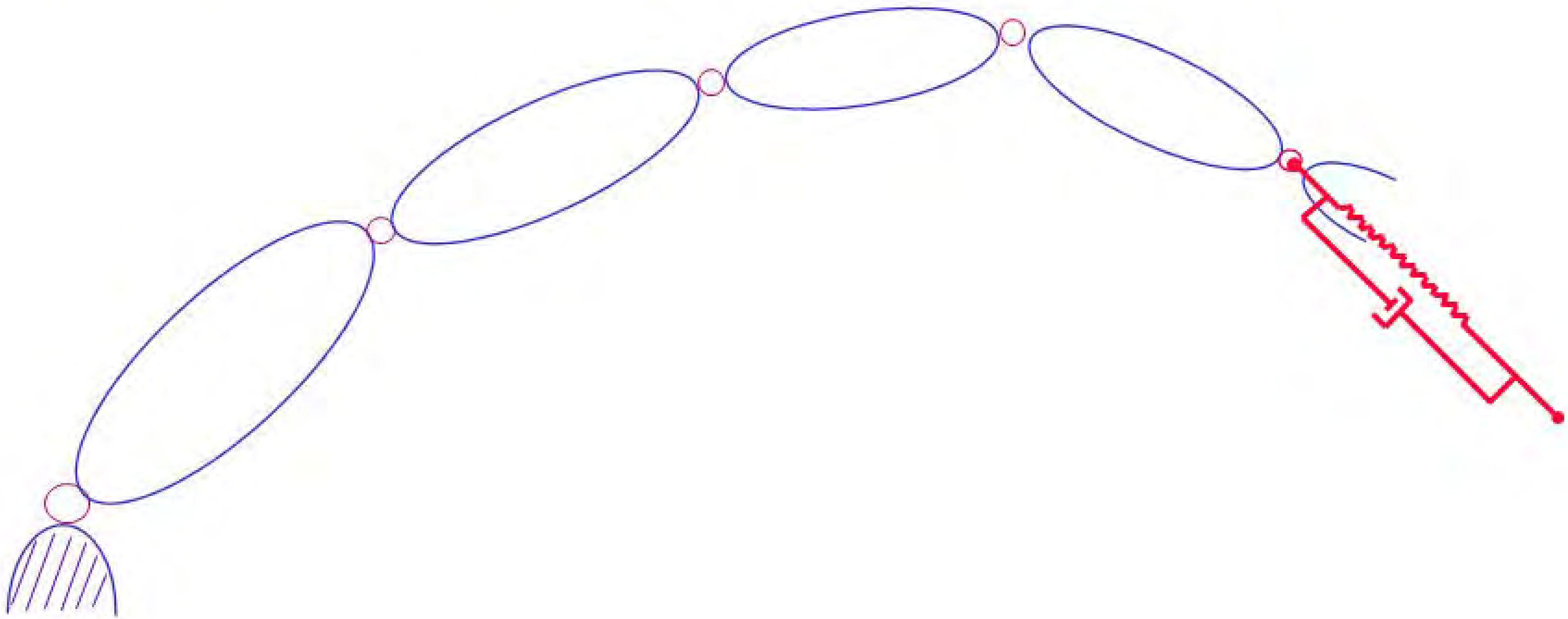
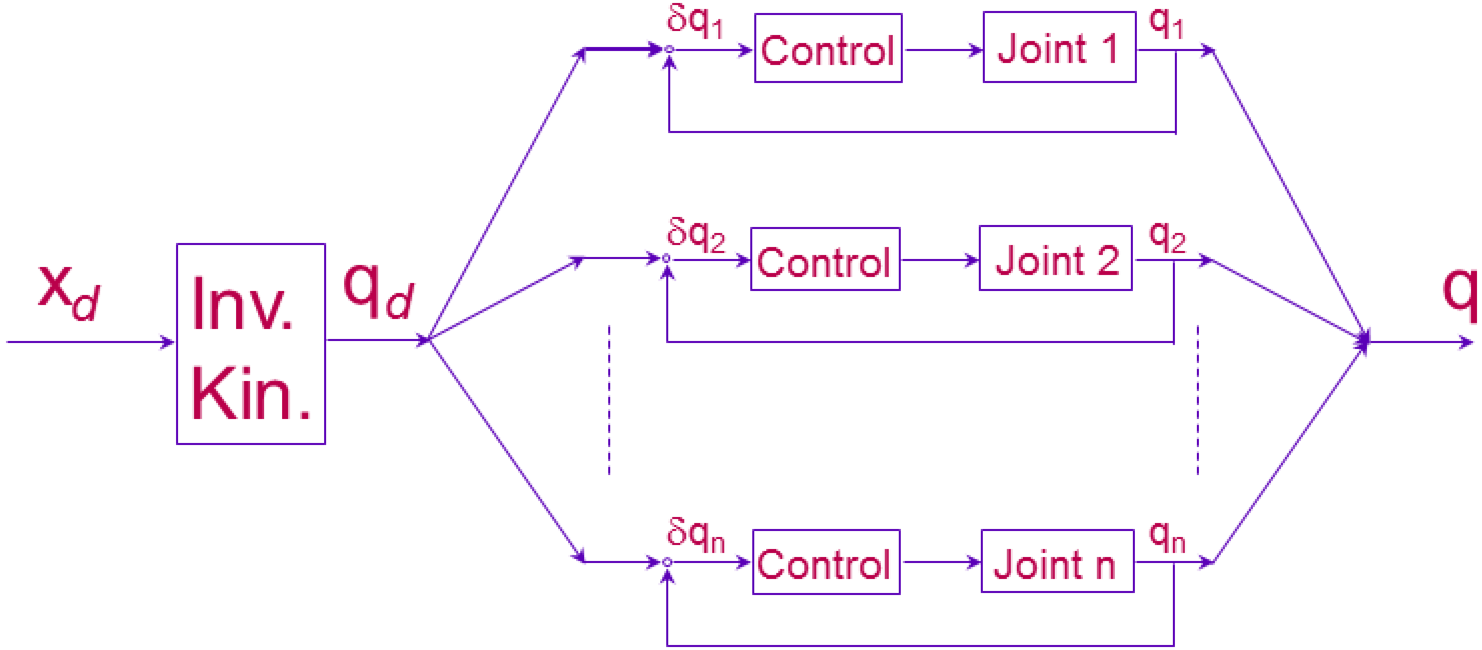
Video Segment

