Video Segment

SCHAFT: DARPA Robotics Challenge 8 Tasks + Special Walking, 2013

Motion Control

$$m\ddot{x} + b(x,\dot{x}) = f \underset{f = mf' + b}{\Rightarrow} 1.\ddot{x} = f'$$

Goal Position (x_d):

Control: $f' = -k_v'\dot{x} - k_p'(x - x_d)$

Closed-loop System: $1.\ddot{x} + k_v'\dot{x} + k_p'(x - x_d) = 0$

Trajectory Tracking

$x_d(t)$; $\dot{x}_d(t)$; and $\ddot{x}_d(t)$

Control: $f' = \ddot{x}_d - k_v'(\dot{x} - \dot{x}_d) - k_p'(x - x_d)$

Closed-loop System:

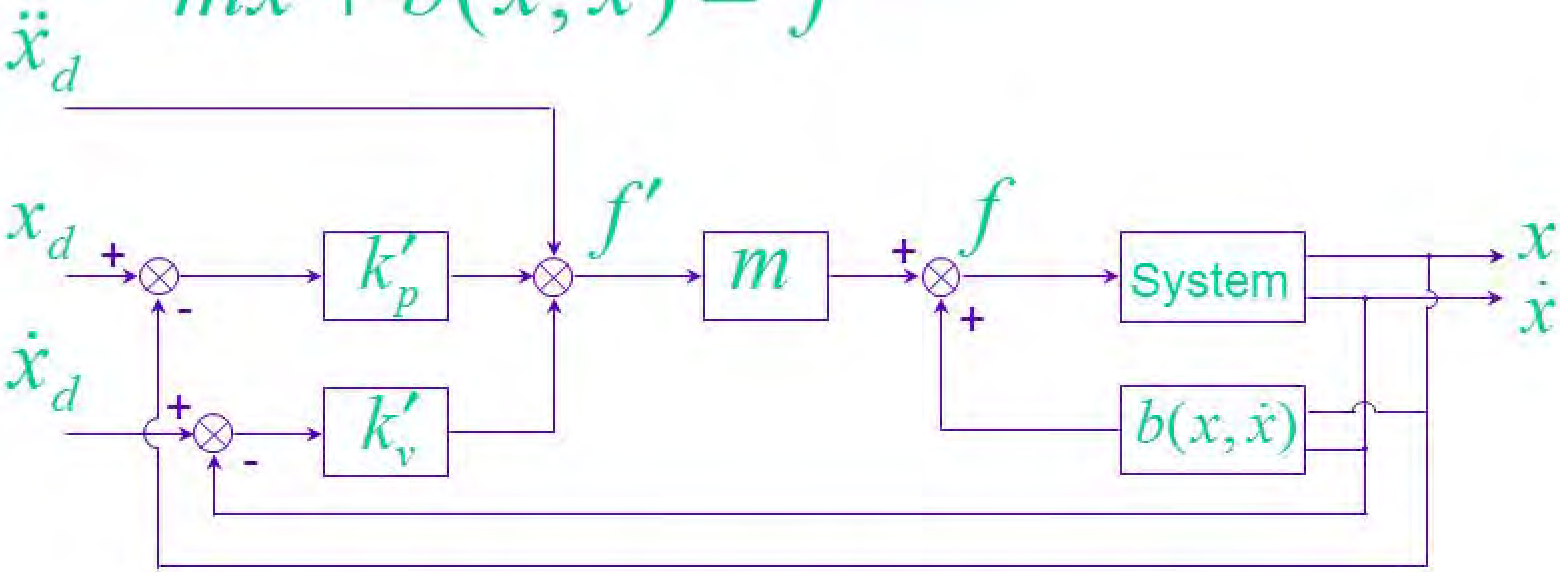
$$(\ddot{x} - \ddot{x}_d) + k_v'(\dot{x} - \dot{x}_d) + k_p'(x - x_d) = 0$$

