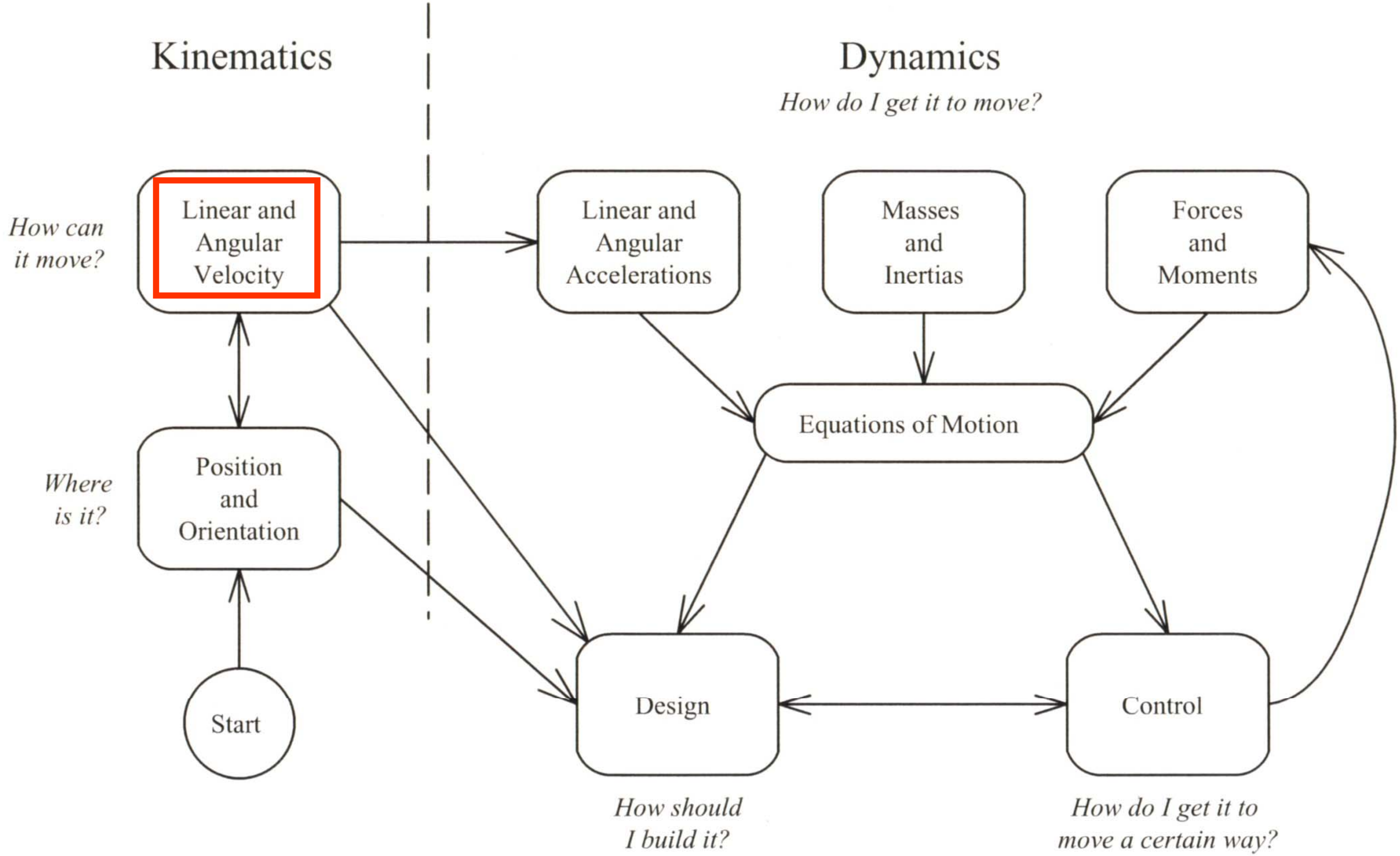
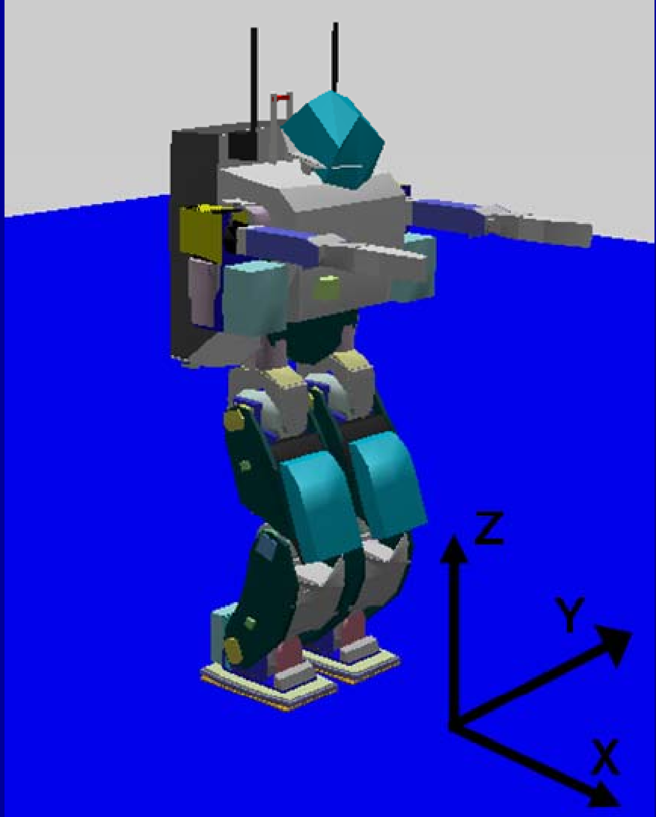
JACOBIANS: velocities and static forces