Valkyrie: NASA's Superhero Robot, IEEE Spectrum, 2013

Task-Oriented Control



Joint Space Control



Resolved Motion Rate Control (Whitney 72)

$$\delta x = J(\theta)\delta\theta$$

Outside singularities

$$\delta\theta = J^{-1}(\theta)\delta x$$

Arm at Configuration θ

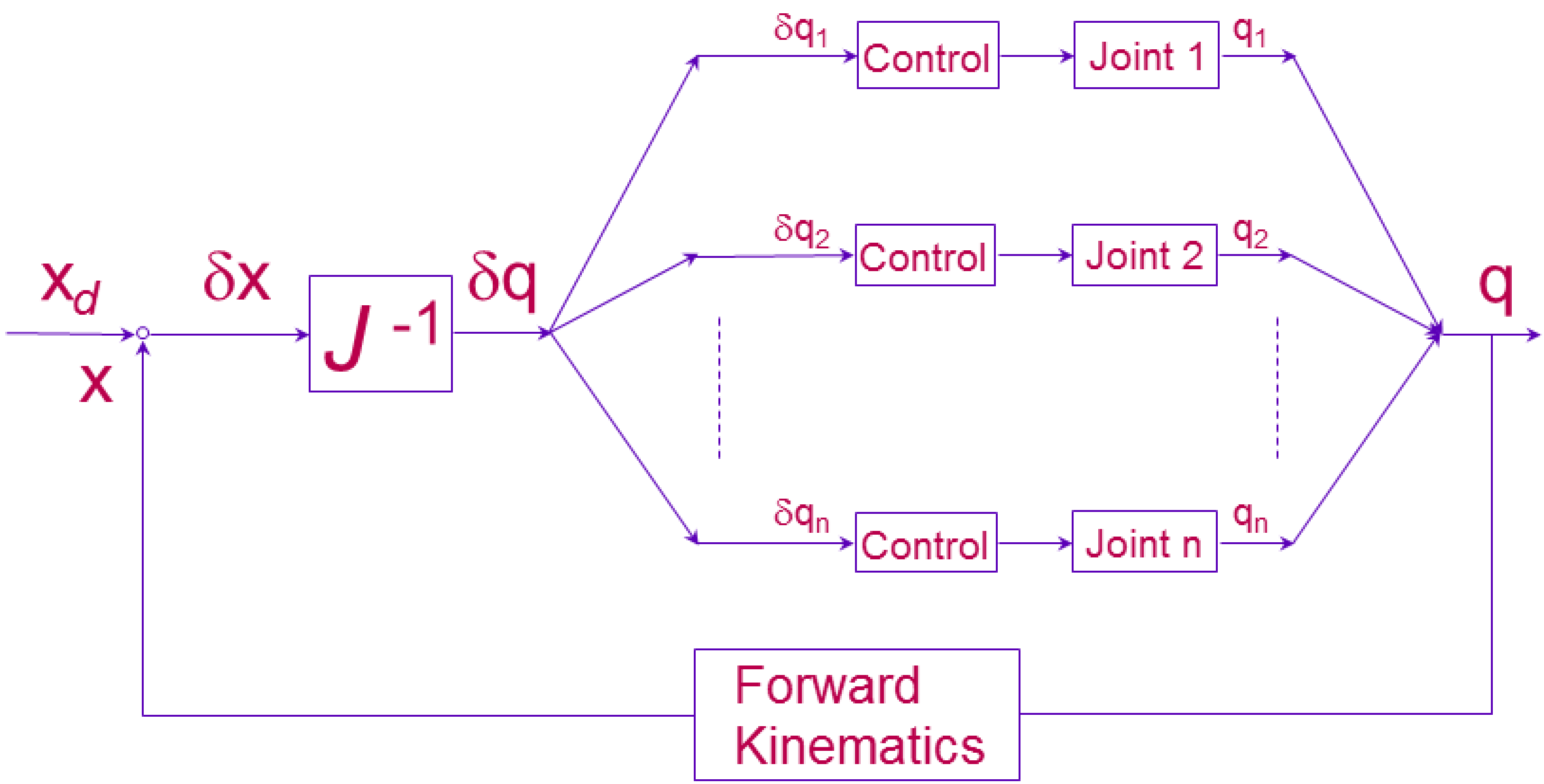
$$x = f(\theta)$$

$$\delta x = x_d - x$$

$$\delta\theta = J^{-1}\delta x$$

$$\boxed{\theta^{+} = \theta + \delta\theta}$$

Resolved Motion Rate Control



Natural Systems

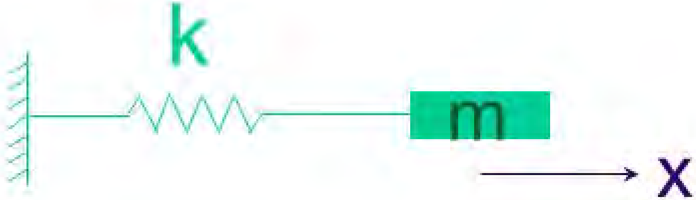
Conservative Systems

$$\frac{d}{dt}\left(\frac{\partial (K-V)}{\partial \dot{x}}\right)-\frac{\partial (K-V)}{\partial x}=0$$

$$K=\frac{1}{2}m\dot{x}^2$$

Natural Systems

Conservative Forces



$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right)-\frac{\partial K}{\partial x}=-\frac{\partial V}{\partial x}$$

$$m\ddot{x} = F = -kx$$

$$-\frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right)$$

Potential Energy of a spring

$$V = Work = \int_x^0 (-kx)\delta x = \frac{1}{2}kx^2$$

Natural Systems

Conservative Forces

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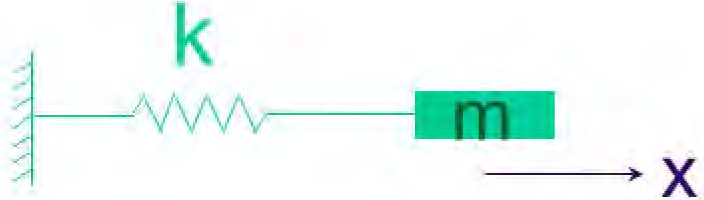
Natural Systems

Conservative Systems



$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial \dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=0$$