with $e \equiv x - x_d$

$$\ddot{e} + k_v'\dot{e} + k_p'e = 0$$

Disturbance Rejection

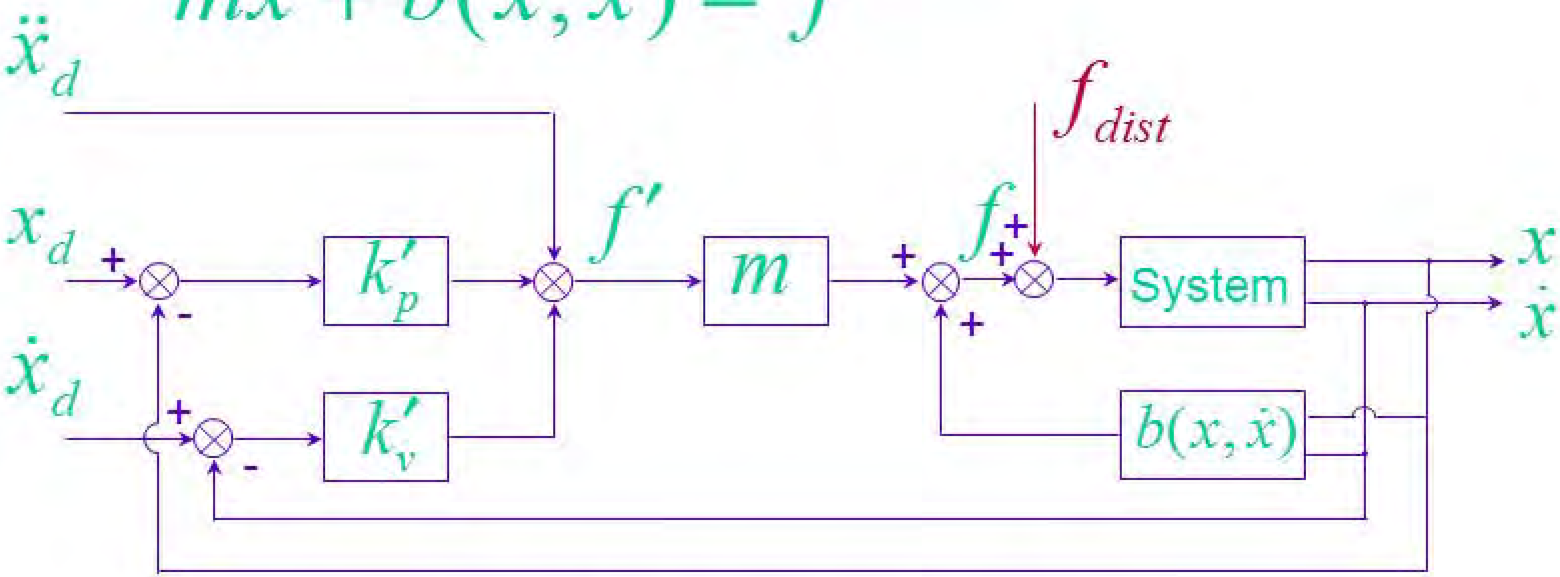
$$m\ddot{x} + b(x,\dot{x}) = f$$



$$\ddot{e} + k'_v\dot{e} + k'_p e = 0$$

Disturbance Rejection

$$m\ddot{x} + b(x,\dot{x}) = f$$



$$m\ddot{x} + b(x,\dot{x}) = f + f_{dist}$$

bounded

$$\{\forall t \, |f_{dist}| < a\}$$

Control

$$f = mf' + b(x,\dot{x})$$

Closed loop

$$\boxed{\ddot{e} + k'_v\dot{e} + k'_p e = \frac{f_{dist}}{m}}$$

Steady-State Error

$$\ddot{e} + k_v'\dot{e} + k_p'e = \frac{f_{dist}}{m}$$

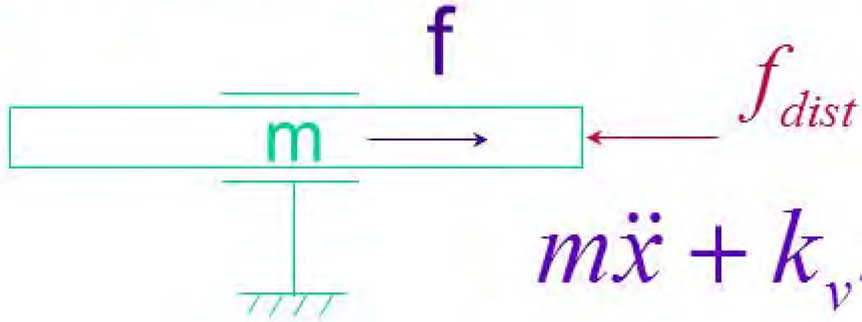
The steady-state $\left(\dot{e} = \ddot{e} = 0\right):$

$$k_p'e = \frac{f_{dist}}{m}$$

$$e = \frac{f_{dist}}{mk_p'} = \frac{f_{dist}}{k_p}$$

Closed loop position gain (stiffness)

Steady-State Error - Example



$$m\ddot{x} + k_v\dot{x} + k_p(x - x_d) = 0$$

$$k_p(x - x_d) = f_{dist}$$

$$x = x_d + \frac{f_{dist}}{k_p}$$

$$\Delta x$$

$$f_{dist} = k_p \Delta x$$

$$\Delta x = \frac{f_{dist}}{k_p}$$

Closed Loop Stiffness

PID (adding Integral action)

System $m\ddot{x} + b(x, \dot{x}) = f + f_{dist}$

Control $f = mf' + b(x, \dot{x})$

$$f' = \ddot{x}_d - k'_v(\dot{x} - \dot{x}_d) - k'_p(x - x_d) - k'_i \int (x - x_d) dt$$

Closed-loop System

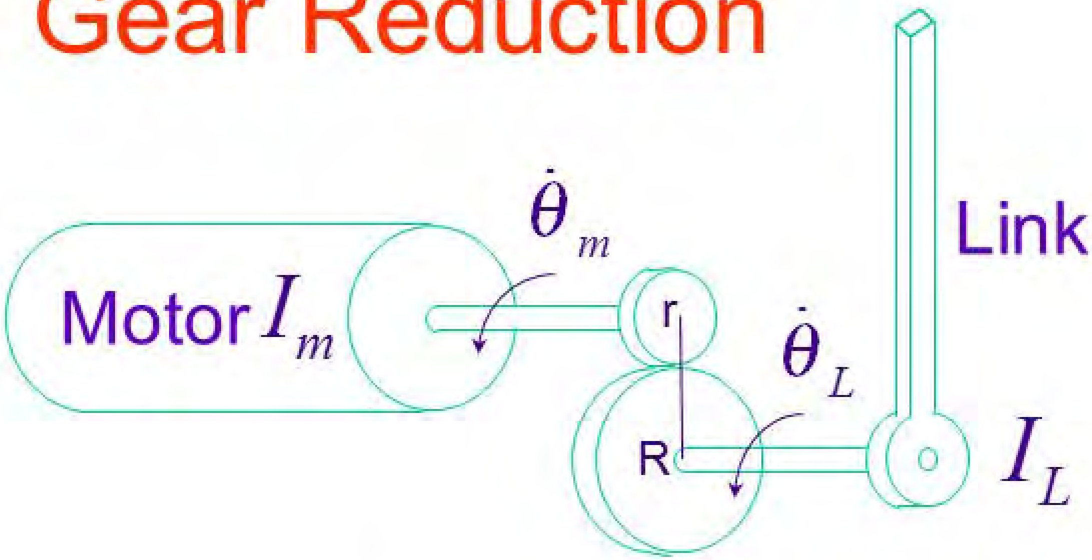
$$\ddot{e} + k'_v \dot{e} + k'_p e + k'_i \int e dt = \frac{f_{dist}}{m}$$

