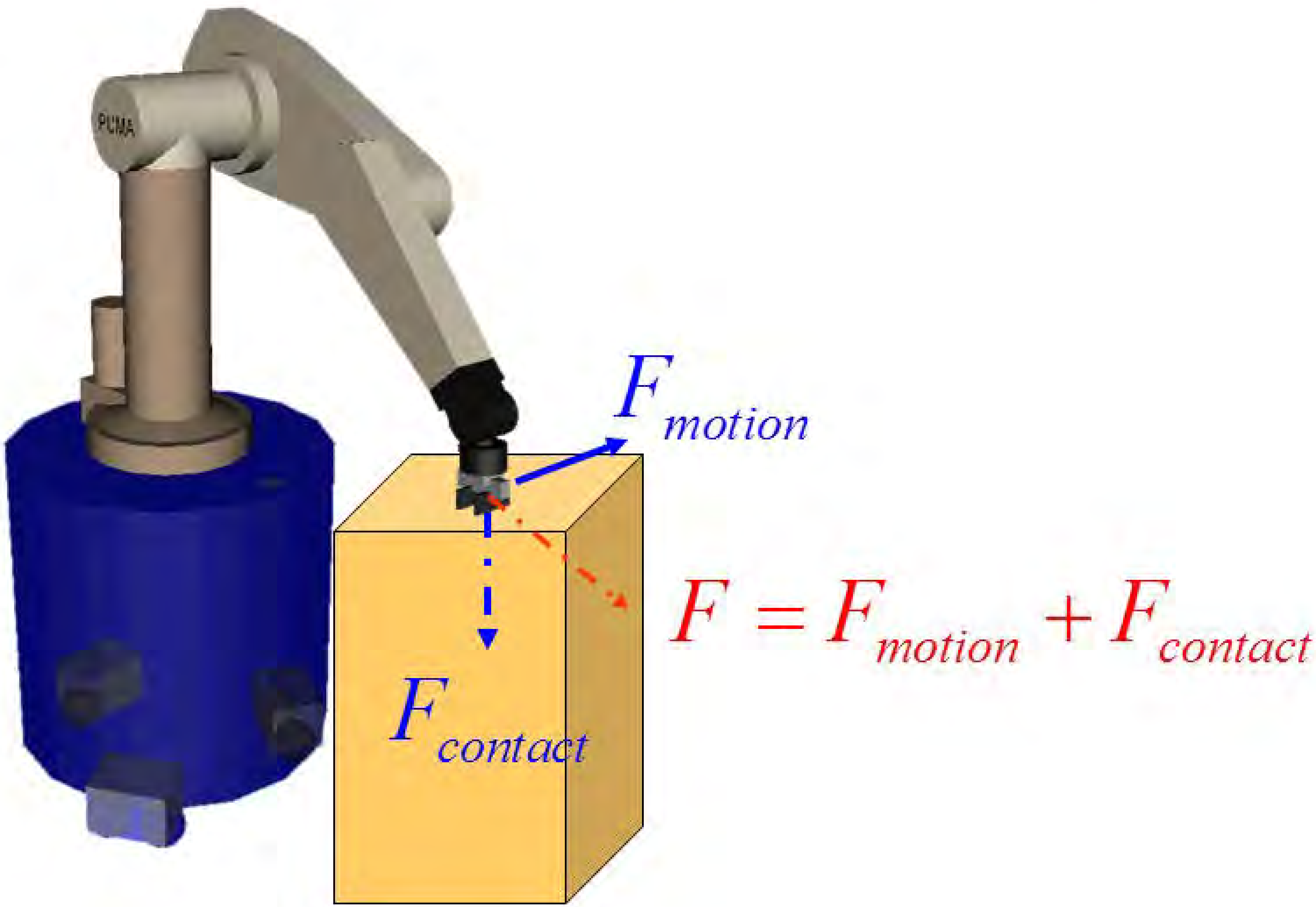
Operational Space Control



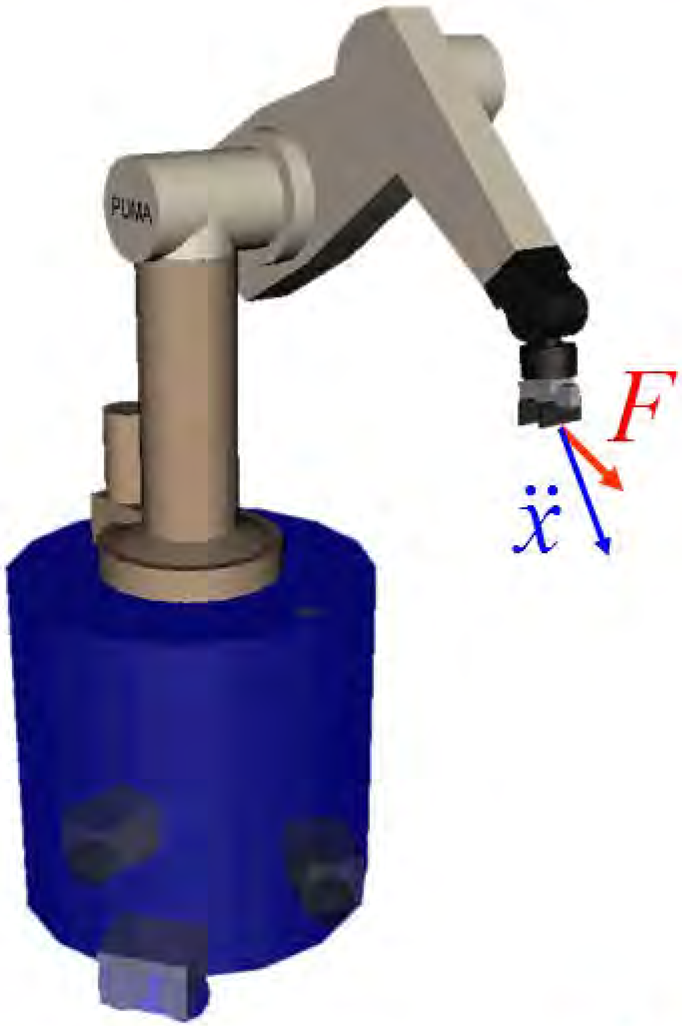
$$F = -\nabla V(x_{Goal})$$

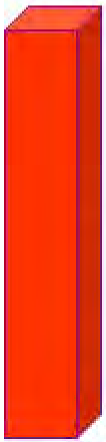
$$\tau = J^T F$$

Unified Motion & Force Control



Operational Space Dynamics

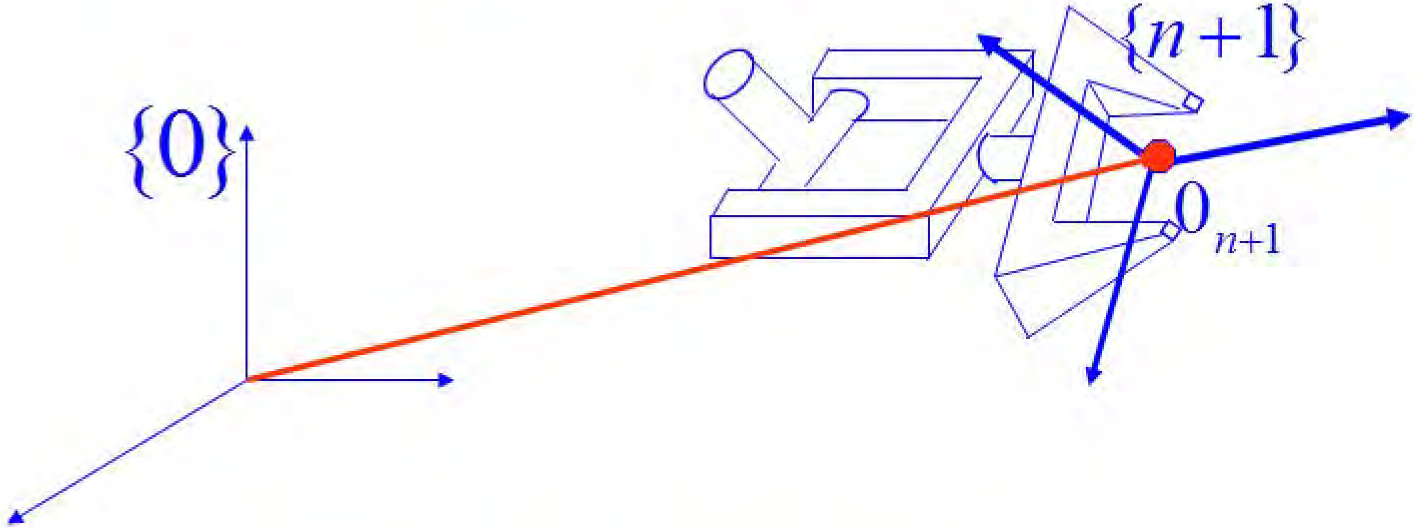




$$M_x \ddot{x} + V_x + G_x = F$$

$$F = F[\hat{M}_x, \hat{V}_x, \hat{G}_x, V(x_{Goal})]$$

Task-Oriented Equations of Motion



Non-Redundant Manipulator ; $n = m$

$$x = (x_1 \, x_2 \ldots x_m)^T$$

$$q = (q_1 \, q_2 \ldots q_n)^T$$

Equations of Motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F$$

$$x = \begin{pmatrix} x \\ y \\ z \\ \alpha \\ \beta \\ \gamma \end{pmatrix}$$

with

$$L(x, \dot{x}) = K(x, \dot{x}) - U(x)$$

Operational Space Dynamics

$$M_x(x)\ddot{x} + V_x(x,\dot{x}) + G_x(x) = F$$

x : End-Effector Position and Orientation