$$K=\frac{1}{2}m\dot{x}^2$$

$$V=\frac{1}{2}kx^2$$

$$m\ddot{x}+kx=0$$

Natural Systems

Conservative Systems

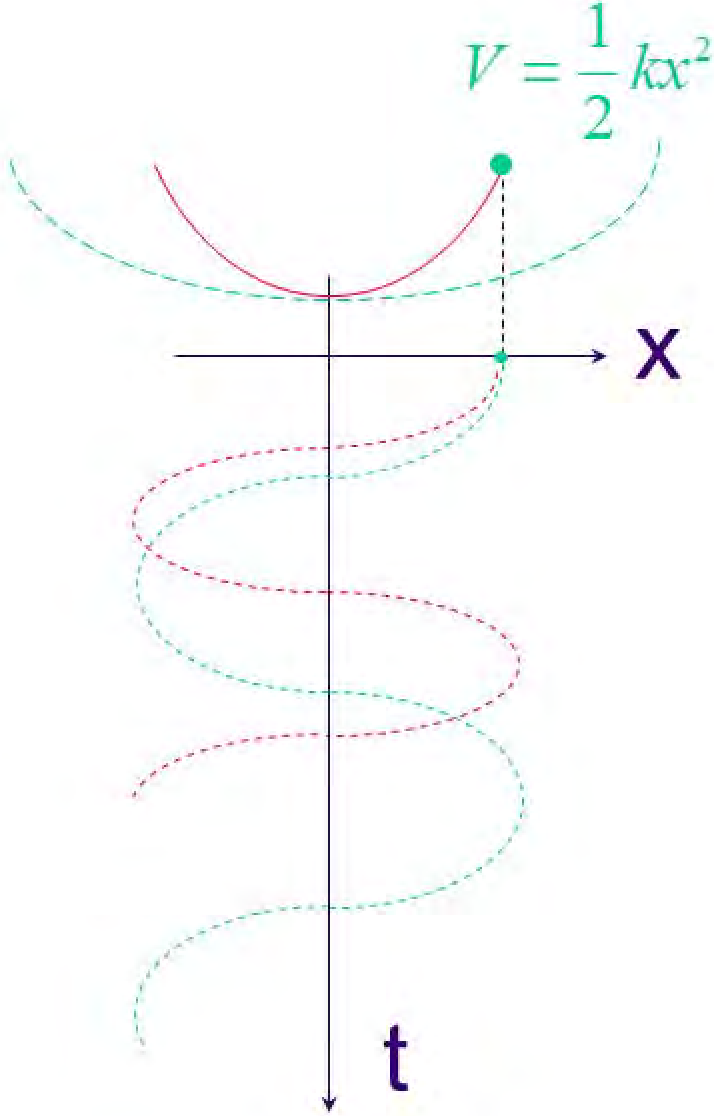
$$m\ddot{x} + kx = 0$$

Frequency increases
with stiffness
and inverse mass

Natural Frequency $\omega_n = \sqrt{\frac{k}{m}}$

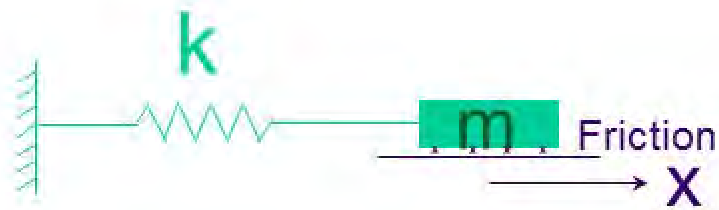
