Movie Segment

The Curiosity Mars Rover.

Steven Lee, Jet Propulsion Laboratory, 2010.

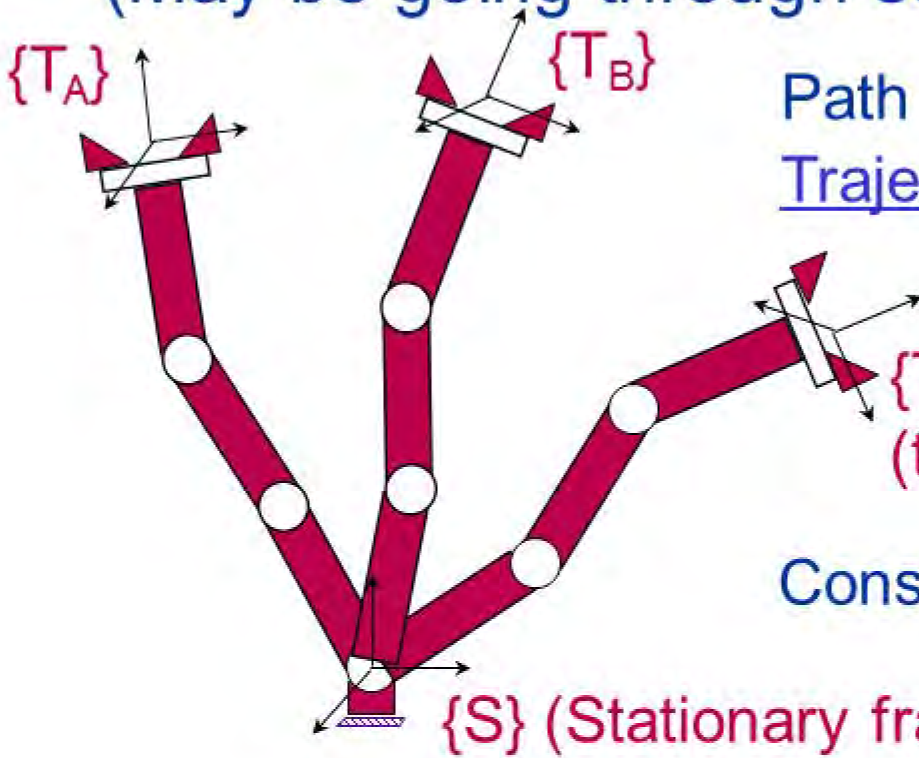
Trajectory Generation

Trajectory Generation

Basic Problem:

Move the manipulator arm from some initial position $\{T_A\}$ to some desired final position $\{T_C\}$.

(May be going through some via point $\{T_B\}$)



Path points : Initial, final and via points

Trajectory : Time history of position, velocity and acceleration for each DOF

Constraints: Spatial, time, smoothness

Solution Spaces :

Joint space

- Easy to go through via points
  (Solve inverse kinematics at all path points and plan)
- No problems with singularities
- Less calculations
- Can not follow straight line

Cartesian space

- We can track a shape
  (for orientation : equivalent axes, Euler angles,…
- More expensive at run time
  (after the path is calculated need joint angles
  in a lot of points)
- Discontinuity problems

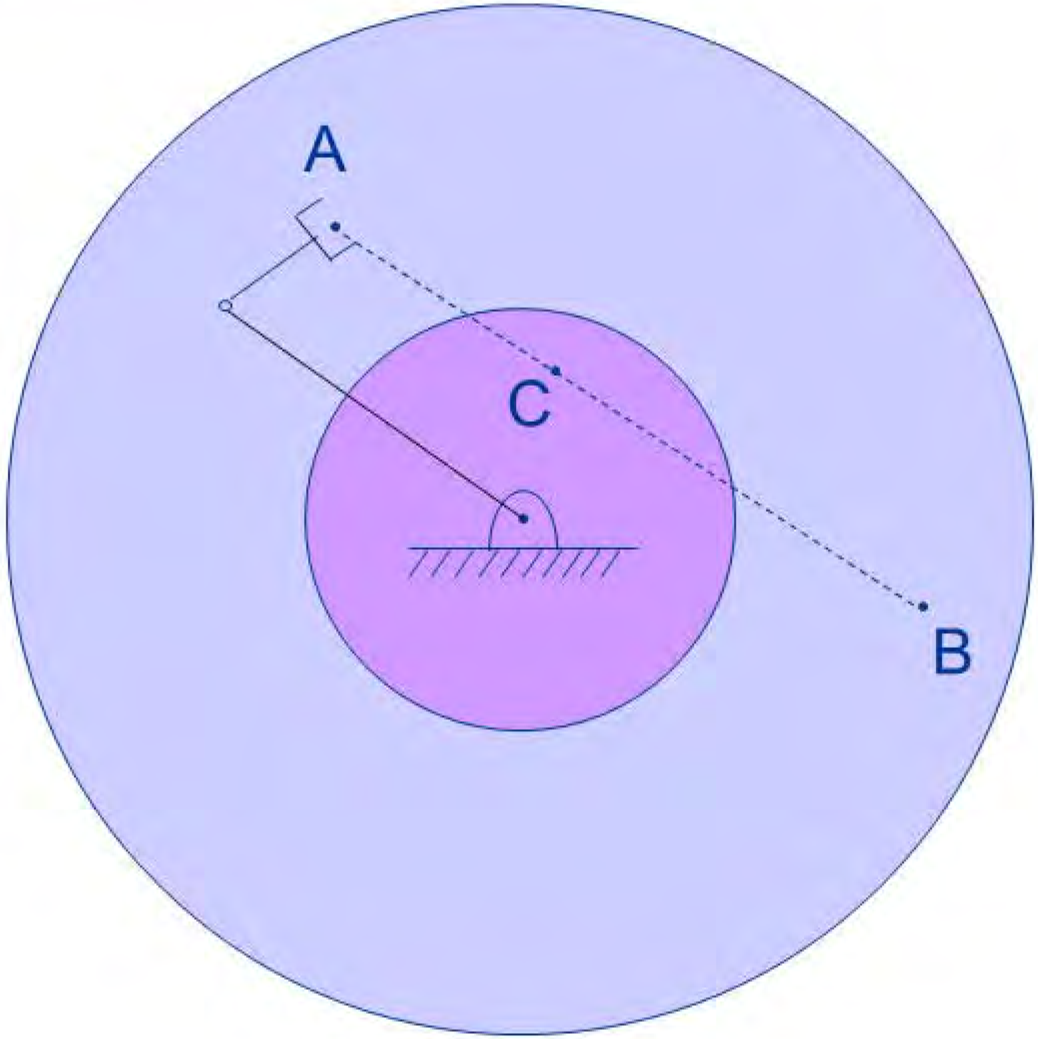
Cartesian planning difficulties :