$M_x(x)$: End-Effector Kinetic Energy Matrix

$V_x(x,\dot{x})$: End-Effector Centrifugal and Coriolis forces

$G_x(x)$: End-Effector Gravity forces

F : End-Effector Generalized forces

Joint Space/Task Space Relationships

Kinetic Energy

$$K_x(x,\dot{x}) \equiv K_q(q,\dot{q})$$

$$\frac{1}{2}\dot{x}^T M_x(x)\dot{x} \equiv \frac{1}{2}\dot{q}^T M(q)\dot{q}$$

Using $\dot{x} = J(q)\dot{q}$

$$\frac{1}{2}\dot{q}^T (J^T M_x J)\,\dot{q} \equiv \frac{1}{2}\dot{q}^T M\,\dot{q}$$

Joint Space/Task Space Relationships

$$M_x(x) = J^{-T}(q)\,M(q)\,J^{-1}(q)$$

$$V_x(x,\dot{x}) = J^{-T}(q)\,V(q,\dot{q}) - M_x(q)\,h(q,\dot{q})$$

$$G_x(x) = J^{-T}(q)\,G(q)$$

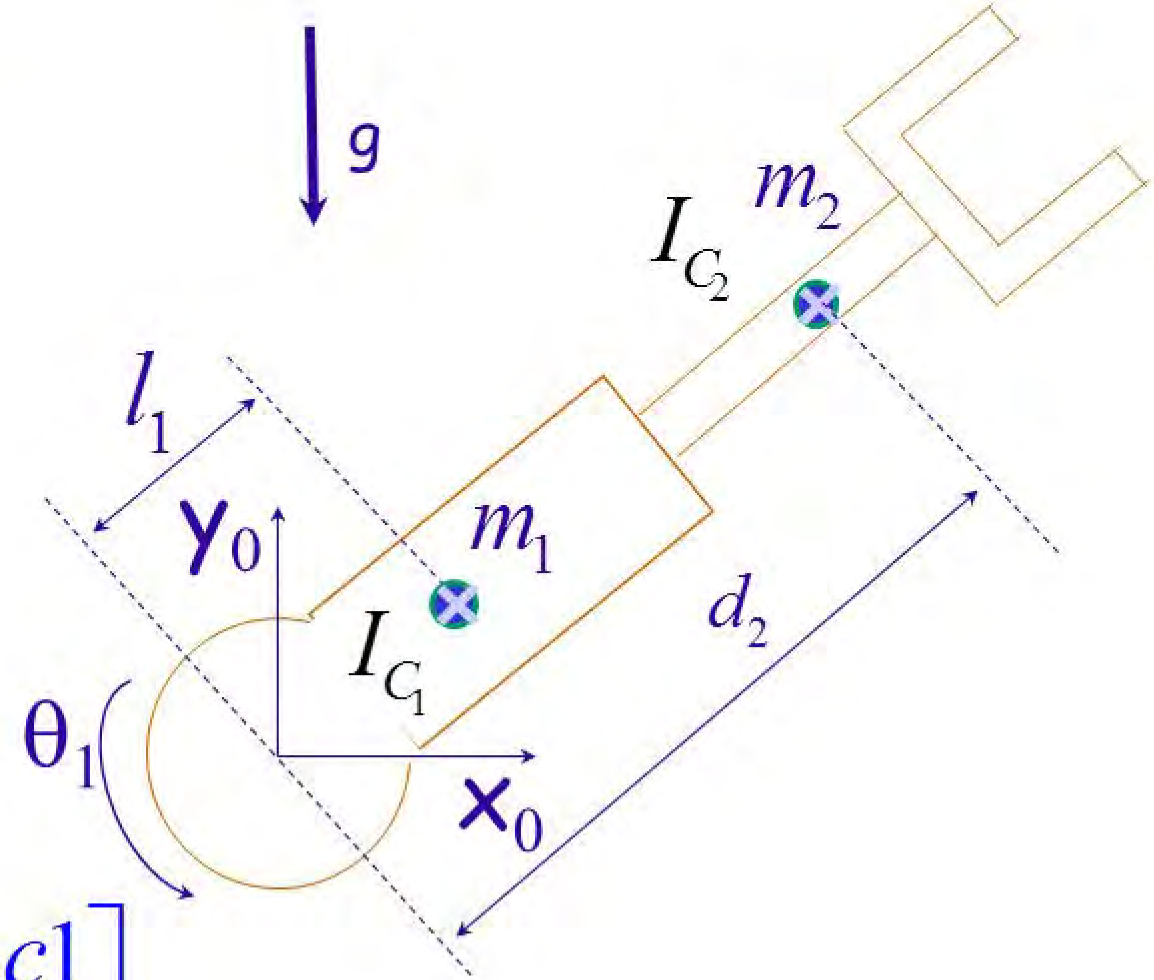
where $h(q,\dot{q}) \doteq \dot{J}(q)\dot{q}$

Example

$$q_2 = d_2$$

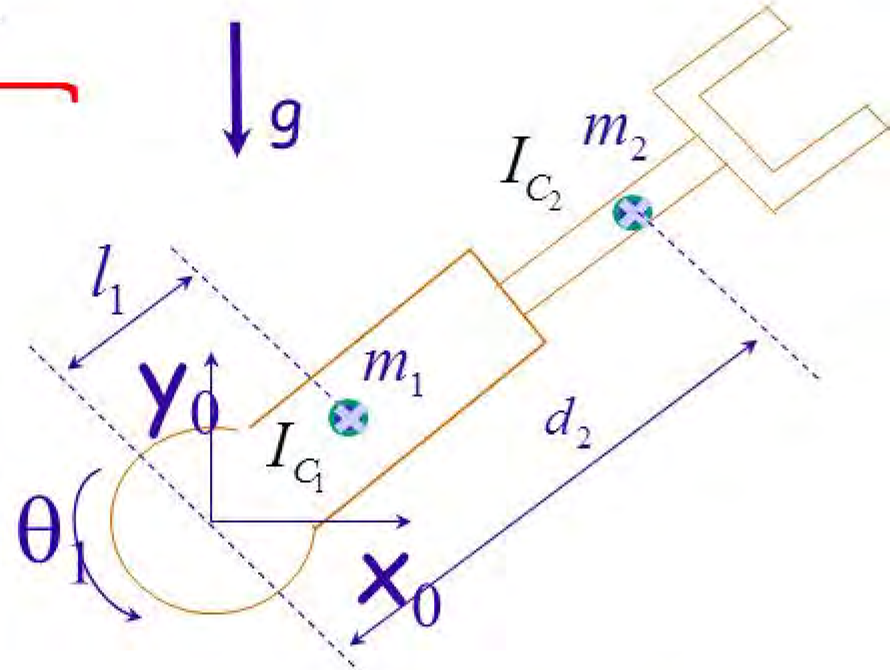
$$x = \begin{bmatrix} d_2 c1 \\ d_2 s1 \end{bmatrix}$$