$$\ddot{x} + \omega_n^2 x = 0$$

$$x(t) = c \cos(\omega_n t + \phi)$$



Natural Systems

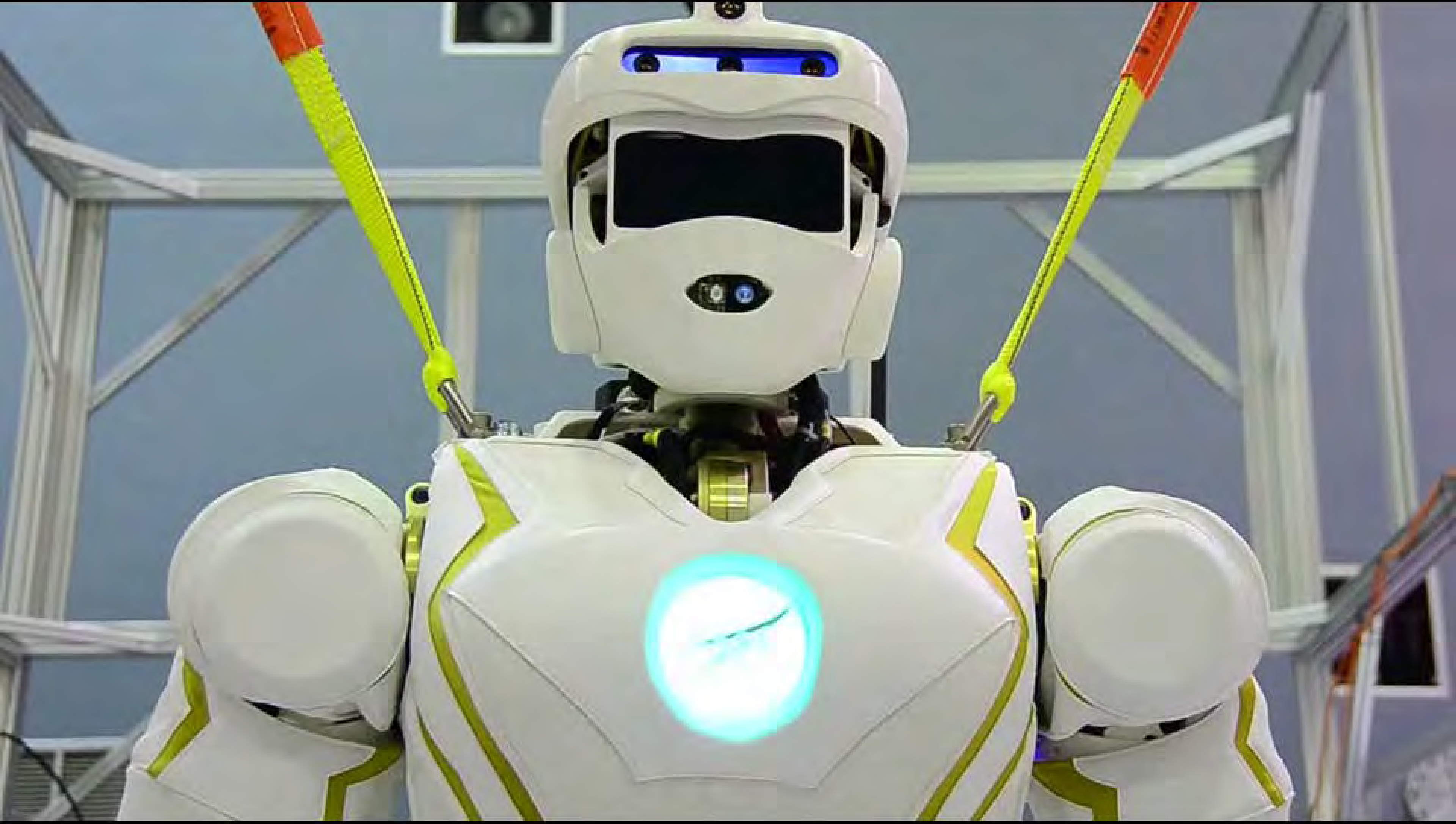
Dissipative Systems



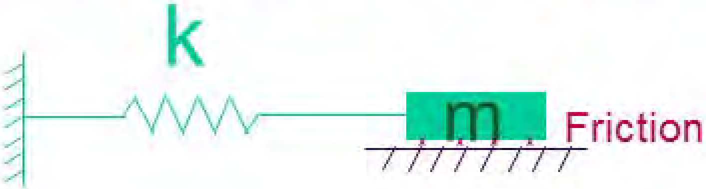
$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial\dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=f_{friction}$$