constant

$$\dddot{e} + k'_v \ddot{e} + k'_p \dot{e} + k'_i e = 0$$

Steady-state Error $\boxed{e = 0}$

Gear Reduction



Gear ratio $\eta = \dfrac{R}{r}$

$$\dot{\theta}_L = (\frac{1}{\eta})\dot{\theta}_m$$

$$\tau_L = \eta \tau_m$$

$$\tau_m = I_m \ddot{\theta}_m + \frac{1}{\eta}(I_L \ddot{\theta}_L) + b_m \dot{\theta}_m + \frac{1}{\eta} b_L \dot{\theta}_L$$

$$\ddot{\theta}_L = \frac{1}{\eta}\ddot{\theta}_m$$

$$\tau_m = (I_m + \frac{I_L}{\eta^2})\ddot{\theta}_m + (b_m + \frac{b_L}{\eta^2})\dot{\theta}_m$$

$$\tau_L = \underbrace{(I_L + \eta^2 I_m)}_{\text{Effective Inertia}}\ddot{\theta}_L + \underbrace{(b_L + \eta^2 b_m)}_{\text{Effective Damping}}\dot{\theta}_L$$

Effective Inertia

$$I_{eff} = I_L + \eta^2 I_m$$

for a manipulator

$$I_L = I_L(q)$$

$$\eta = 1$$

Direct Drive

Gain Selection

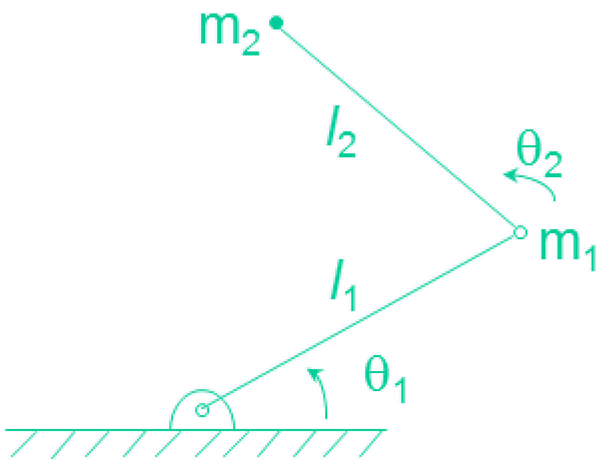
$$k_p = (I_L + \eta^2 I_m)k'_p$$

$$k_v = (I_L + \eta^2 I_m)k'_v$$

Time Optimal Selection