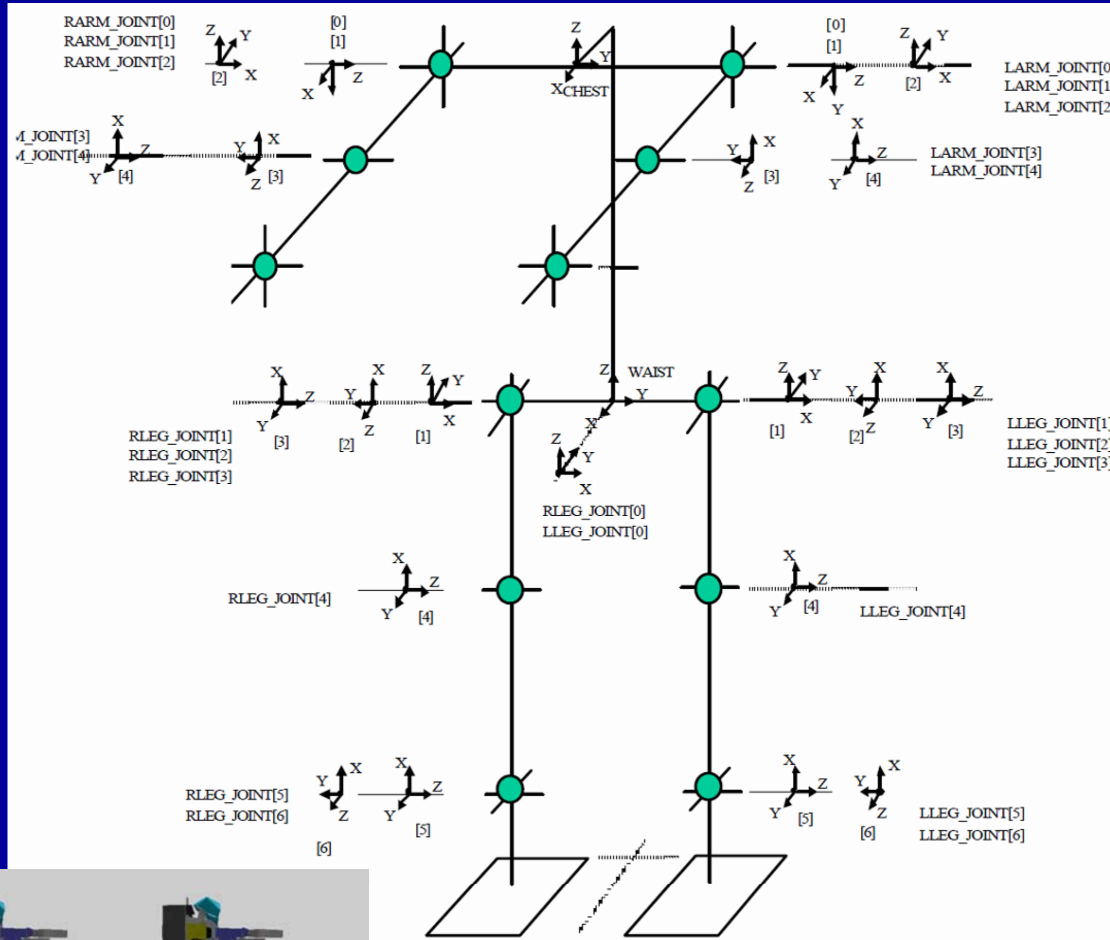
January 6, 2010


Kinematics
Dynamics
How do I get it to move?
How can it move?
Linear and Angular Velocity
Linear and Angular Accelerations
Masses and Inertias
Forces and Moments
Equations of Motion
Where is it?
Position and Orientation
Start
Design
Control
How should I build it?
How do I get it to move a certain way?