$$^0J = \begin{bmatrix} -d_2 s1 & c1 \\ d_2 c1 & s1 \end{bmatrix}$$



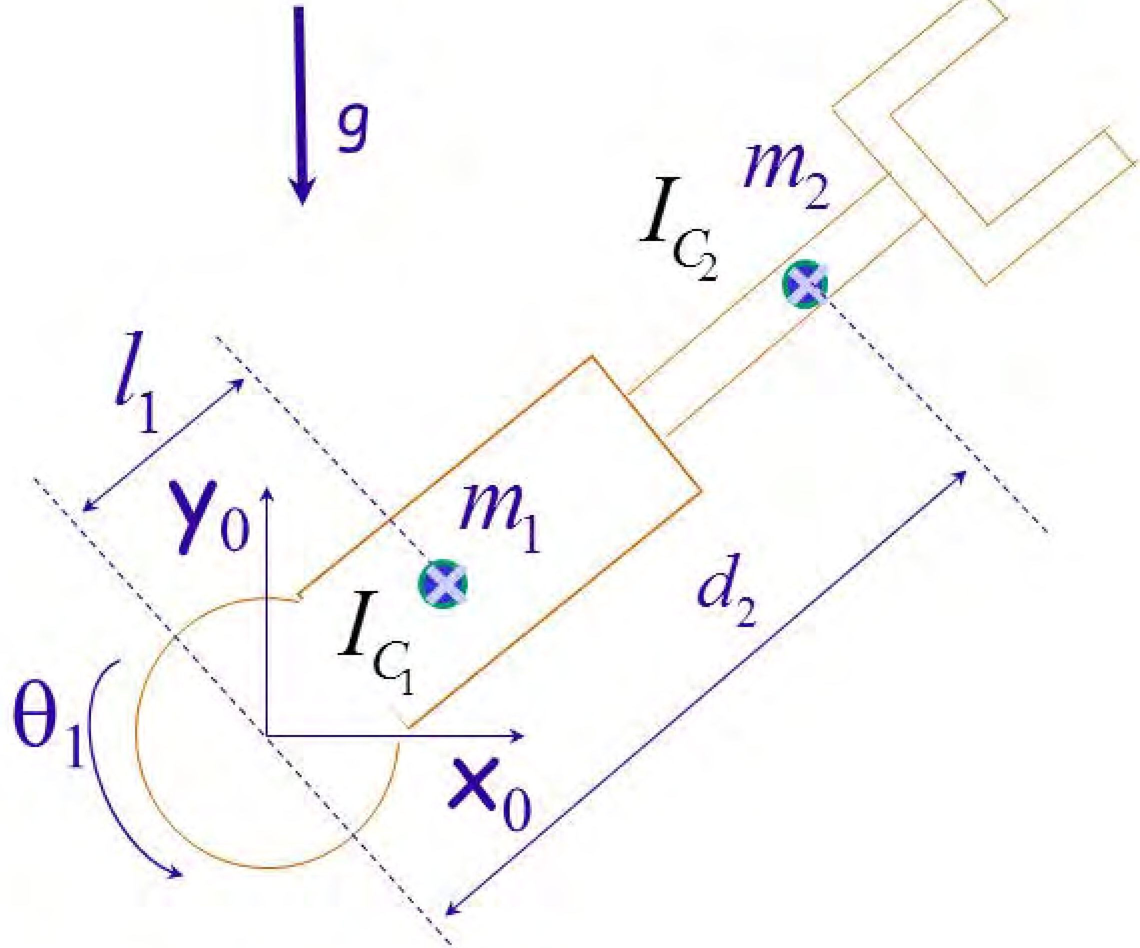
$$ {}^0J = \begin{bmatrix} -d_2 s1 & c1 \\ d_2 c1 & s1 \end{bmatrix} $$

$$ {}^0J = \begin{pmatrix} c1 & -s1 \\ s1 & c1 \end{pmatrix} \overbrace{\begin{pmatrix} 0 & 1 \\ d_2 & 0 \end{pmatrix}}^{{}^1J} $$



$$ {}^1J^{-1} = \begin{pmatrix} 0 & 1/d_2 \\ 1 & 0 \end{pmatrix}; $$

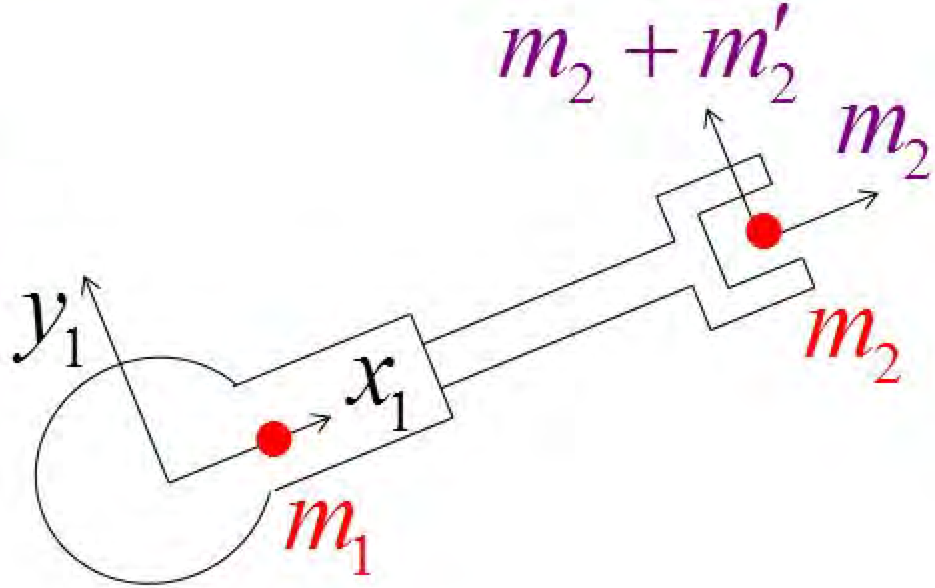
$$ {}^1M_x = \begin{pmatrix} 0 & 1 \\ 1/d_2 & 0 \end{pmatrix} \begin{pmatrix} m_{11} & 0 \\ 0 & m_{22} \end{pmatrix} \begin{pmatrix} 0 & 1/d_2 \\ 1 & 0 \end{pmatrix} $$



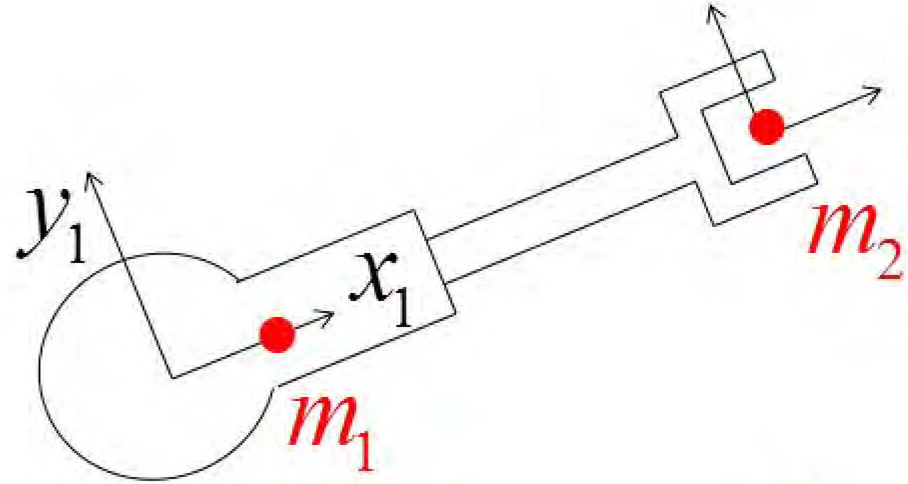
$$M = \begin{bmatrix} m_1 l_1^2 + I_{zz1} + m_2 d_2^2 + I_{zz2} & 0 \\ 0 & m_2 \end{bmatrix}$$

$$m_2' = \frac{I_{zz1} + I_{zz2} + m_1 l_1^2}{d_2^2}$$

$$^1M_x = \begin{pmatrix} m_2 & 0 \\ 0 & m_2 + m_2' \end{pmatrix}$$



$$M = \begin{bmatrix} m_1 l_1^2 + I_{zz1} + m_2 d_2^2 + I_{zz2} & 0 \\ 0 & m_2 \end{bmatrix}$$



$$^0M_x = \begin{pmatrix} c1 & -s1 \\ s1 & c1 \end{pmatrix} \begin{pmatrix} m_2 & 0 \\ 0 & m_2 + m_2' \end{pmatrix} \begin{pmatrix} c1 & s1 \\ -s1 & c1 \end{pmatrix}$$