Viscous friction: $f_{friction}=-b\dot{x}$

$$m\ddot{x}+b\dot{x}+kx=0$$

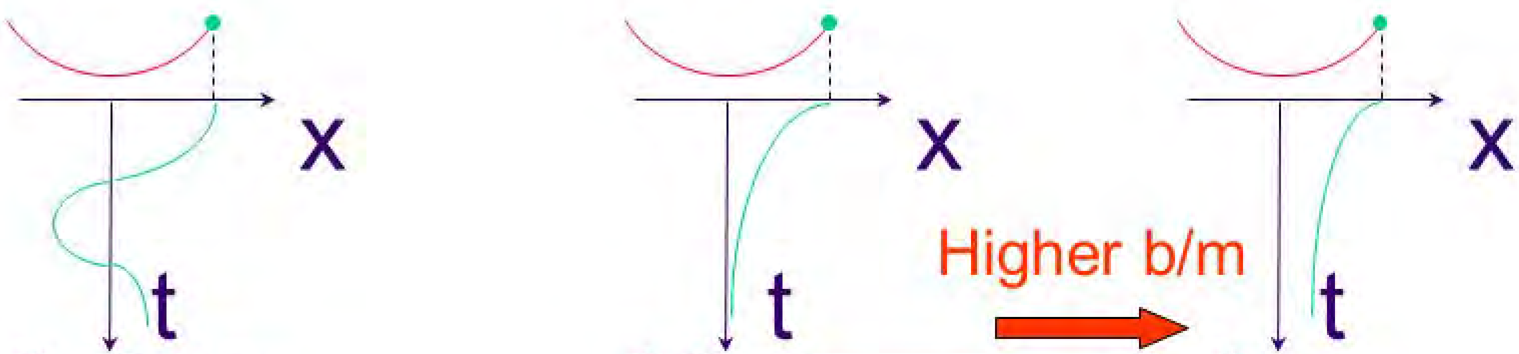


Dissipative Systems



$$m\ddot{x} + b\dot{x} + kx = 0$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$



Oscillatory damped

Critically damped

$$\frac{b}{m} = 2\omega_n$$

Over damped

2$^{\underline{d}}$ order systems

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\frac{b/m}{2\omega_n} \cdot 2\omega_n$$

$$\omega_n{}^2$$

Natural damping ratio

Critically damped when $b/m=2\omega_n$

$$\xi_n = \frac{b}{2\omega_n m} = \frac{b}{2\sqrt{km}}$$

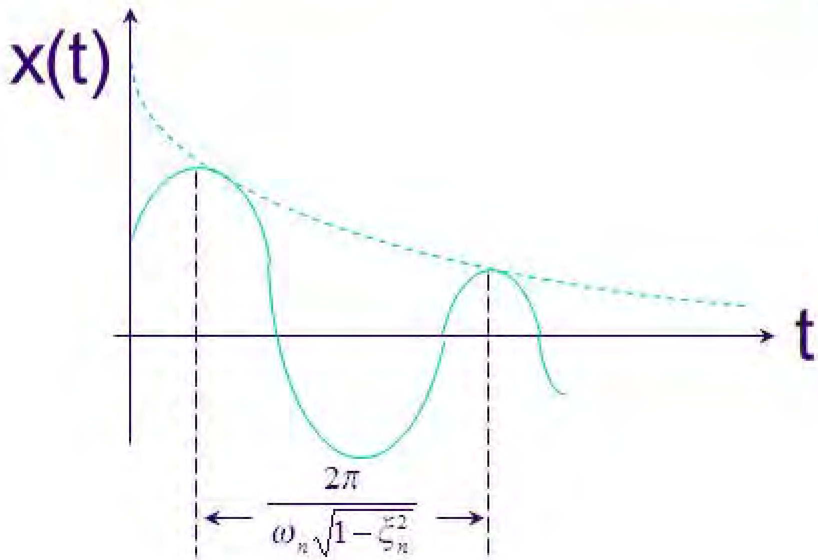
Critically damped system: $\xi_n = 1 \ \ (b = 2\sqrt{km})$

Time Response

$$\ddot{x} + 2\xi_n \omega_n \dot{x} + \omega_n^2 x = 0$$

Natural frequency $\omega_n = \sqrt{\dfrac{k}{m}}$; $\xi_n = \dfrac{b}{2\sqrt{km}}$ Natural damping ratio

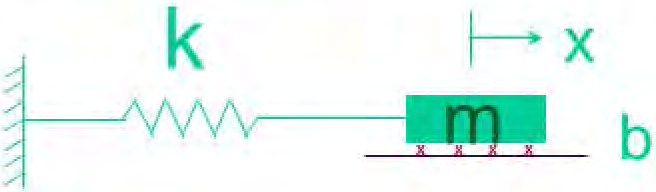
$$x(t) = ce^{-\xi_n \omega_n t} \cos(\underbrace{\omega_n \sqrt{1-\xi_n^2}}_{\omega} t + \phi)$$