Initial and Goal Points are

________________.

Intermediate points (C) are

________________.

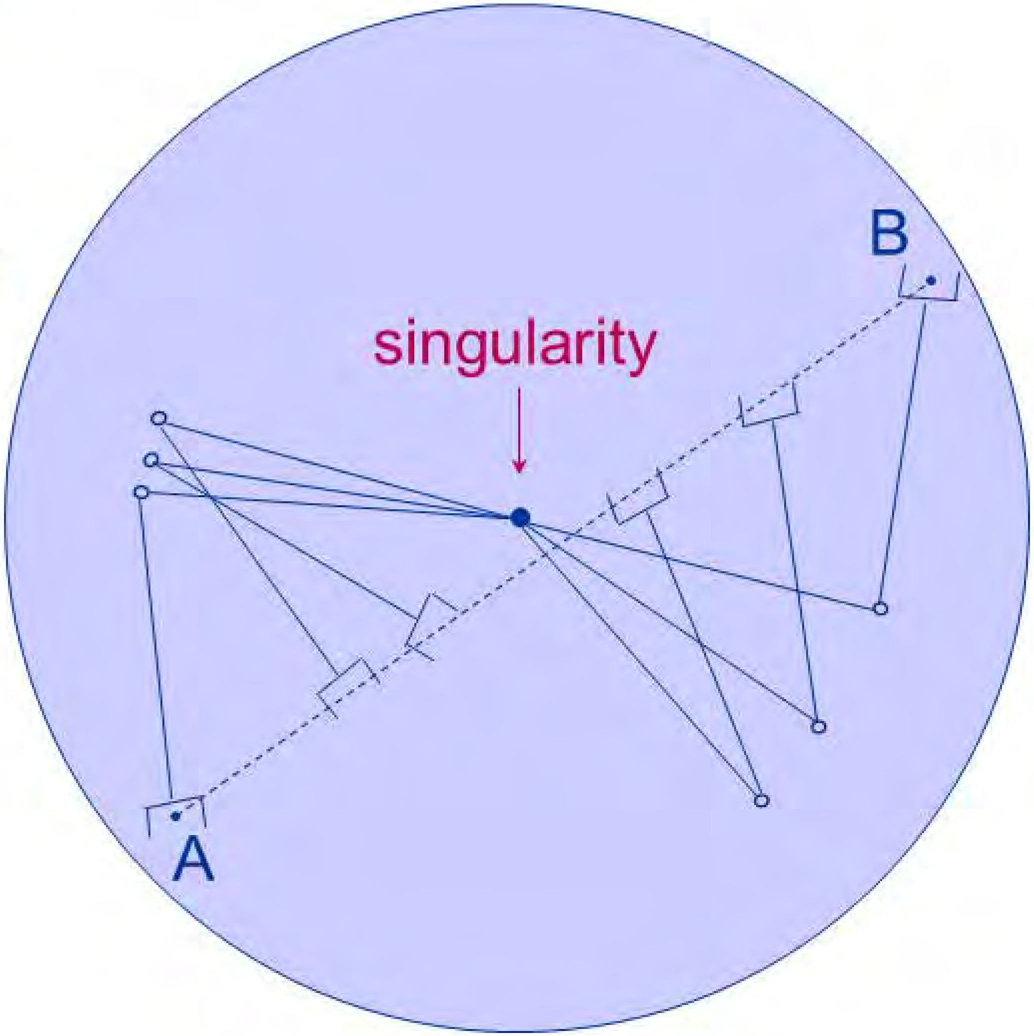


Cartesian planning difficulties :