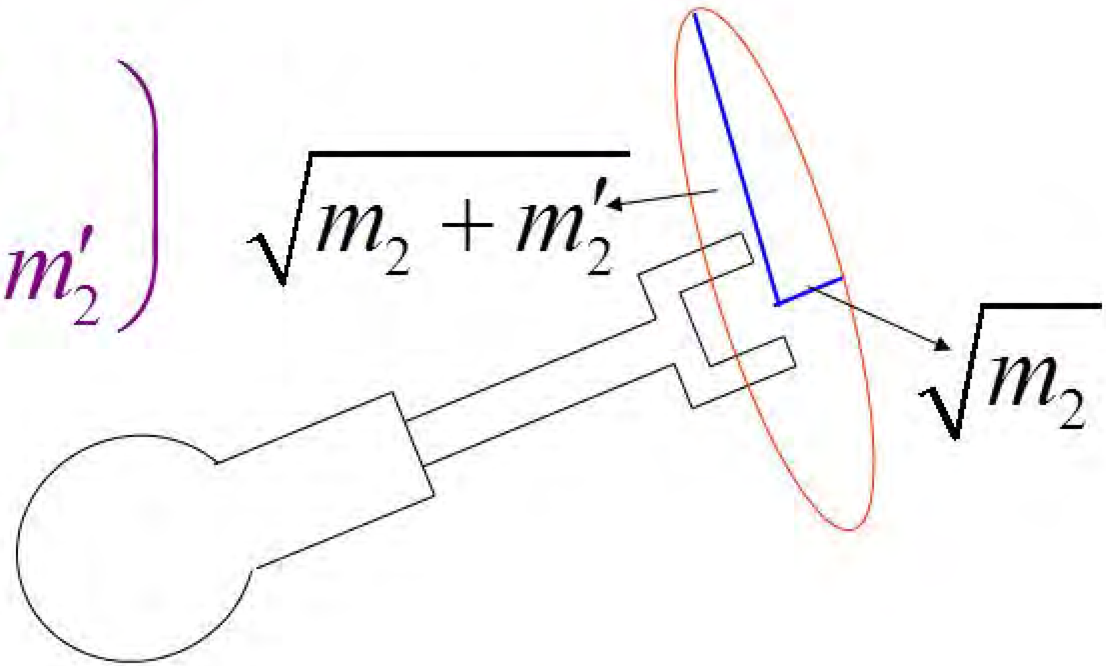
Task Space

$$^0M_x = \begin{pmatrix} m_2 + m_2' \, s1^2 & -m_2' s\, c1 \\ -m_2' s\, c1 & m_2 + m_2' \, c1^2 \end{pmatrix}$$

Joint Space

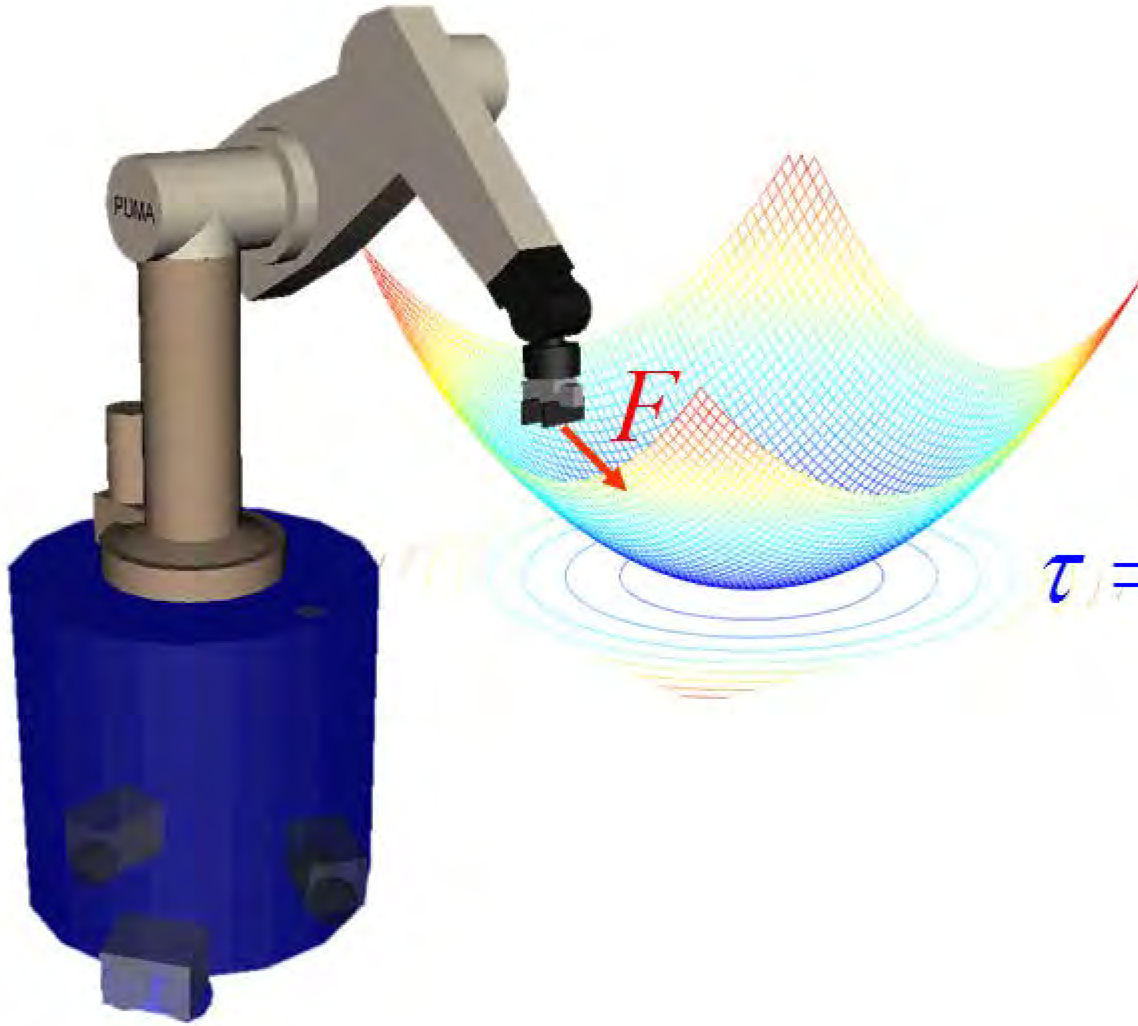
$$M = \begin{bmatrix} m_1 l_1^2 + I_{zz1} + m_2 d_2^2 + I_{zz2} & 0 \\ 0 & m_2 \end{bmatrix}$$

$$ {}^{1}M_{x} = \begin{pmatrix} m_2 & 0 \\ 0 & m_2 + m_2' \end{pmatrix} $$



$$ {}^{0}\Lambda = \begin{pmatrix} m_2 + m_2'\, s1^2 & -m_2' s\, c1 \\ -m_2' s\, c1 & m_2 + m_2'\, c1^2 \end{pmatrix} $$

End-Effector Control



$$\tau = J^T(q)F$$

Passive Systems (Stability)

$$V_{goal} = \frac{1}{2} k_p \left(x - x_g\right)^T \left(x - x_g\right)$$

System $$\frac{d}{dt}\left(\frac{\partial(K - V)}{\partial \dot{x}}\right) - \frac{\partial(K - V)}{\partial x} = F$$

$$\Downarrow \quad F = -\frac{\partial}{\partial X}\left(V_{goal} - \hat{V}\right)$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial\left(K - V_{goal}\right)}{\partial x} = 0$$