$$\hat{I}_L = \frac{1}{4}(\sqrt{I_{L_{\min}}} + \sqrt{I_{L_{\max}}})^2$$

Manipulator Control

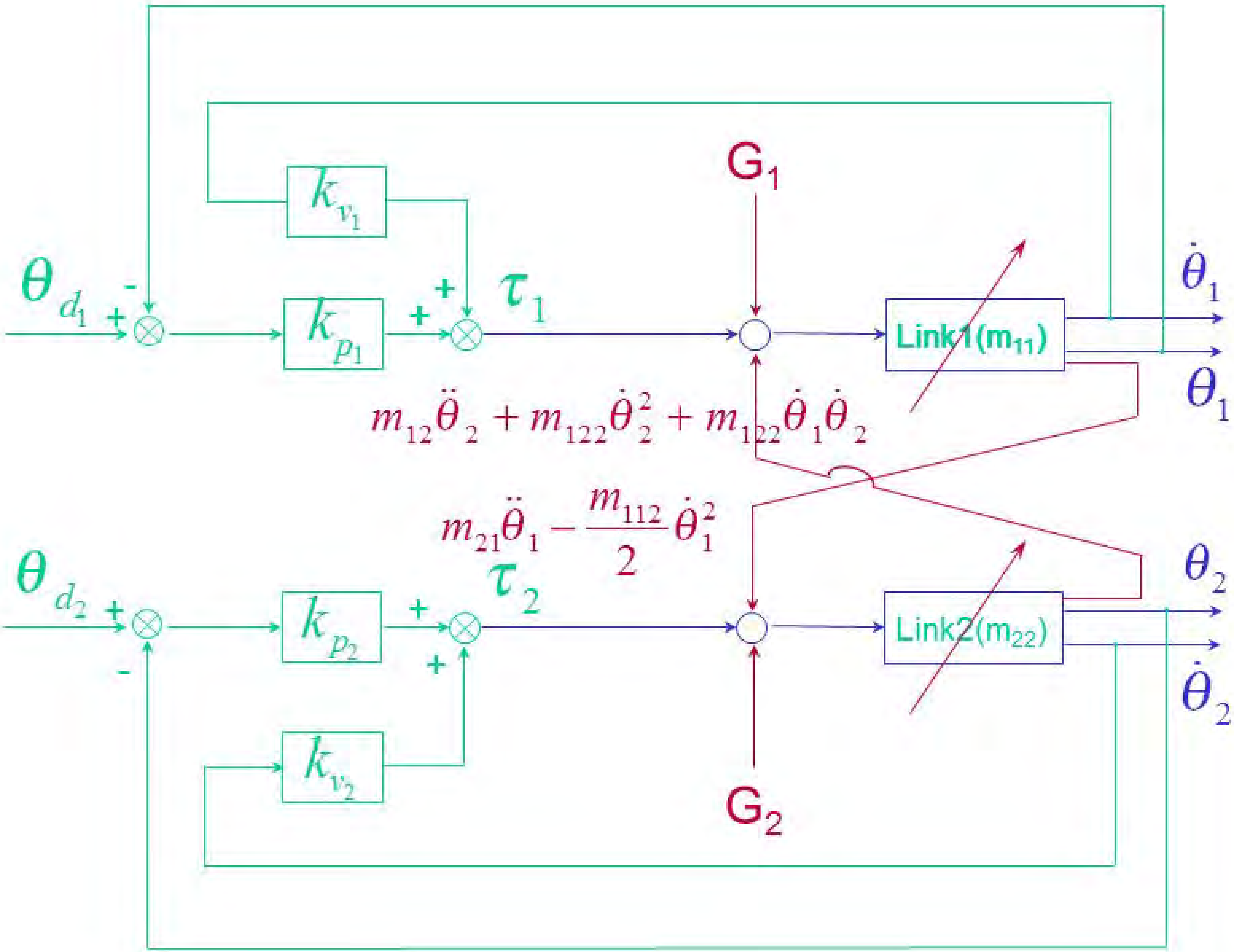


$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) = \tau$$

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} + \begin{pmatrix} m_{112} \\ 0 \end{pmatrix}\begin{pmatrix} \dot{\theta}_1 & \dot{\theta}_2 \end{pmatrix} + \begin{pmatrix} 0 & m_{122} \\ -\dfrac{m_{112}}{2} & 0 \end{pmatrix}\begin{pmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{pmatrix} + \begin{pmatrix} G_1 \\ G_2 \end{pmatrix} = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix}$$

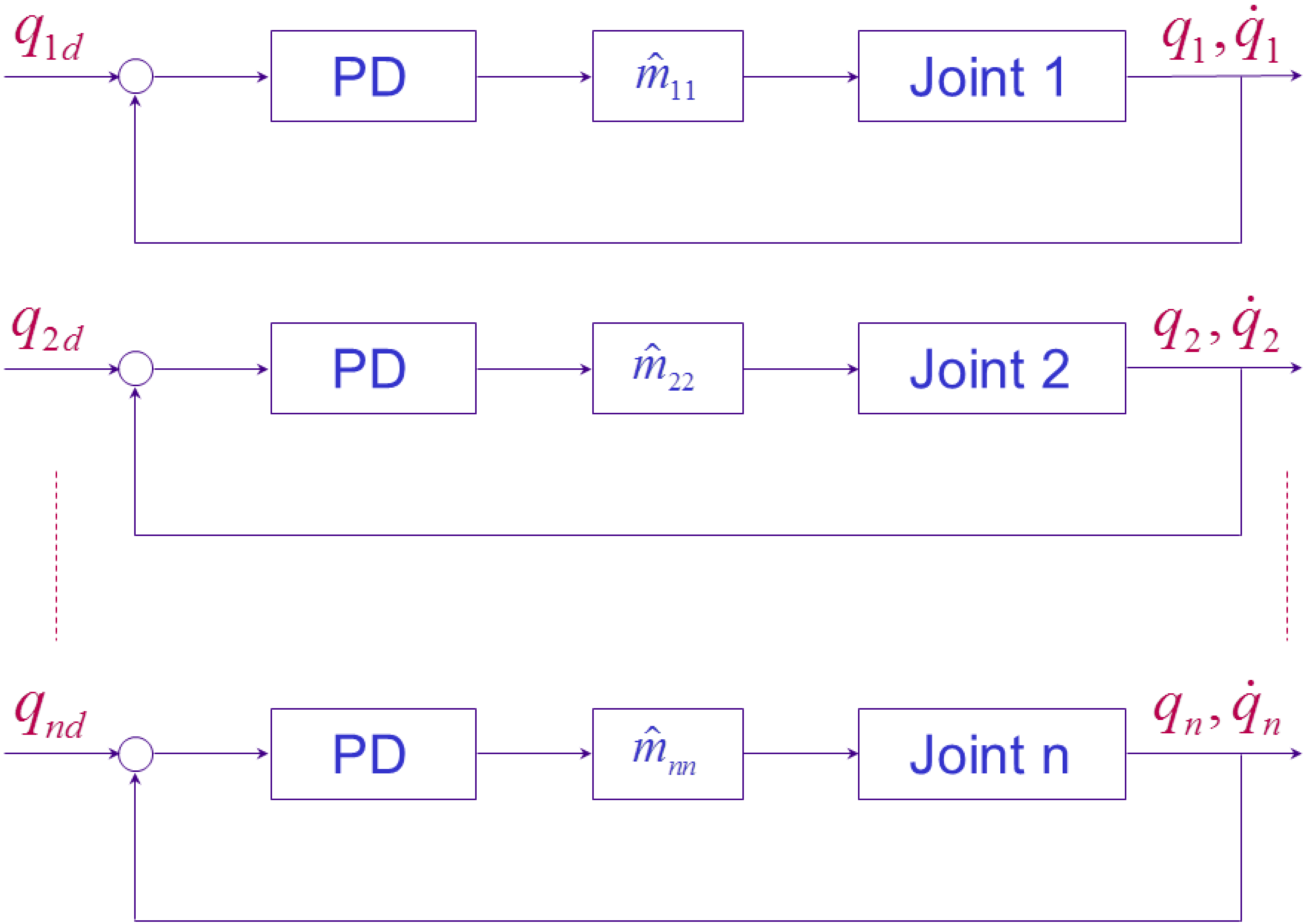
$$\underline{m_{11}\ddot{\theta}_1} + m_{12}\ddot{\theta}_2 + m_{112}\dot{\theta}_1\dot{\theta}_2 + m_{122}\dot{\theta}_2^2 + G_1 = \underline{\tau_1}$$

$$\underline{m_{22}\ddot{\theta}_2} + m_{21}\ddot{\theta}_1 - \frac{m_{112}}{2}\dot{\theta}_1^2 + G_2 = \underline{\tau_2}$$