damped Natural frequency

$$\omega = \omega_n \sqrt{1-\xi_n^2}$$

Example



$$m = 2.0$$
$$b = 4.8$$
$$k = 8.0$$

$$m\ddot{x} + b\dot{x} + kx = 0$$

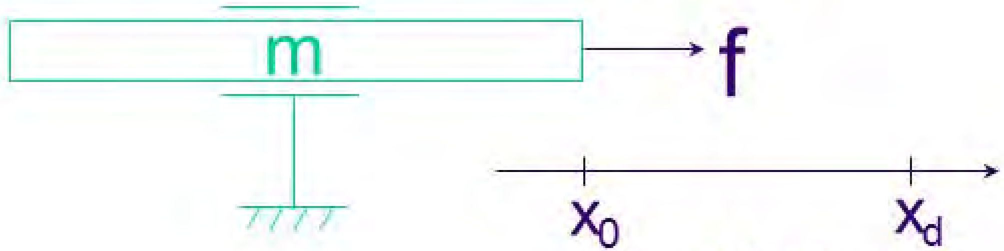
what is the "damped Natural frequency"

$$\omega = \omega_n \sqrt{1 - \xi_n^2}$$

$$\omega_n = \sqrt{\frac{k}{m}} = 2\ ; \quad \xi_n = \frac{b}{2\sqrt{km}} = 0.6$$

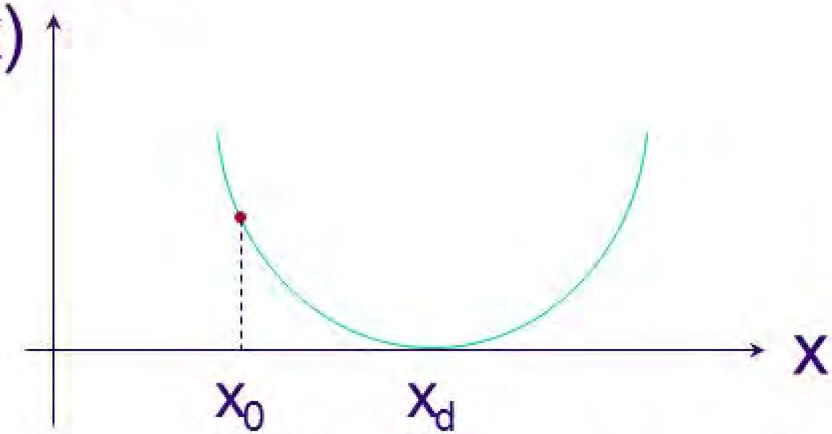
$$\omega = 2\sqrt{1 - 0.36} = 1.6$$

1-dof Robot Control



$$m\ddot{x} = f$$

Potential Field



$$V(x) > 0, x \neq x_d$$

$$V(x) = 0, x = x_d$$

$$V(x) = \frac{1}{2}k_p(x - x_d)^2 \; ; f = -\nabla V(x) = -\frac{\partial V}{\partial x}$$

$$m\ddot{x} = -\frac{\partial}{\partial x}[\frac{1}{2}k_p(x - x_d)^2] \; ; \; m\ddot{x} + k_p(x - x_d) = 0$$

Position gain

Passive Systems (Stability)

$$V_{goal} = \frac{1}{2} k_p \left(x - x_g\right)^T \left(x - x_g\right)$$

System $$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial K}{\partial x} = f$$

$$\Downarrow \quad f = -\frac{\partial V_{goal}}{\partial X}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial \left(K - V_{goal}\right)}{\partial x} = 0$$

Conservative Forces

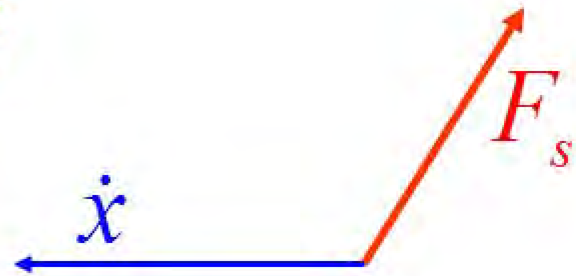
Stable

Asymptotic Stability

a system $$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right)-\frac{\partial\left(K-V_{goal}\right)}{\partial x}=0 \quad F_s$$

is asymptotically stable if

$$\boxed{F_s^T\,\dot{x} < 0 \quad ; \; for \;\; \dot{x} \neq 0}$$



$$F_s = -k_v\dot{x} \;\; \rightarrow \;\; k_v > 0$$

Control

$$F = -k_p\left(x - x_{goal}\right) - k_v\dot{x}$$

Proportional-Derivative Control (PD)

$$m\ddot{x} = f = -k_p(x - x_d) - k_v\dot{x}$$

$$m\ddot{x} + k_v\dot{x} + k_p(x - x_d) = 0$$

Velocity gain (k_v) — Position gain (k_p)

$$1.\ \ddot{x} + \frac{k_v}{m}\dot{x} + \frac{k_p}{m}(x - x_d) = 0$$

$$1.\ \ddot{x} + 2\xi\omega\dot{x} + \omega^2(x - x_d) = 0$$

$$\xi = \frac{k_v}{2\sqrt{k_p m}}$$ closed loop damping ratio

$$\omega = \sqrt{\frac{k_p}{m}}$$ closed loop frequency

Gains

$$k_p = m\omega^2$$

$$k_v = m(2\xi\omega)$$

Gain Selection

$$\text{set} \begin{pmatrix} \xi \\ \omega \end{pmatrix} \rightarrow \begin{array}{l} k_p = m\omega^2 \\ k_v = m(2\xi\omega) \end{array}$$