Conservative Forces

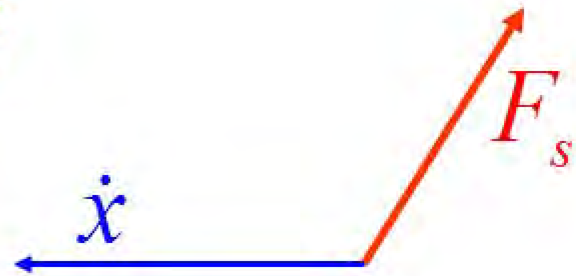
Stable

Asymptotic Stability

a system
$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial\left(K - V_{goal}\right)}{\partial x} = 0 \quad F_s$$

is asymptotically stable if

$$F_s^T \dot{x} < 0 \quad ; \; for \;\; \dot{x} \neq 0$$



$$F_s = -k_v \dot{x} \;\; \rightarrow \;\; k_v > 0$$

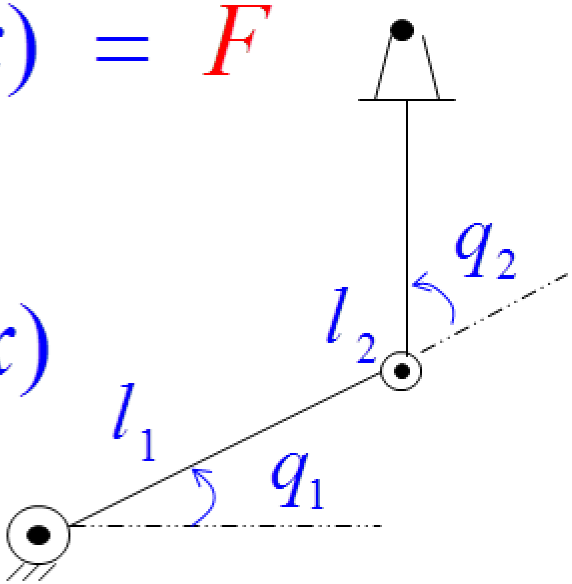
Control

$$F = -k_p\left(x - x_{goal}\right) + \hat{G}_x - k_v \dot{x}$$

Example 2-d.o.f arm: Non-Dynamic Control

$$M_x(x)\ddot{x} + V_x(x, \dot{x}) + G_x(x) = F$$

$$F = -k_p\left(x - x_g\right) - k_v\,\dot{x} + \hat{G}(x)$$



Course Evaluation

http://axess.stanford.edu

$$\left(m_1^* c^2 12 + m_2\right)\ddot{x} + m_1^* \ddot{y} + V_{x1} = -k_p\left(x - x_g\right) - k_v \dot{x}$$

$$\left(m_1^* c^2 12 + m_2\right)\ddot{y} + m_1^* \ddot{x} + V_{x2} = -k_p\left(y - y_g\right) - k_v \dot{y}$$

Closed loop behavior

$$m_{11}(q)\ddot{x} + k_v \dot{x} + k_p\left(x - x_g\right) = -\left(m_1^* \ddot{y} + V_{x1}\right)$$

$$m_{22}(q)\ddot{y} + k_v \dot{y} + k_p\left(y - y_g\right) = -\left(m_1^* \ddot{x} + V_{x2}\right)$$

Nonlinear Dynamic Decoupling

Model

$$M_x(x)\ddot{x} + V_x(x,\dot{x}) + G_x(x) = F$$

Control Structure

$$F = \hat{M}(x)F' + \hat{V}_x(x,\dot{x}) + \hat{G}_x(x)$$

Decoupled System

$$I\,\ddot{x} = F'$$

with $\tau = J^T F$

Perfect Estimates

$$I\,\ddot{x} = F\,'$$

$F\,'$ input of decoupled end-effector

Goal Position Control

$$F' = -k_v'\,\dot{x} - k_p'\left(x - x_g\right)$$

Closed Loop

$$I\,\ddot{x} + k_v'\dot{x} + k_p'(x - x_g) = 0$$

Trajectory Tracking

Trajectory: $x_d\,,\ \dot{x}_d\,,\ \ddot{x}_d$

$$F' = I\,\ddot{x}_d - k_v^{'}(\dot{x} - \dot{x}_d) - k_p^{'}(x - x_d)$$

$$(\ddot{x} - \ddot{x}_d) + k_v^{'}(\dot{x} - \dot{x}_d) + k_p^{'}(x - x_d) = 0$$

or

$$\ddot{\varepsilon}_x + k_v^{'}\dot{\varepsilon}_x + k_p^{'}\varepsilon_x = 0$$

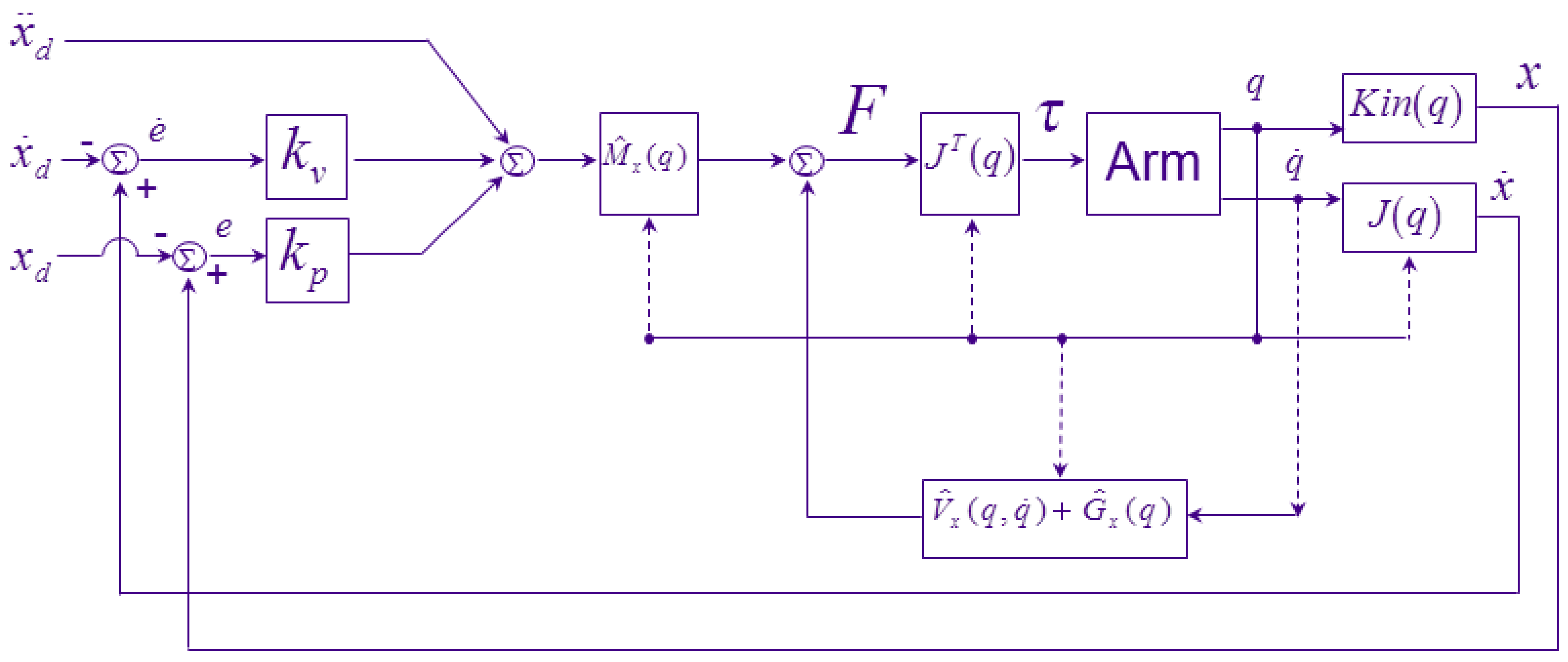
with $\varepsilon_x = x - x_d$

In joint space

$$\ddot{\varepsilon}_q + k_v^{'}\dot{\varepsilon}_q + k_p^{'}\varepsilon_q = 0$$

with $\varepsilon_q = q - q_d$

Task-Oriented Control

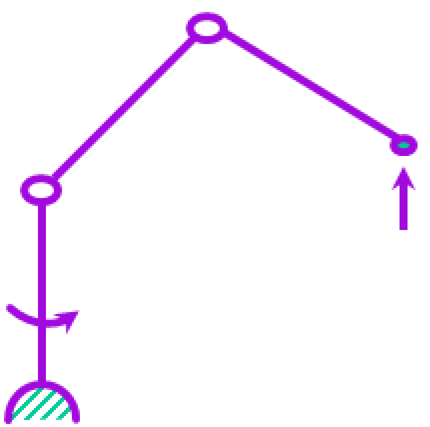


Compliance

$$I\ddot{x} = F'$$

$$F' = -\begin{pmatrix} k'_{p_x} & 0 & 0 \\ 0 & k'_{p_y} & 0 \\ 0 & 0 & k'_{p_z} \end{pmatrix}(x - x_d) - k'_v \dot{x}$$