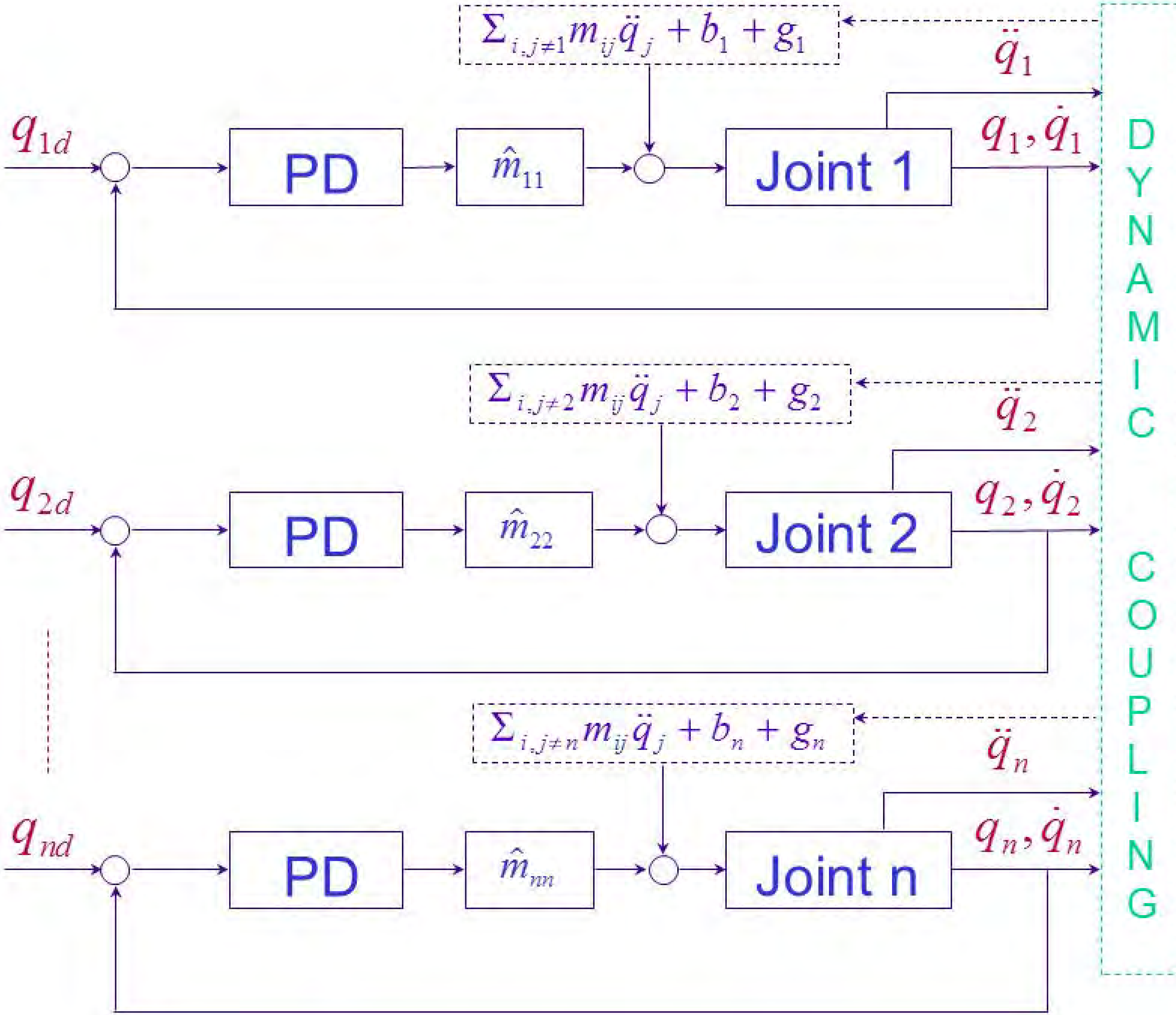


θ_{d_1}
k_{v_1}
k_{p_1}
τ_1
G_1
Link1(m_{11})
$\dot{\theta}_1$
θ_1
$m_{12}\ddot{\theta}_2 + m_{122}\dot{\theta}_2^2 + m_{122}\dot{\theta}_1\dot{\theta}_2$
$m_{21}\ddot{\theta}_1 - \frac{m_{112}}{2}\dot{\theta}_1^2$
θ_{d_2}
τ_2
k_{p_2}
k_{v_2}
Link2(m_{22})
θ_2
$\dot{\theta}_2$
G_2


RANGER
XP
POLARIS



q_{1d}
PD
$\hat{m}_{11}$
Joint 1
$q_1, \dot{q}_1$
q_{2d}
PD
$\hat{m}_{22}$
Joint 2
$q_2, \dot{q}_2$
q_{nd}
PD
$\hat{m}_{nn}$
Joint n
$q_n, \dot{q}_n$