Approaching

________________,

some joint velocities go to ∞ causing deviation from the path

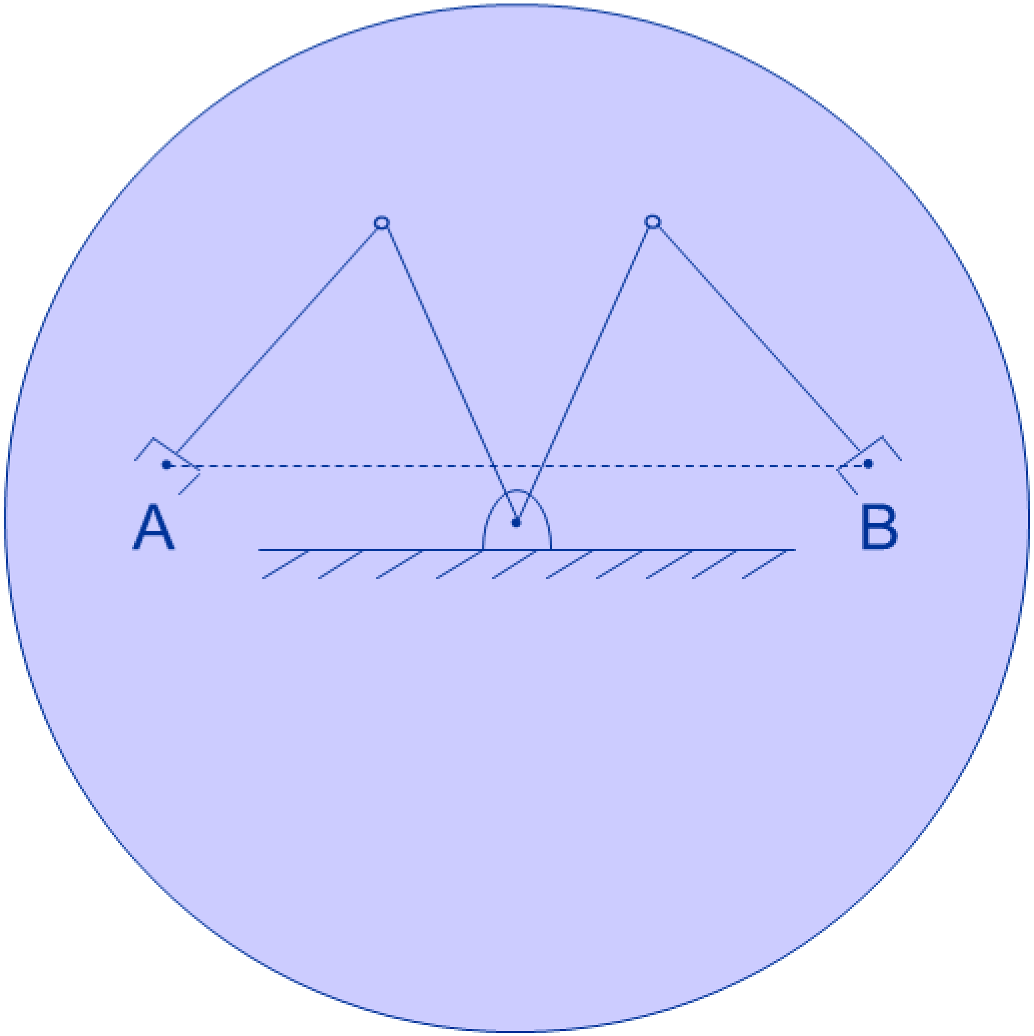


Cartesian planning difficulties :

Start point (A) and goal point (B) are reachable in

____

joint space solutions (The middle points are reachable from below.)



Actual planning in any space:

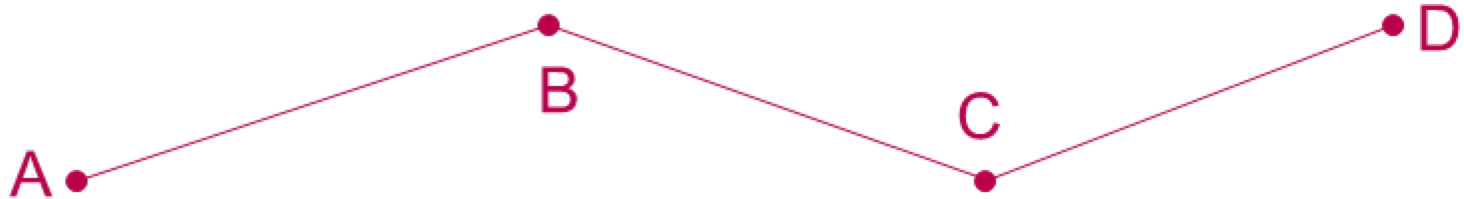
Assume one generic variable u

(can be x, y, z, orientation - α, β, γ)

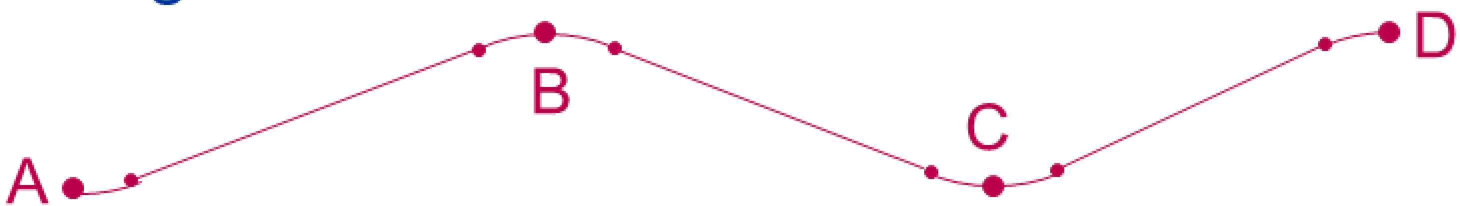
joint variables direction cosines

Candidate curves :

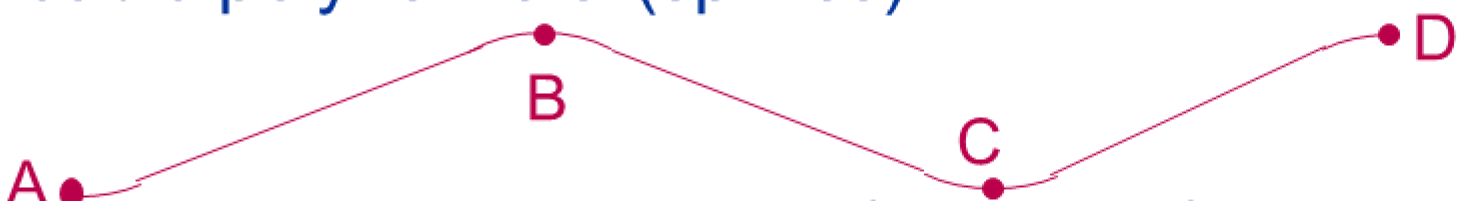
straight line (discontinuous velocity at path points)