set to zero



$$\ddot{x} + k'_v \dot{x} + k'_{px}(x - x_d) = 0$$

$$\ddot{y} + k'_v \dot{y} + k'_{py}(y - y_d) = 0$$

$$\ddot{z} + k'_v \dot{z} = 0$$

Compliance along Z

Stiffness

$$\ddot{z} + k'_v \dot{z} + k'_{p_z}(z - z_d) = 0$$

determines stiffness along $\underline{z}$

Closed-Loop Stiffness: $\hat{M}_x k'_p = k_p$

$$F = K_x(x - x_d)$$

$$\tau = J^T F = J^T K_x \Delta x = (J^T K_x J)\Delta\theta = K_\theta \Delta\theta$$

$$\boxed{K_\theta = J^T(\theta) K_x J(\theta)}$$

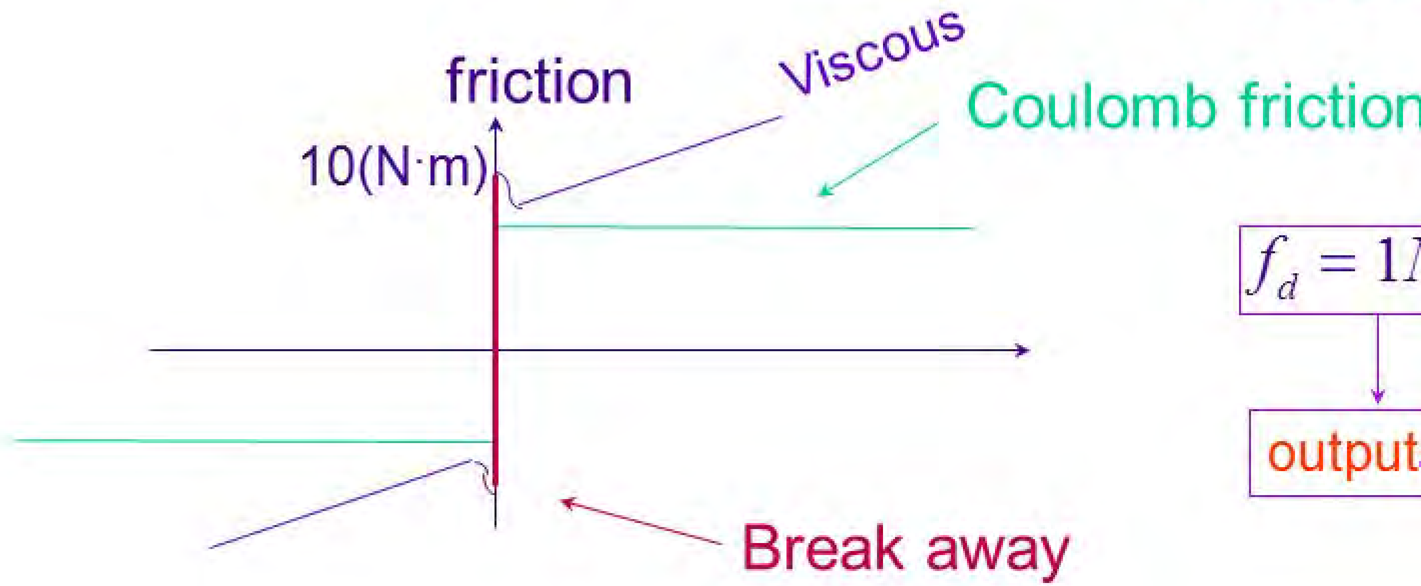
Force Control

1-d.o.f.

$$m\ddot{x} = f$$

set $f = f_d$

Problem



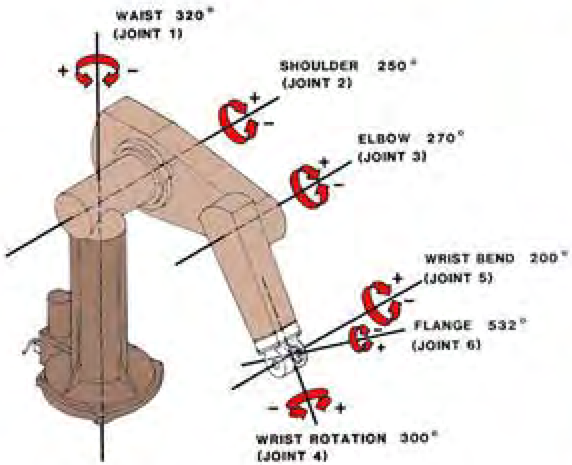
$f_d = 1Nm$

↓

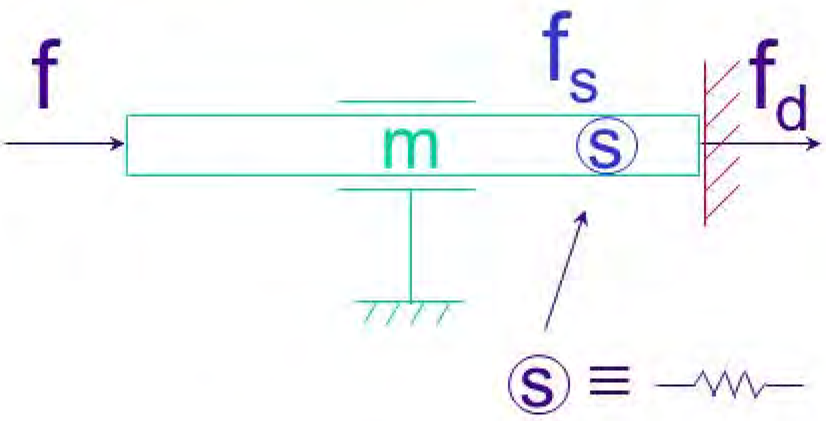
output≃0

CS225A - Experimental Robotics

Moved the Fall Quarter



Force Sensing



$$; f_s = k_s x$$

$$m\ddot{x} + \underbrace{k_s x}_{f_s} = f$$

At static Equilibrium

$$f_s = f_d \Rightarrow f = f_d$$

Dynamics

$$m\ddot{x} + k_s x = f_d + f_{Dynamic}$$

Dynamics

$$m\ddot{x} + \underline{k_s x} = f$$

$\downarrow f_s$

$$\frac{m}{k_s}\ddot{f}_s + f_s = f$$

$$f_s = k_s x$$

$$\dot{f}_s = k_s \dot{x}$$

$$\ddot{f}_s = k_s \ddot{x}$$

Control

$$f_d + \frac{m}{k_s}(-k'_{p_f}(f_s - f_d) - k'_{v_f}\dot{f}_s)$$

Closed Loop

$$\frac{m}{k_s}[\ddot{f}_s + k'_{v_f}\dot{f}_s + k'_{p_f}(f_s - f_d)] + f_s = f_d$$

Steady-State error

$$\frac{m}{k_s}(\ddot{f}_s + k'_{v_f}\dot{f}_s + k'_{p_f}(f_s - f_d)) + (f_s - f_d) = 0$$