$\Sigma_{i,j\neq 1} m_{ij}\ddot{q}_j + b_1 + g_1$
$\ddot{q}_1$
q_{1d}
PD
$\hat{m}_{11}$
Joint 1
$q_1, \dot{q}_1$
$\Sigma_{i,j\neq 2} m_{ij}\ddot{q}_j + b_2 + g_2$
$\ddot{q}_2$
q_{2d}
PD
$\hat{m}_{22}$
Joint 2
$q_2, \dot{q}_2$
$\Sigma_{i,j\neq n} m_{ij}\ddot{q}_j + b_n + g_n$
$\ddot{q}_n$
q_{nd}
PD
$\hat{m}_{nn}$
Joint n
$q_n, \dot{q}_n$
DYNAMIC
COUPLING

PD Control Stability

$$M(q)\ddot{q}+B(q)[\dot{q}\dot{q}]+C(q)[\dot{q}^2]+G(\theta)=\tau$$

$$\tau=-k_p(q-q_d)-k_v\dot{q}$$

$$V_d=1/2k_p(q-q_d)^2$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right)-\frac{\partial K}{\partial q}+\frac{\partial V_s}{\partial q}=\cancel{\tau}\frac{\partial V_d}{\partial q}-k_v\dot{q}$$

PD Control Stability

$$M(q)\ddot{q}+B(q)[\dot{q}\dot{q}]+C(q)[\dot{q}^2]+G(\theta)=\tau$$

$$\tau=-k_p(q-q_d)-k_v\dot{q}$$

$$V_d=1/2k_p(q-q_d)^T(q-q_d)$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right)-\frac{\partial K}{\partial q}+\frac{\partial (V_s-V_d)}{\partial q}=\tau_s$$

$\tau_s=-k_v\dot{q}$ with $\boxed{\tau_s^T\dot{q}<0 \ for\ \dot{q}\neq 0;\ k_v>0}$

Performance

High Gains $\longrightarrow$ better disturbance rejection

Gains are limited by

structural flexibilities

time delays (actuator-sensing)

sampling rate

$$\omega_n \le \frac{\omega_{res}}{2} \longleftarrow \text{lowest structural flexibility}$$

$$\omega_n \le \frac{\omega_{delay}}{3} \longleftarrow \text{largest delay} \left(\frac{2\pi}{\tau_{delay}}\right)$$

$$\omega_n \le \frac{\omega_{sampling-rate}}{5}$$

Nonlinear Dynamic Decoupling

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) = \tau$$

$$\tau = \hat{M}(\theta)\underline{\tau'} + \hat{V}(\theta,\dot{\theta}) + \hat{G}(\theta)$$

$$\mathbf{1}.\ddot{\theta} = (M^{-1}\hat{M})\tau' + M^{-1}[(V-\hat{V}) + (G-\hat{G})]$$

with perfect estimates

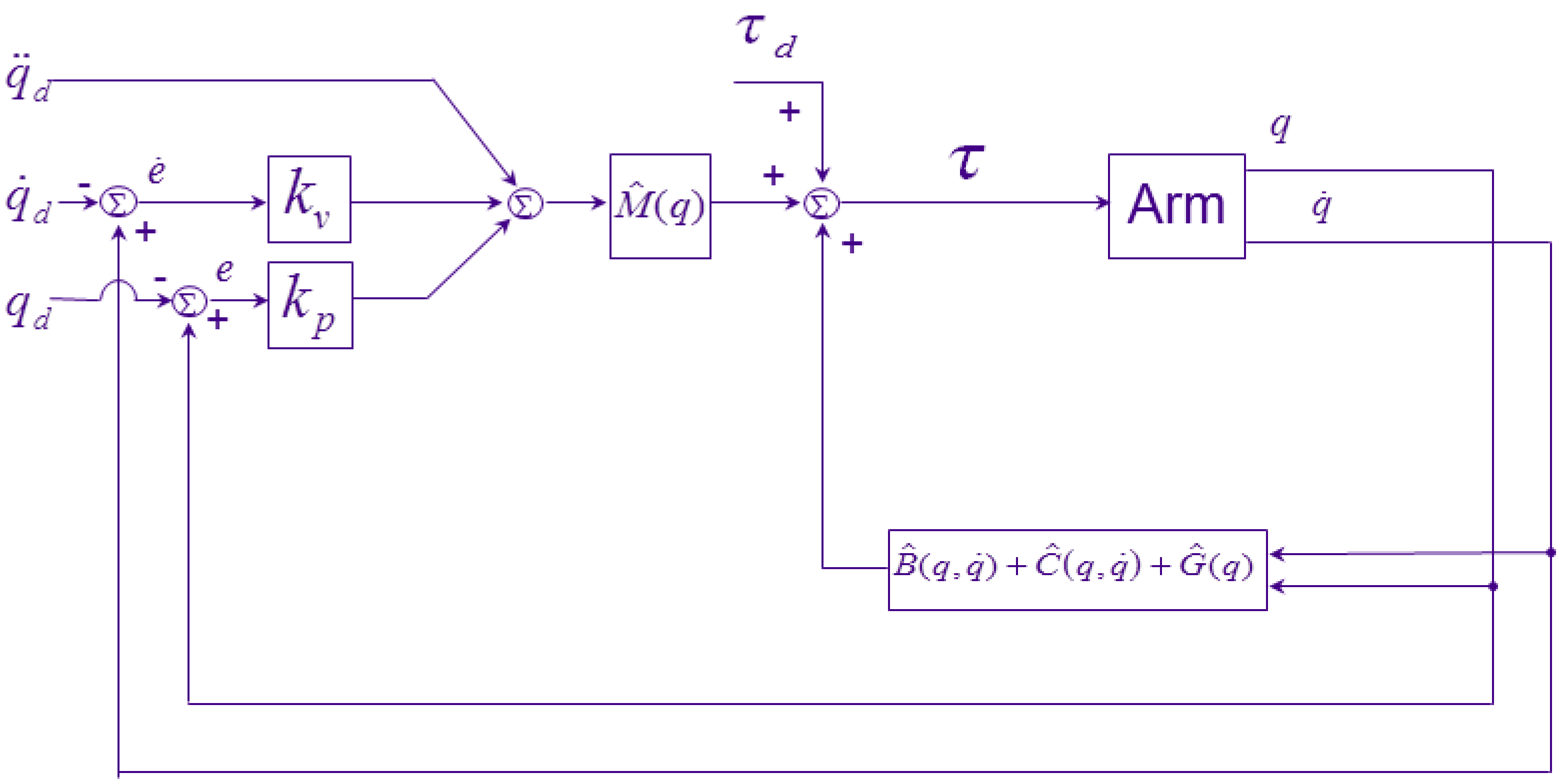
$$\mathbf{1}.\ddot{\theta} = \tau' + \varepsilon(t)$$