straight line with blends

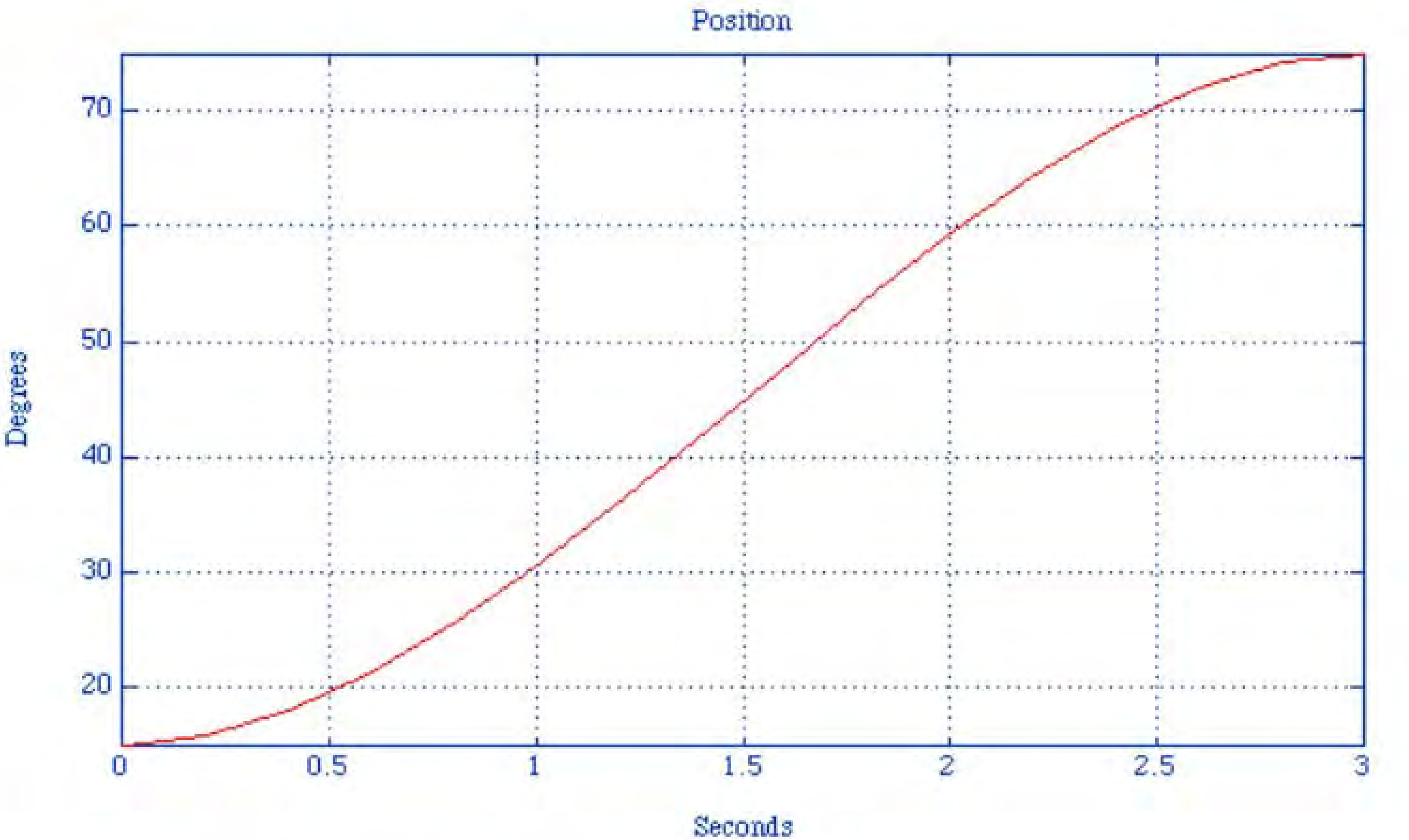


cubic polynomials (splines)