Unit mass system

$$k'_p = \omega^2$$

$$k'_v = 2\xi\omega$$

m - mass system

$$k_p = m\, k'_p$$

$$k_v = m\, k'_v$$

Control Partitioning

$$m\ddot{x} = f \quad \Longrightarrow \quad m\ (1.\ddot{x}) = m\ \ f'$$

$$f = -k_v\dot{x} - k_p(x - x_d)$$

$$f = m[\underbrace{-k'_v\dot{x} - k'_p(x - x_d)}_{f'}] = m f'$$

$$m\ddot{x} = m f'$$

$1.\ddot{x} = f'$ unit mass system

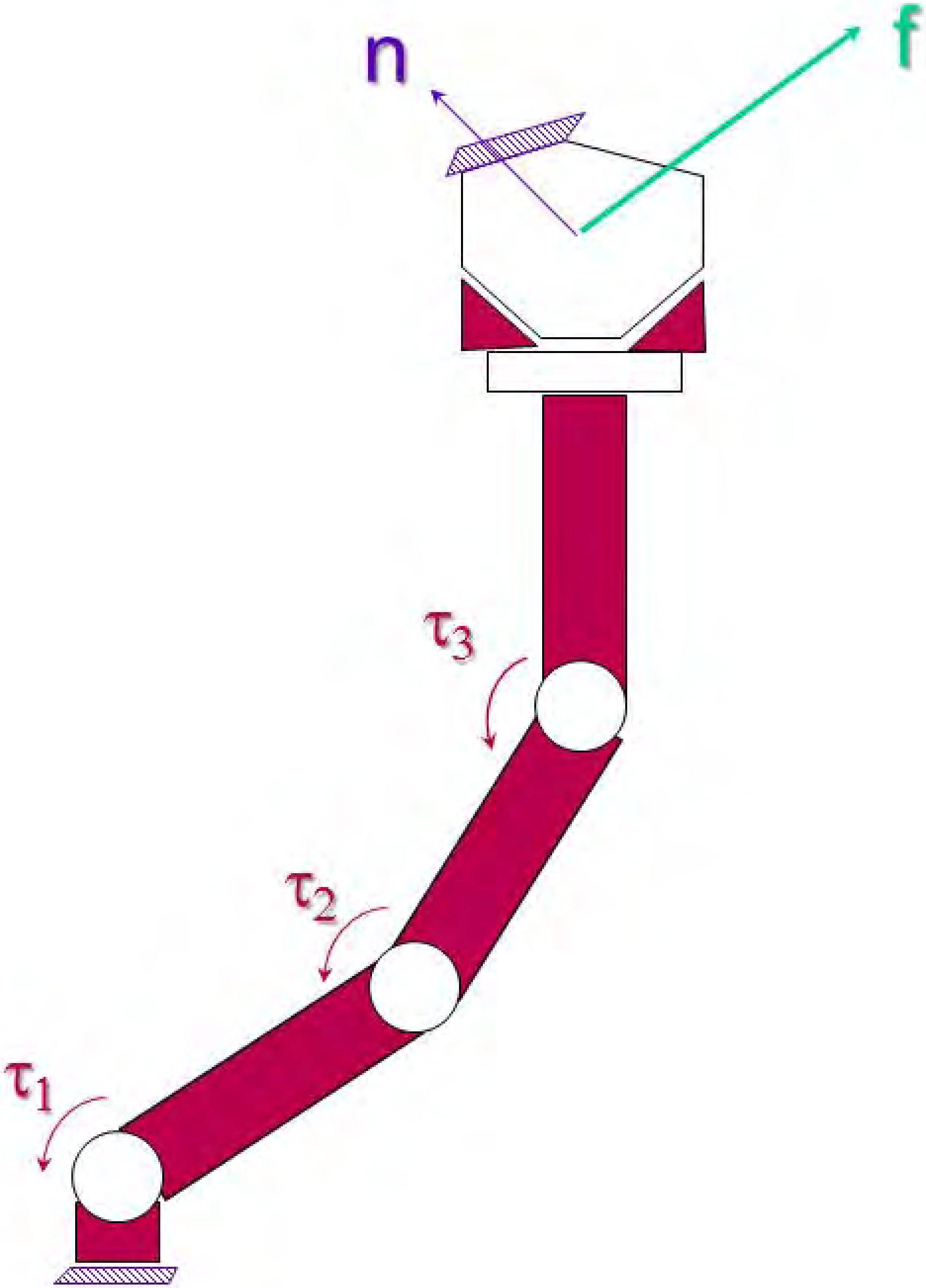
$$1.\ddot{x} + k'_v\dot{x} + k'_p(x - x_d) = 0$$

$$k'_v \Leftarrow 2\xi\omega \qquad k'_p \Leftarrow \omega^2$$

Robot Control

Control

- Natural Systems
- PID Control
- Joint-Space Dynamic Control
- Task-Oriented Control
- Force Control