τ': input of the unit-mass systems

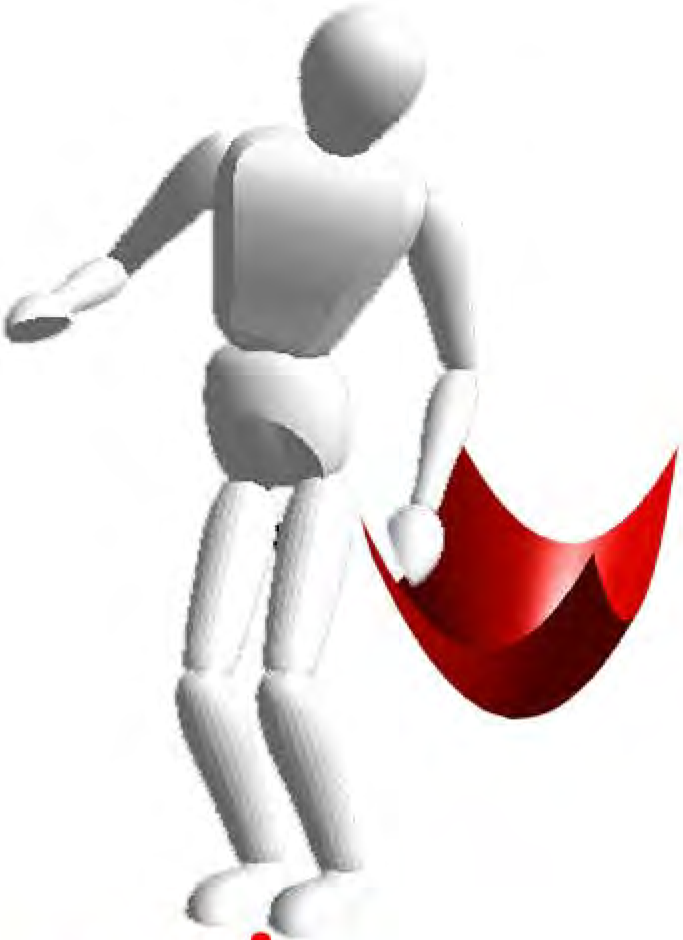
$$\tau' = \ddot{\theta}_d - k'_v(\dot{\theta} - \dot{\theta}_d) - k'_p(\theta - \theta_d)$$

Closed-loop

$$\ddot{E} + k'_v\dot{E} + k'_p E = 0 + \varepsilon(t)$$



$\ddot{q}_d$
$\dot{q}_d$
q_d
$\dot{e}$
e
k_v
k_p
$\hat{M}(q)$
τ_d
τ
Arm
q
$\dot{q}$
$\hat{B}(q,\dot{q}) + \hat{C}(q,\dot{q}) + \hat{G}(q)$