higher order polynomials (quintic,…) or other curves

Single Cubic Polynomial

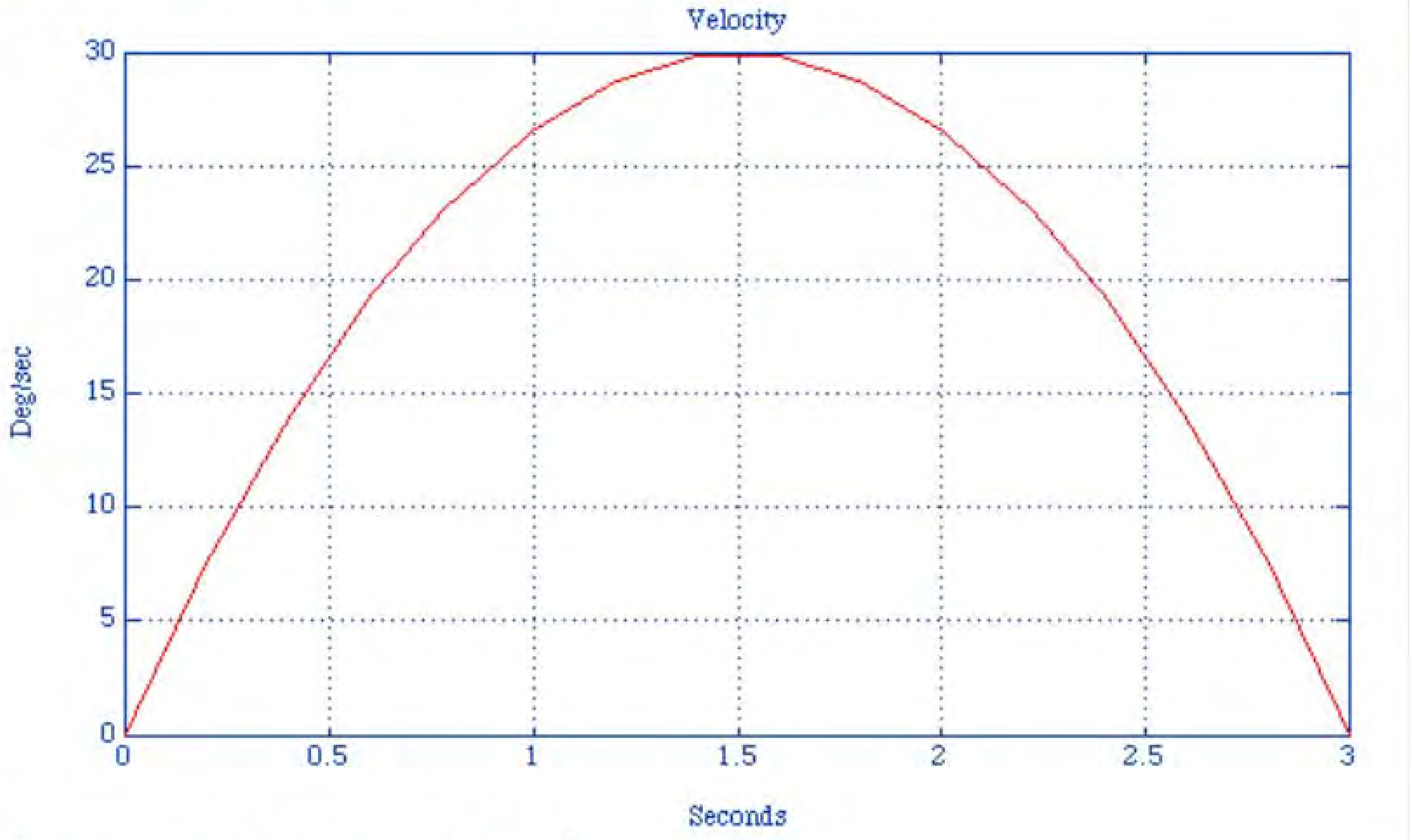


$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Initial Conditions:

$$\theta(0) = \_\_ \ ; \quad \theta(t_f) = \_\_$$

Single Cubic Polynomial



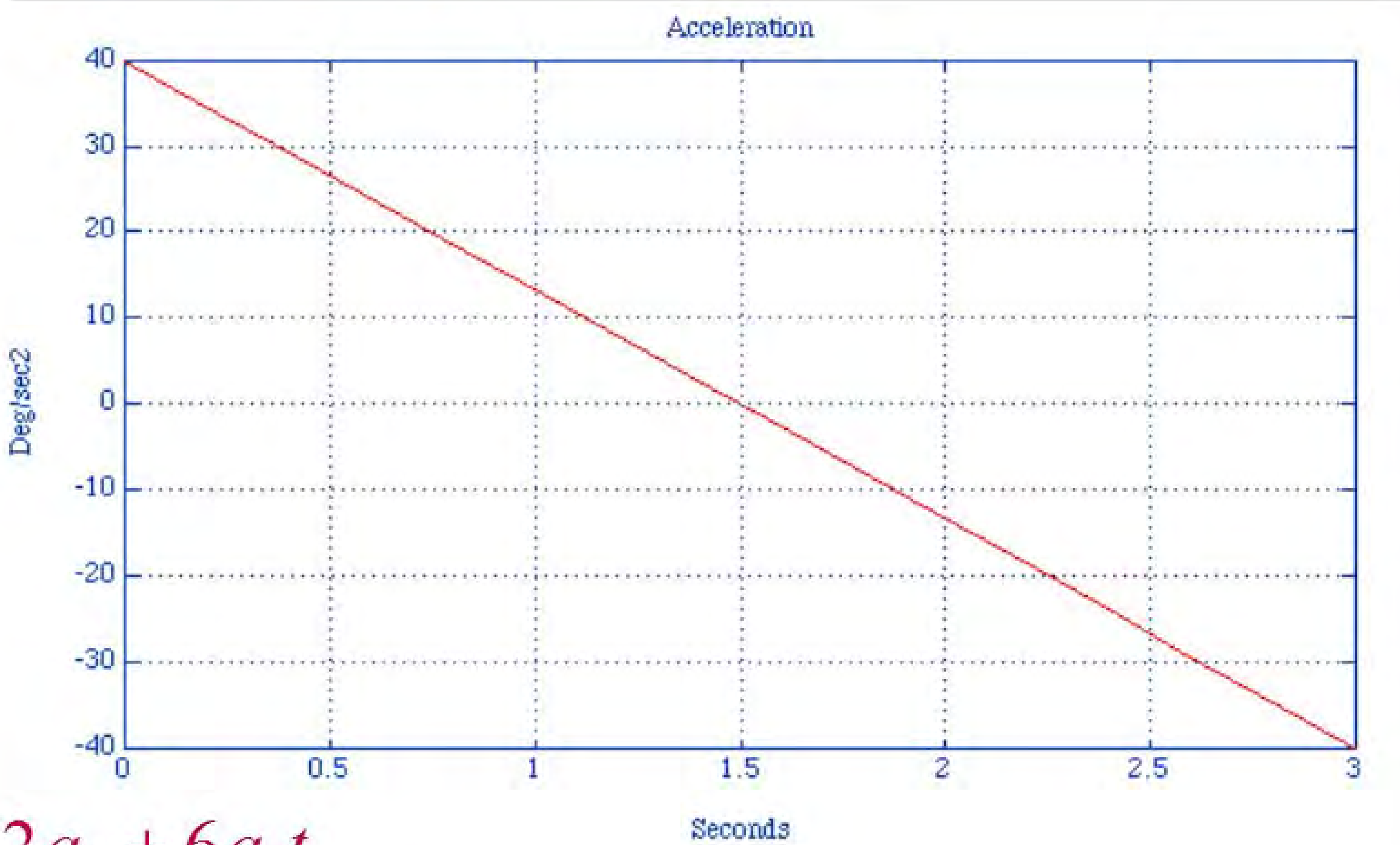
$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