Introduction

- Static positioning problems in Chapters 3 and 4
- Linear and angular velocity of a rigid body
- Static forces acting on a rigid body


Z
Y
X


RARM_JOINT[0]
RARM_JOINT[1]
RARM_JOINT[2]
CHEST
LARM_JOINT[0]
LARM_JOINT[1]
LARM_JOINT[2]
LARM_JOINT[3]
LARM_JOINT[4]
WAIST
RLEG_JOINT[1]
RLEG_JOINT[2]
RLEG_JOINT[3]
LLEG_JOINT[1]
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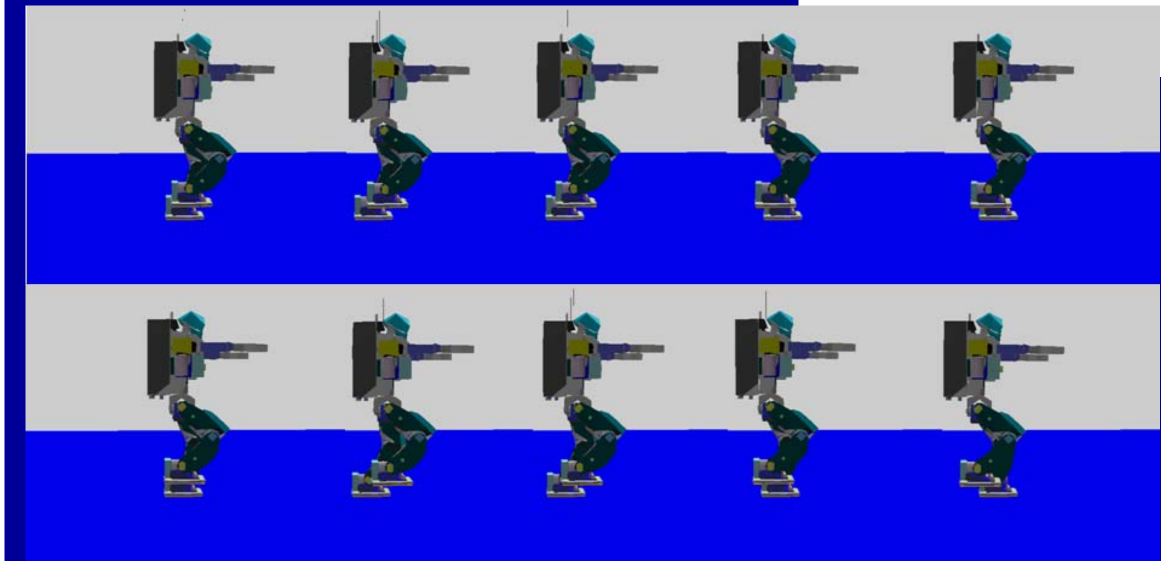
LLEG_JOINT[6]

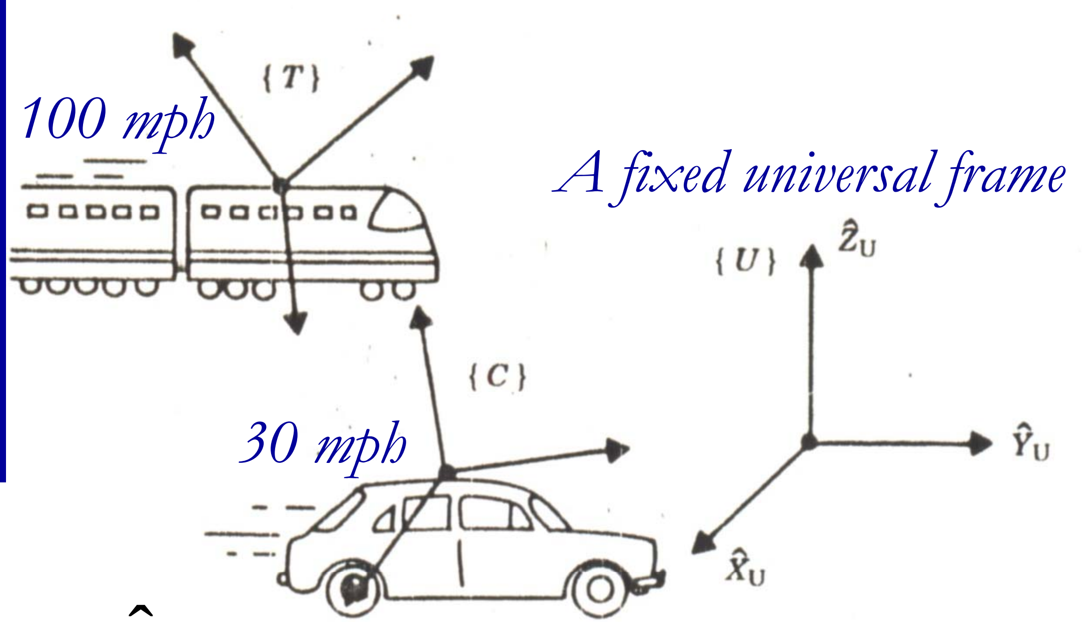


Differentiation of position vectors

$$^{B}V_{Q}=\frac{d}{dt}{}^{B}Q=\lim_{\Delta t\to 0}\frac{{}^{B}Q(t+\Delta t)-{}^{B}Q(t)}{\Delta t}.$$