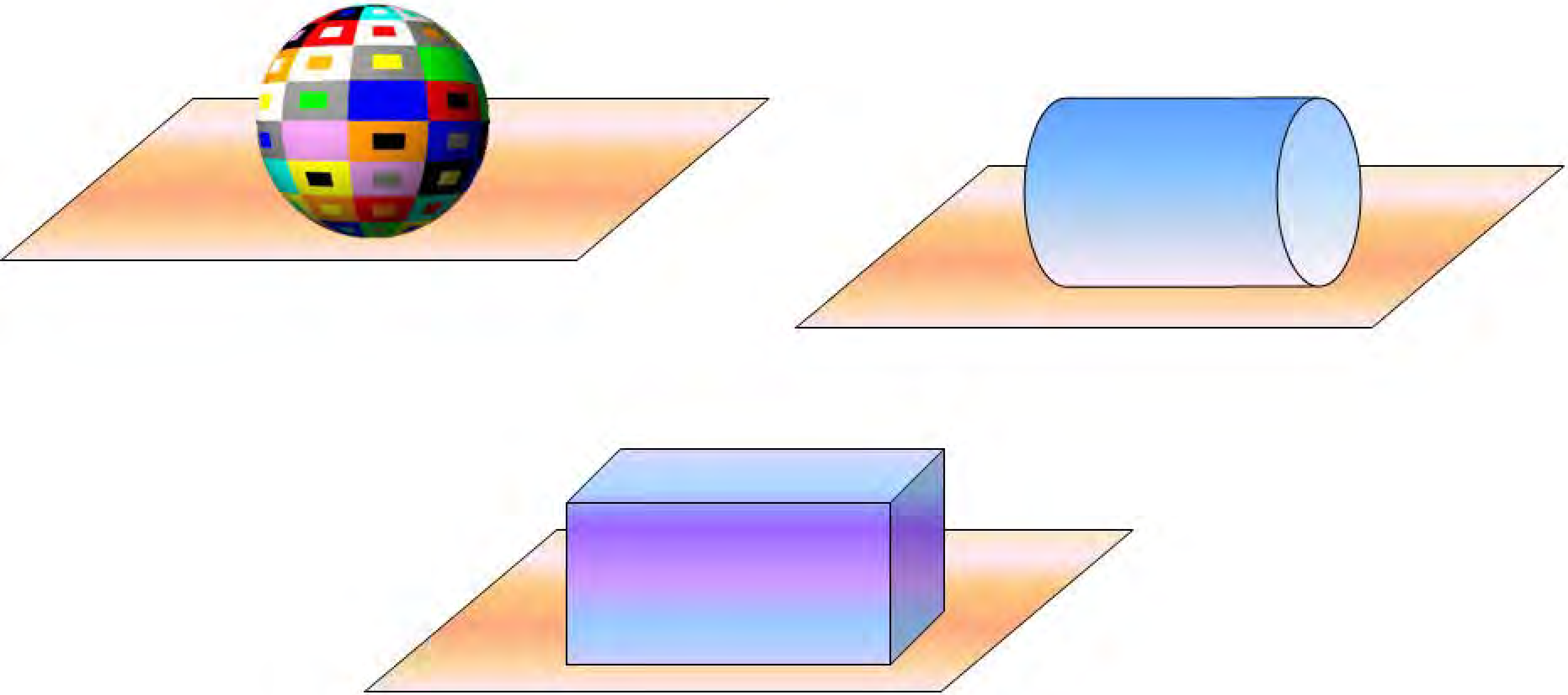
f_{dist}

$$\ddot{f}_s = \dot{f}_s = 0$$

$$(\frac{mk'_{p_f}}{k_s} + 1)e_f = f_{dist}$$

$$e_f = \frac{f_{dist}}{1 + \frac{mk'_{p_f}}{k_s}}$$

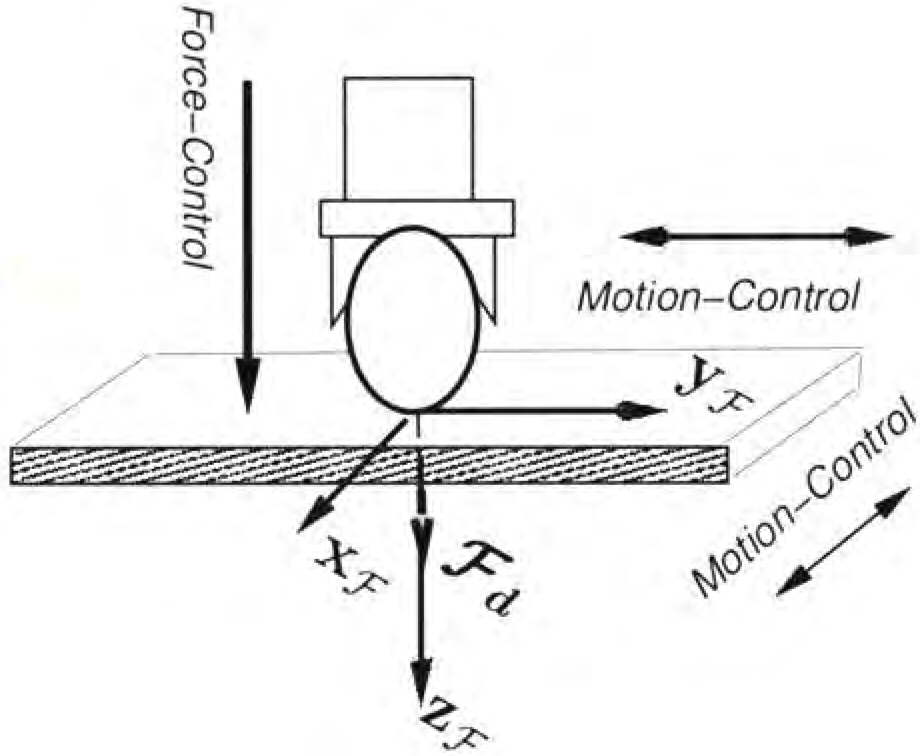
Task Description



Task Specification

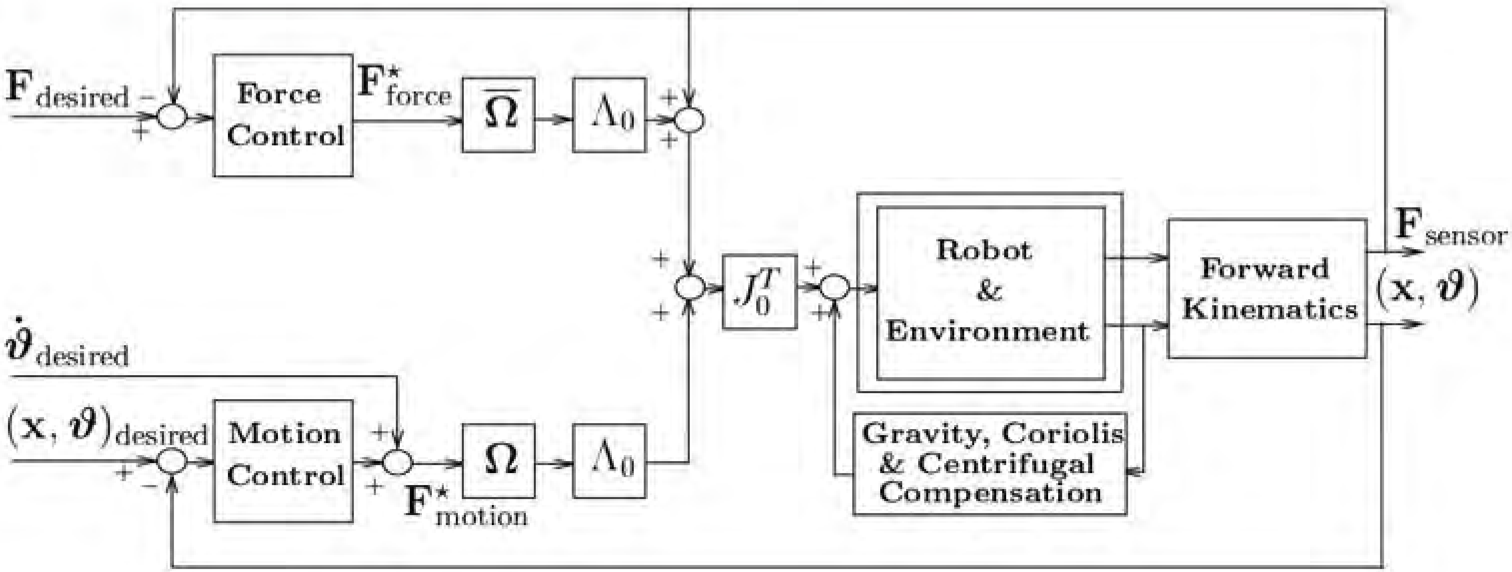
$$F = \Omega F_{motion} + \bar{\Omega} F_{force}$$