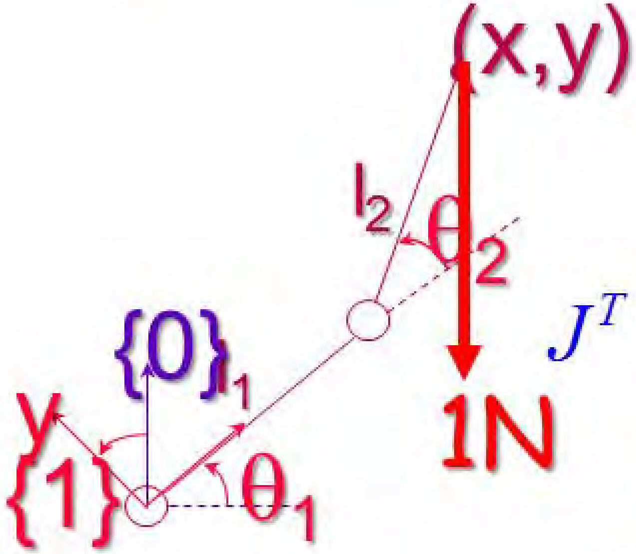
n
f
τ3
τ2
τ1

Velocity/Force Duality

$$\dot{x} = J\,\dot{\theta}$$

$$\tau = J^T F$$

Example (Static Forces)



$$J = \begin{pmatrix} -(l_1 S1 + l_2 S12) & -l_2 S12 \\ l_1 C1 + l_2 C12 & l_2 C12 \end{pmatrix}$$

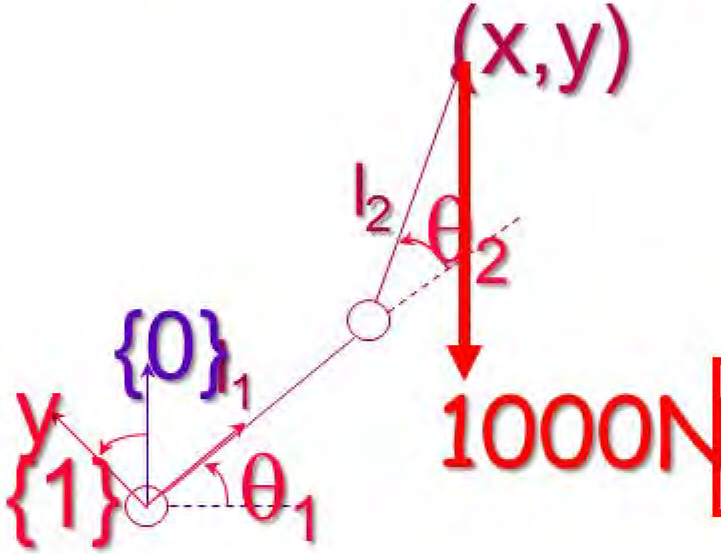
$$J^T = \begin{pmatrix} -(l_1 S1 + l_2 S12) & l_1 C1 + l_2 C12 \\ -l_2 S12 & l_2 C12 \end{pmatrix}$$

$$\tau = J^T F$$

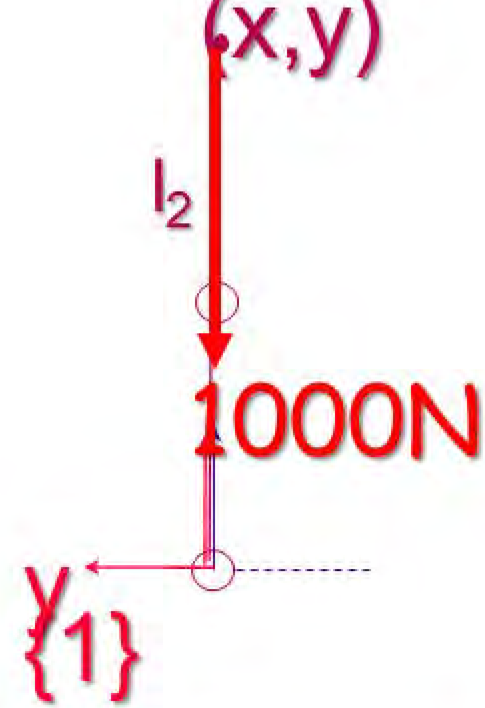
$$l_1 = l_2 = 1; \quad \theta_1 = 0; \ \theta_2 = 60^\circ$$

$$\tau = \begin{pmatrix} -(l_1 S1 + l_2 S12) & l_1 C1 + l_2 C12 \\ -l_2 S12 & l_2 C12 \end{pmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -\begin{bmatrix} l_1 C1 + l_2 C12 \\ l_2 C12 \end{bmatrix} = -\begin{bmatrix} 3/2 \\ 1/2 \end{bmatrix}$$

Example (Static Forces)



$$\tau = J^T F$$



$$\tau = \begin{pmatrix} -\left(l_1 S1 + l_2 S12\right) & l_1 C1 + l_2 C12 \\ -l_2 S12 & l_2 C12 \end{pmatrix} \begin{bmatrix} 0 \\ -1K \end{bmatrix} = \begin{bmatrix} l_1 C1 + l_2 C12 \\ l_2 C12 \end{bmatrix} (-1K) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$l_1 = l_2 = 1; \quad \theta_1 = 90; \theta_2 = 0^\circ$$

Joint-Space Control