Initial Conditions:

$$\dot{\theta}(0) = \_\_; \quad \dot{\theta}(t_f) = \_\_$$

Starts and ends at rest

Single Cubic Polynomial



$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$

$$\dddot{\theta}(t) = 6a_3 \text{ (constant)}$$

Solution : $\theta(t) = \theta_0 + \frac{3}{t_f^2}(\theta_f - \theta_0)t^2 + \left(-\frac{2}{t_f^3}\right)(\theta_f - \theta_0)t^3$

Cubic Polynomials with via points

- If we come to rest at each point
  use formula from previous slide
- For continuous motion (no stops)
  need velocities at intermediate points:

$\dot{\theta}(0) = \_\_\_$

$\dot{\theta}(t_f) = \_\_\_$

Initial Conditions

Solution : $a_0 = \theta_0$

$$a_1 = \dot{\theta}_0$$

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f$$

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0) + \frac{1}{t_f^2}(\dot{\theta}_f + \dot{\theta}_0)$$

How to find $\dot{\theta}_0, \dot{\theta}_f, \ldots$ (velocities at via points)

Three examples:

- if we know Cartesian linear and angular velocities

$$\rightarrow \text{ use } J^{-1}: \boxed{\dot{\theta} = J^{-1}\begin{pmatrix} \mathbf{v} \\ \omega \end{pmatrix}}$$

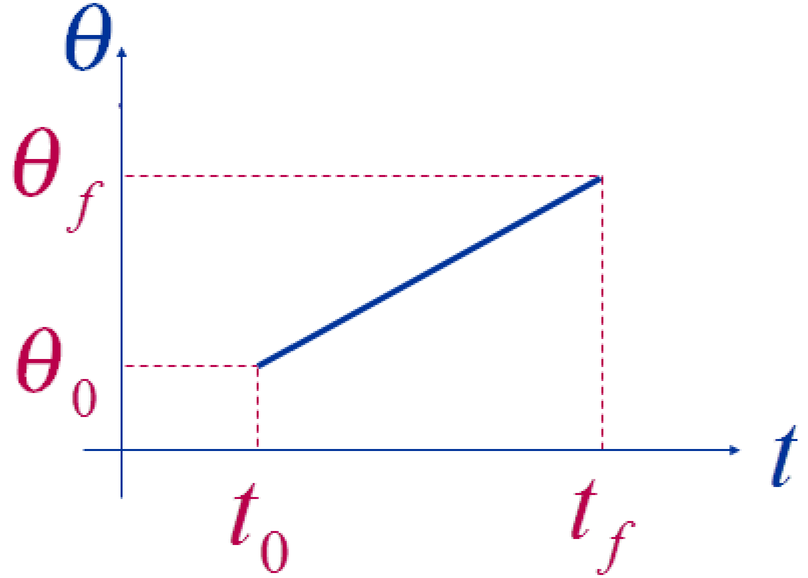
- the system chooses reasonable velocities using heuristics (average of 2 sides etc.)
- the system chooses them for continuous

velocity $\dot{\theta}_1(t_f) =$ ______ and

acceleration $\ddot{\theta}_1(t_f) =$ ______

Linear interpolation:

Straight line



$$\theta(t) = a_0 + a_1 t$$

2 conditions : $\theta(t_0) = \theta_0$

$$\theta(t_f) = \theta_f$$

Discontinuous velocity - can not be controlled

Linear interpolation:

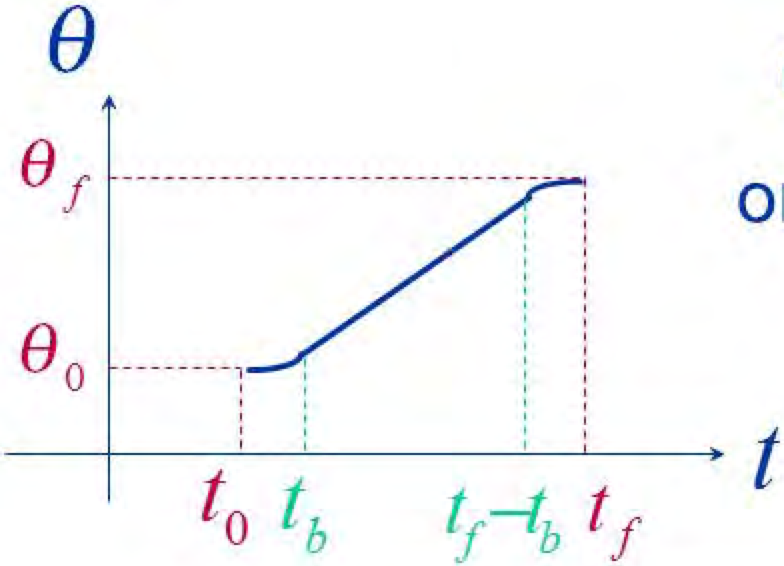
Parabolic blend

$$\theta(t) = \frac{1}{2}at^2$$ at blend regions

Linear velocity $\dot{\theta}(t) = at$

Constant acceleration $\ddot{\theta}(t) = a$

or $$\boxed{\theta(t) = \frac{1}{2}\ddot{\theta}t^2}$$ at blend regions

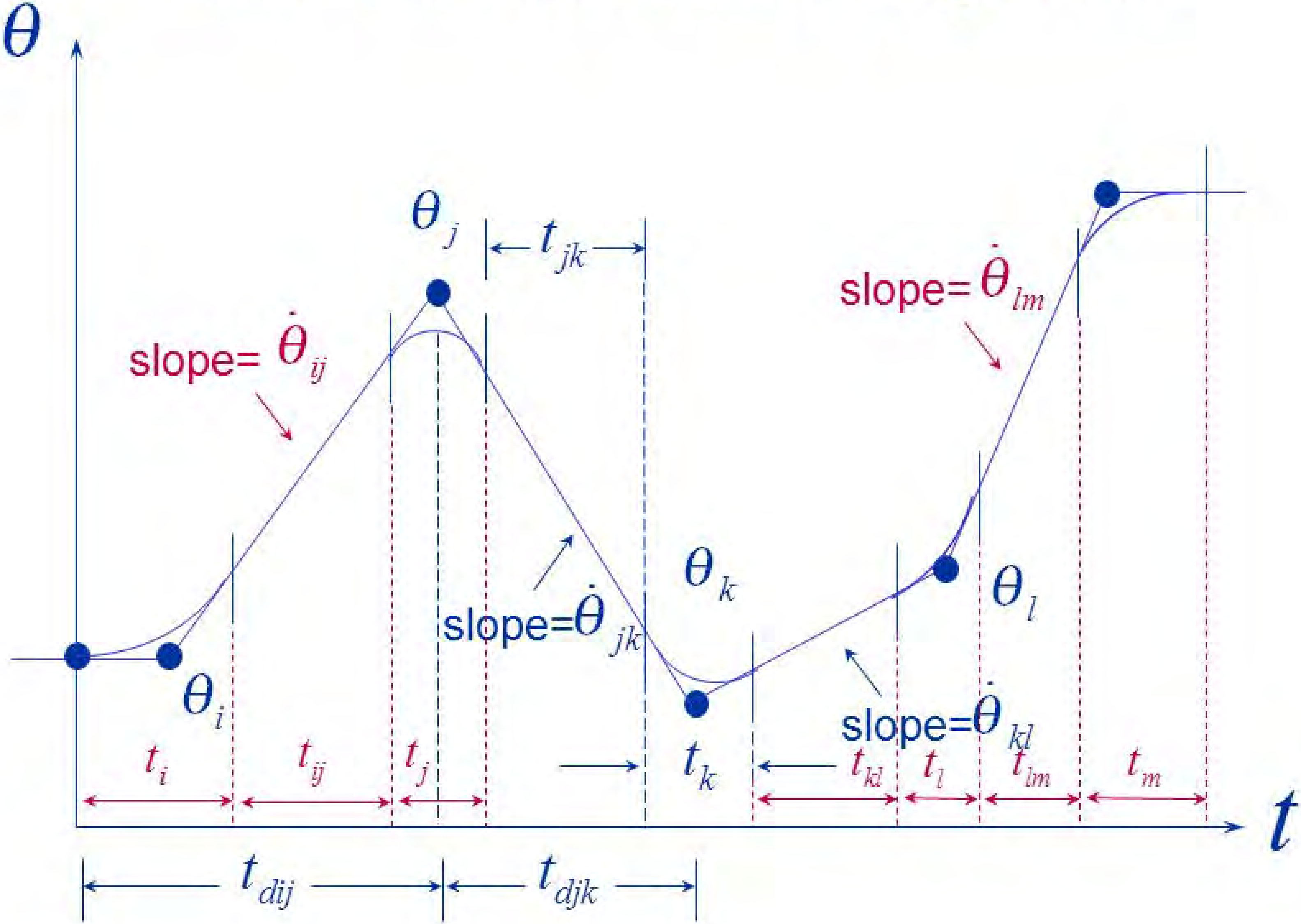


From continuous velocity:

$$t_b = \frac{t}{2} - \frac{\sqrt{\ddot{\theta}^2 t^2 - 4\ddot{\theta}(\theta_f - \theta_0)}}{2\ddot{\theta}}$$

where $t = t_f - t_0$ desired duration of motion

Linear Interpolation with blends for several segments



Given:

- positions u_i, u_j, u_k, u_l, u_m
- desired time durations $t_{dij}, t_{djk}, t_{dkl}, t_{dlm}$
- the magnitudes of the accelerations: $|\ddot{u}_i|, |\ddot{u}_j|, |\ddot{u}_k|, |\ddot{u}_l|$

Compute:

- blends times t_i, t_j, t_k, t_l, t_m
- straight segment times $t_{ij}, t_{jk}, t_{kl}, t_{lm}$
- slopes (velocities) $\dot{u}_{ij}, \dot{u}_{jk}, \dot{u}_{kl}, \dot{u}_{lm}$
- signed accelerations

Formulas (6.30-6.41)

System usually calculates or uses default values for accelerations.
The system can also calculate desired time durations based on default velocities.