Selection matrix



$$\Omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \ \bar{\Omega} = I - \Omega$$

Unified Motion & Force Control



Two decoupled Subsystems

$$\Omega \dot{\vartheta} = \Omega F^*_{motion}$$

$$\bar{\Omega} \dot{\vartheta} = \bar{\Omega} F^*_{force}$$

Control

- Natural Systems
- PID Control
- Joint-Space Dynamic Control
- Task-Oriented Control
- Force Control

Performance

High Gains ⟶ better disturbance rejection

Gains are limited by

- structural flexibilities
- time delays (actuator-sensing)
- sampling rate

$$\omega_n \leq \frac{\omega_{res}}{2}$$ ← lowest structural flexibility

$$\omega_n \leq \frac{\omega_{delay}}{3}$$ ← largest delay $\left(\frac{2\pi}{\tau_{delay}}\right)$

$$\omega_n \leq \frac{\omega_{sampling-rate}}{5}$$

Nonlinear Dynamic Decoupling

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) = \tau$$

$$\tau = \hat{M}(\theta)\underline{\tau'} + \hat{V}(\theta,\dot{\theta}) + \hat{G}(\theta)$$

$$\mathbf{1}.\ddot{\theta} = (M^{-1}\hat{M})\tau' + M^{-1}[(V - \hat{V}) + (G - \hat{G})]$$

with perfect estimates

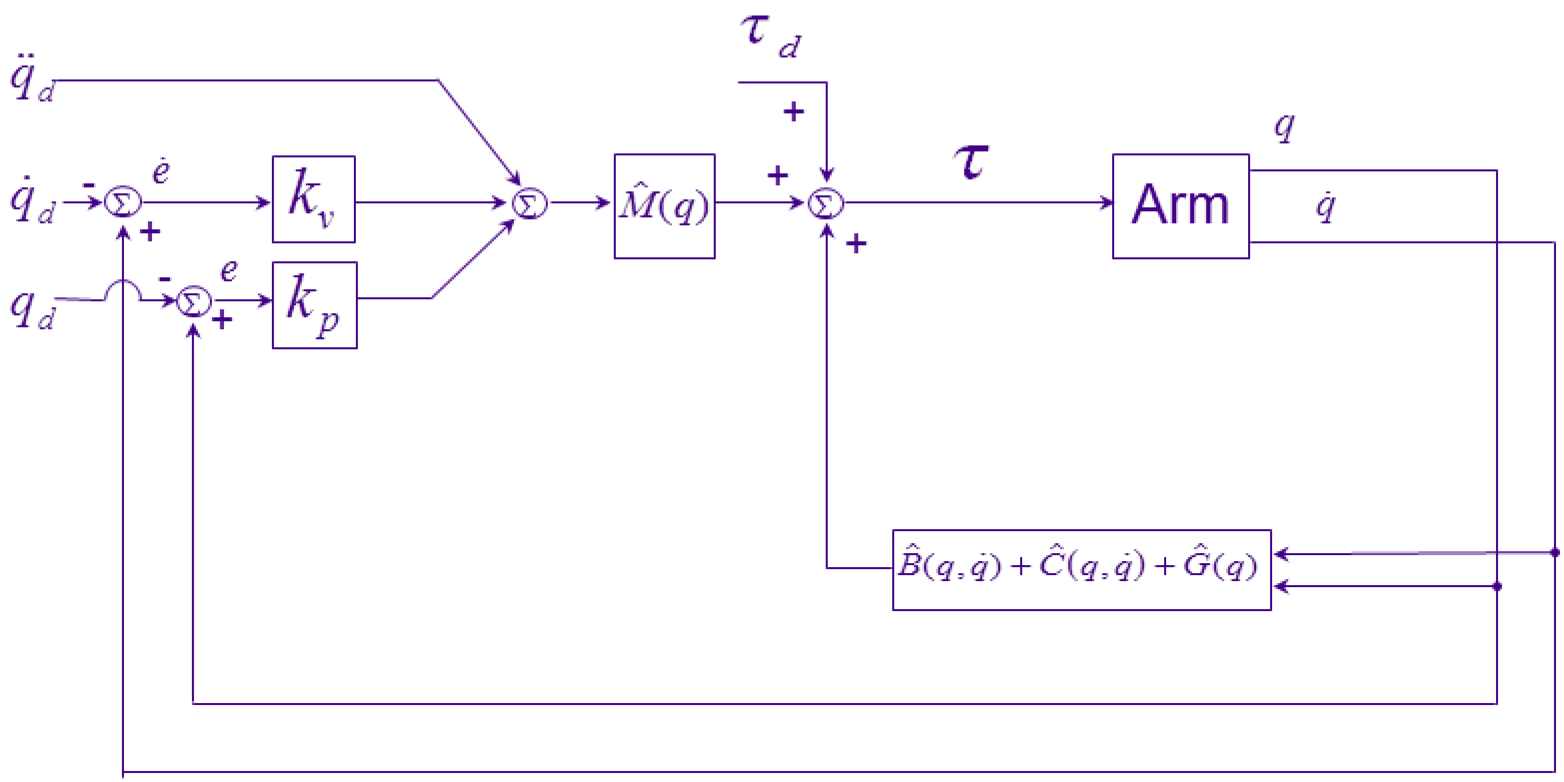
$$\mathbf{1}.\ddot{\theta} = \tau' + \varepsilon(t)$$

τ': input of the unit-mass systems

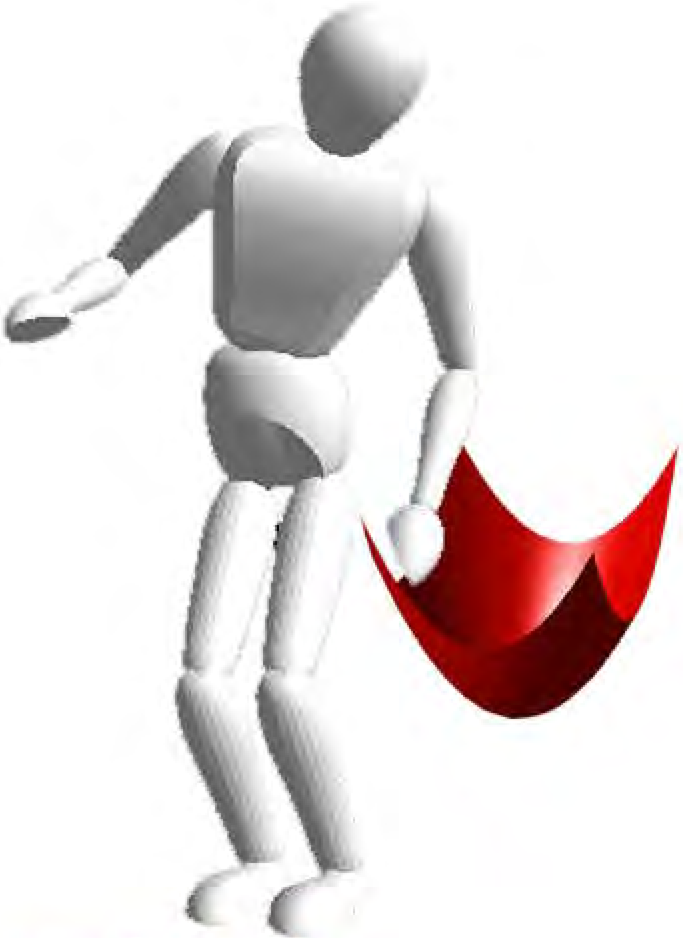
$$\tau' = \ddot{\theta}_d - k'_v(\dot{\theta} - \dot{\theta}_d) - k'_p(\theta - \theta_d)$$

Closed-loop

$$\ddot{E} + k'_v\dot{E} + k'_p E = 0 + \varepsilon(t)$$



$\ddot{q}_d$
$\dot{q}_d$
q_d
$\dot{e}$
e
k_v
k_p
$\hat{M}(q)$
τ_d
τ
Arm
q
$\dot{q}$
$\hat{B}(q,\dot{q}) + \hat{C}(q,\dot{q}) + \hat{G}(q)$



Task Oriented Control