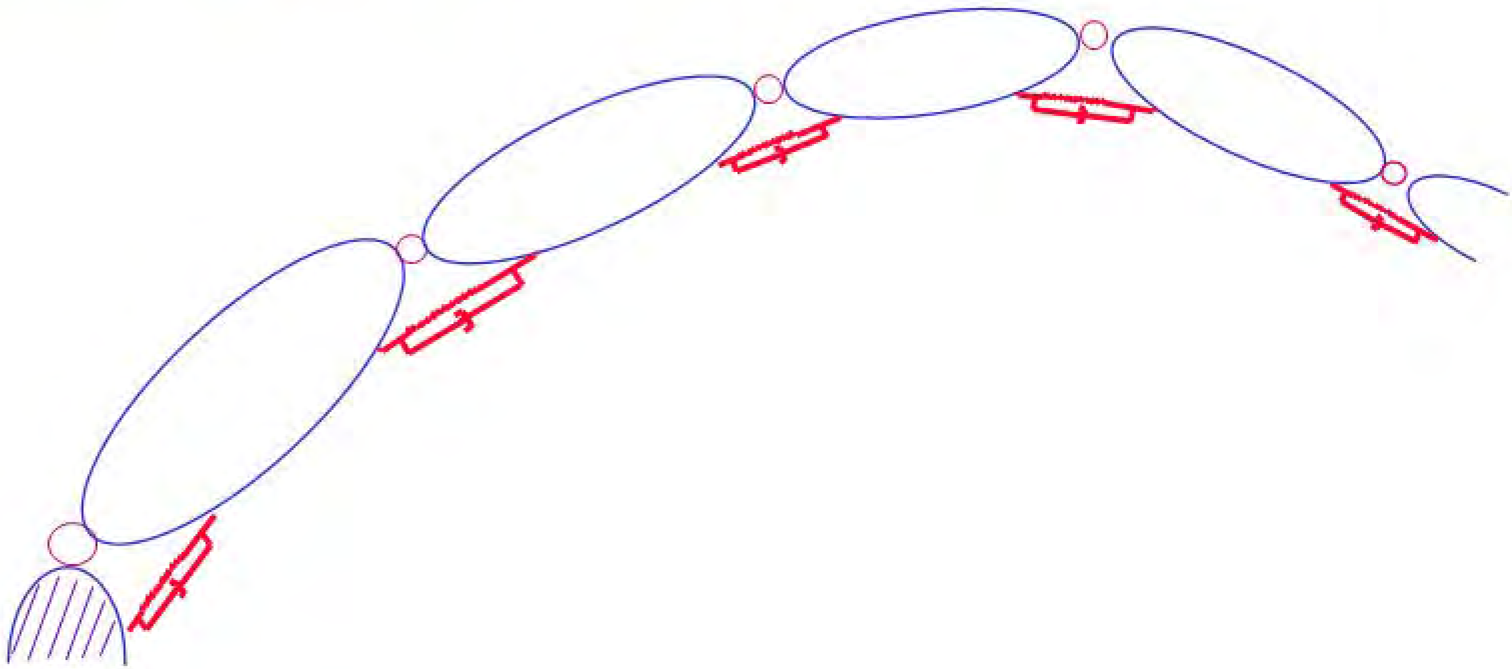


Robot Control

Control

- Natural Systems
- PID Control
- Joint-Space Dynamic Control
- Task-Oriented Control
- Force Control

Joint-Space Control



Proportional-Derivative Control (PD)

$$m\ddot{x} = f = -k_p(x - x_d) - k_v\dot{x}$$

$$m\ddot{x} + k_v\dot{x} + k_p(x - x_d) = 0$$

Velocity gain

Position gain

$$1.\ \ddot{x} + \frac{k_v}{m}\dot{x} + \frac{k_p}{m}(x - x_d) = 0$$

$$1.\ \ddot{x} + 2\xi\omega\dot{x} + \omega^2(x - x_d) = 0$$

$$\xi = \frac{k_v}{2\sqrt{k_p m}}$$ closed loop damping ratio

$$\omega = \sqrt{\frac{k_p}{m}}$$ closed loop frequency

Gains

$$k_p = m\omega^2$$

$$k_v = m(2\xi\omega)$$

Gain Selection

$$\text{set} \begin{pmatrix} \xi \\ \omega \end{pmatrix} \rightarrow \begin{array}{l} k_p = m\omega^2 \\ k_v = m(2\xi\omega) \end{array}$$

Unit mass system

$$k'_p = \omega^2$$

$$k'_v = 2\xi\omega$$

m - mass system

$$k_p = m\, k'_p$$

$$k_v = m\, k'_v$$

Control Partitioning

$$m\ddot{x} = f \quad \Longrightarrow \quad m \;(1.\ddot{x}) = m \;\; f'$$

$$f = -k_v\dot{x} - k_p(x - x_d)$$

$$f = m[\underbrace{-k_v'\dot{x} - k_p'(x - x_d)}_{f'}] = m f'$$

$$m\ddot{x} = m f'$$

$1.\ddot{x} = f'$ unit mass system

$$1.\ddot{x} + \underset{\substack{\Uparrow \\ 2\xi\omega}}{k_v'}\dot{x} + \underset{\substack{\Uparrow \\ \omega^2}}{k_p'}(x - x_d) = 0$$

Non Linearities

$$m\ddot{x} + b(x, \dot{x}) = f$$

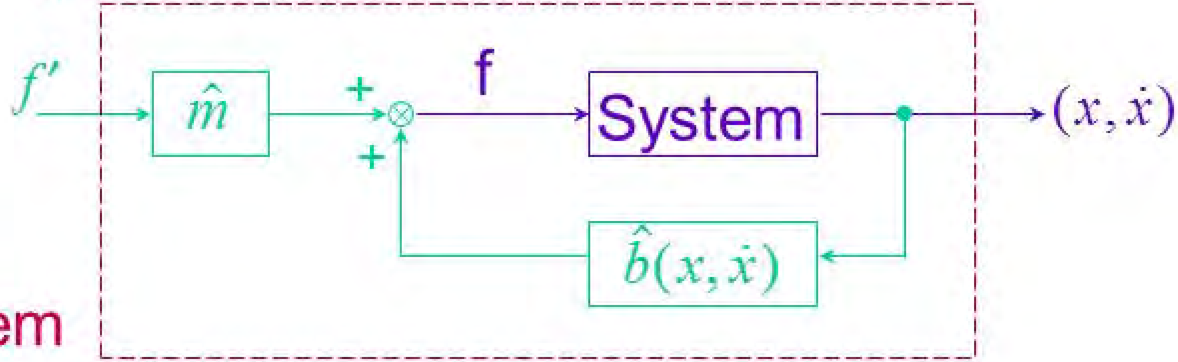
Control Partitioning

$$f = \alpha f' + \beta$$

with
$$\alpha = \hat{m}$$
$$\beta = \hat{b}(x, \dot{x})$$

$$m\ddot{x} + b(x, \dot{x}) = \hat{m}f' + \hat{b}(x, \dot{x})$$

$$\Longrightarrow \quad 1.\ddot{x} = f'$$



Unit mass system