Inside segments

$$\dot{u}_{jk} = \frac{u_k - u_j}{t_{djk}}$$

$$\ddot{u}_k = sign(\dot{u}_{kl} - \dot{u}_{jk})\left|\ddot{u}_k\right|$$

$$t_k = \frac{\dot{u}_{kl} - \dot{u}_{jk}}{\ddot{u}_k}$$

$$t_{jk} = t_{djk} - \frac{1}{2}t_j - \frac{1}{2}t_k$$

First segment

$$\ddot{u}_1 = sign(u_2 - u_1)\left|\ddot{u}_1\right|$$

$$t_1 = t_{d12} - \sqrt{t_{d12}^2 - \frac{2(u_2 - u_1)}{\ddot{u}_1}}$$

$$\dot{u}_{12} = \frac{u_2 - u_1}{t_{d12} - \frac{1}{2}t_1}$$

$$t_{12} = t_{d12} - t_1 - \frac{1}{2}t_2$$

Last segment

$$\ddot{u}_n = sign(u_{n-1} - u_n)\left|\ddot{u}_n\right|$$

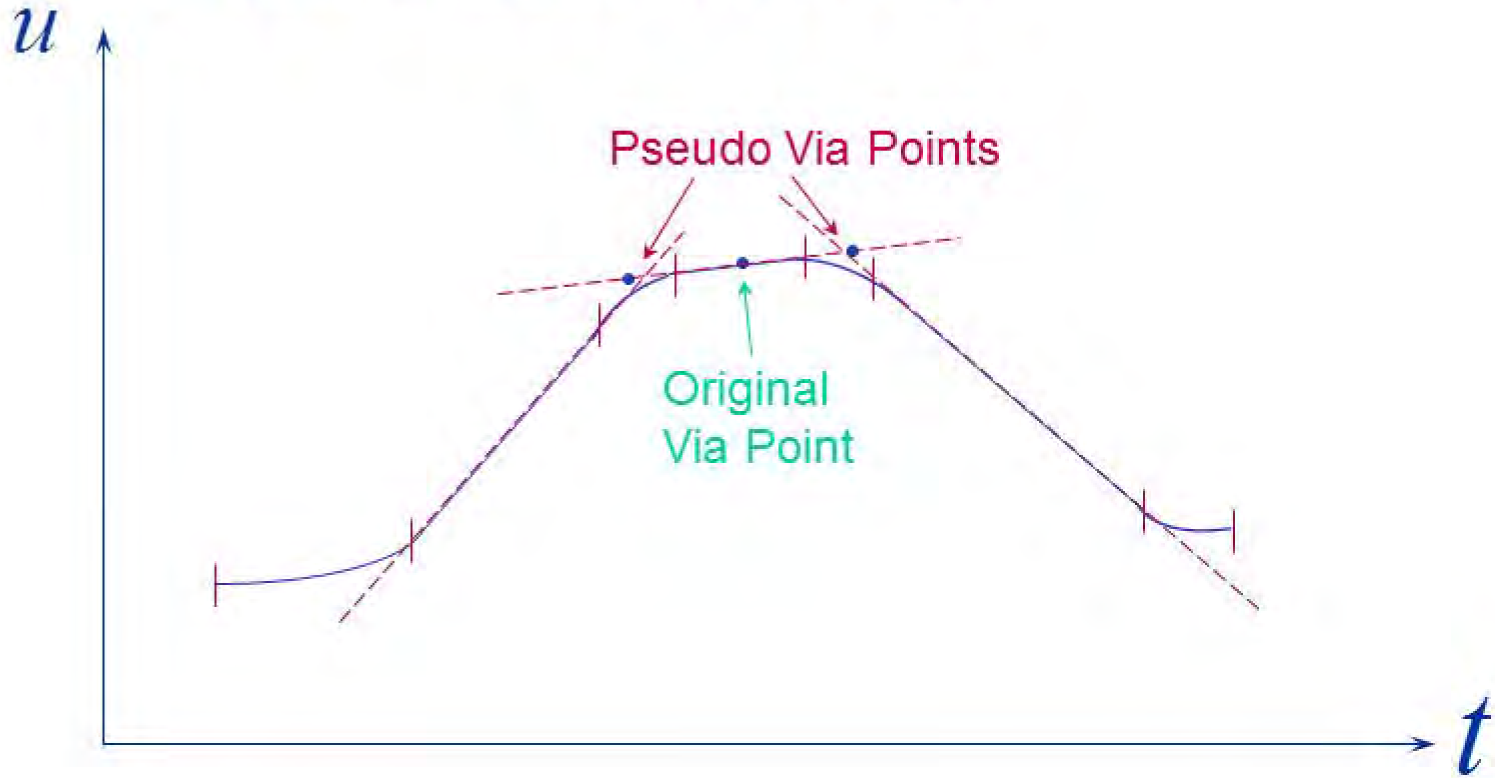
$$t_n = t_{d(n-1)n} - \sqrt{t_{d(n-1)n}^2 - \frac{2(u_n - u_{n-1})}{\ddot{u}_n}}$$

$$\dot{u}_{(n-1)n} = \frac{u_n - u_{n-1}}{t_{d(n-1)n} - \frac{1}{2}t_n}$$

$$t_{(n-1)n} = t_{d(n-1)n} - t_n - \frac{1}{2}t_{n-1}$$

To go through the actual via points:

- Introduce "Pseudo Via Points"



- Use sufficiently high acceleration
- If we want to stop there, simply repeat the via point

Higher Order Polynomials

- For example if given:

6 conditions
$$\begin{cases} \text{position} & (\text{initial } u_0,\ \text{final } u_f) \\ \text{velocity} & (\dot{u}_0, \dot{u}_f) \\ \text{acceleration} & (\ddot{u}_0, \ddot{u}_f) \end{cases}$$

Use quintic: $u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$

and find a_i (i=0 to 5)

Use different functions (exponential, trigonometric,…)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Run Time Path Generation

- trajectory in terms of $\Theta, \dot{\Theta}, \ddot{\Theta}$ fed to the control system
- Path generator computes at path update rate
- In joint space directly:
  - cubic splines -- change set of coefficients at the end of each segment
  - linear with parabolic blends -- check on each update if you are in linear or blend portion and use appropriate formulas for **u**
- In Cartesian space:
  - calculate Cartesian position and orientation at each update point using same formulas
  - convert into joint space using inverse Jacobian and derivatives

  or

  find equivalent frame representation and use inverse kinematics function to find $\Theta, \dot{\Theta}, \ddot{\Theta}$

Trajectory Planning with Obstacles

- Path planning for the whole manipulator
  - Local vs. Global Motion Planning
    - Gross motion planning for relatively uncluttered environments
    - Fine motion planning for the end-effector frame
  - Configuration space (C-space) approach
- Planning for a point robot
  - graph representation of the free space, quadtree
  - Artificial Potential Field method
- Multiple robots, moving robots and/or obstacles