$$^{A}\left({}^{B}V_{Q}\right)={}^{A}_{B}R\,{}^{B}V_{Q}.$$

Example 5.1



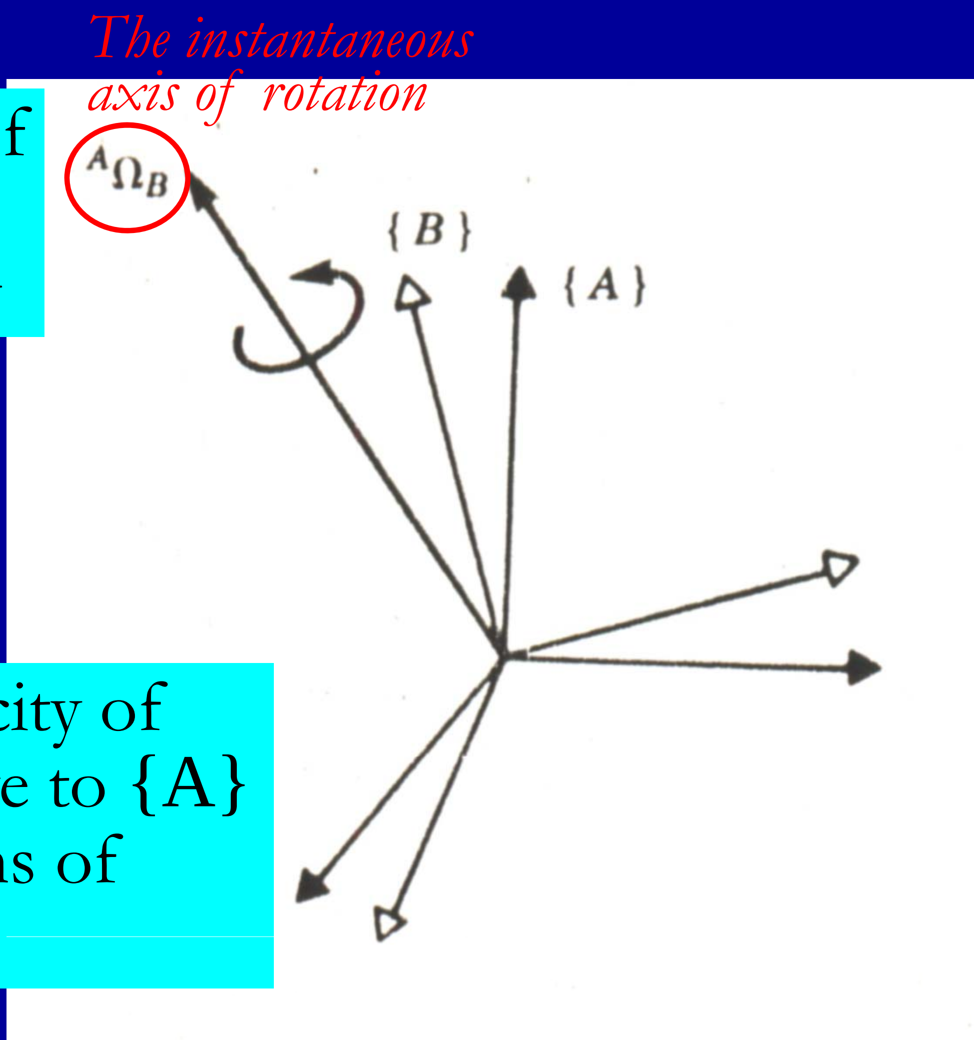
$$^{U}\frac{d}{dt}{}^{U}P_{CORG} = {}^{U}V_{CORG} = v_C = 30\hat{X}.$$

$$^{C}({}^{U}V_{TORG}) = {}^{C}v_T = {}^{C}_{U}Rv_T = {}^{C}_{U}R(100\hat{X}) = {}^{U}_{C}R^{-1}100\hat{X}.$$

$$^{C}({}^{T}V_{CORG}) = {}^{C}_{T}R\,{}^{T}V_{CORG} = {}^{C}_{U}R\,{}^{U}_{T}R\,{}^{T}V_{CORG} = -{}^{U}_{C}R^{-1}\,{}^{U}_{T}R\,70\hat{X}.$$

Angular velocity vector

The rotation of frame {B} relative to {A}



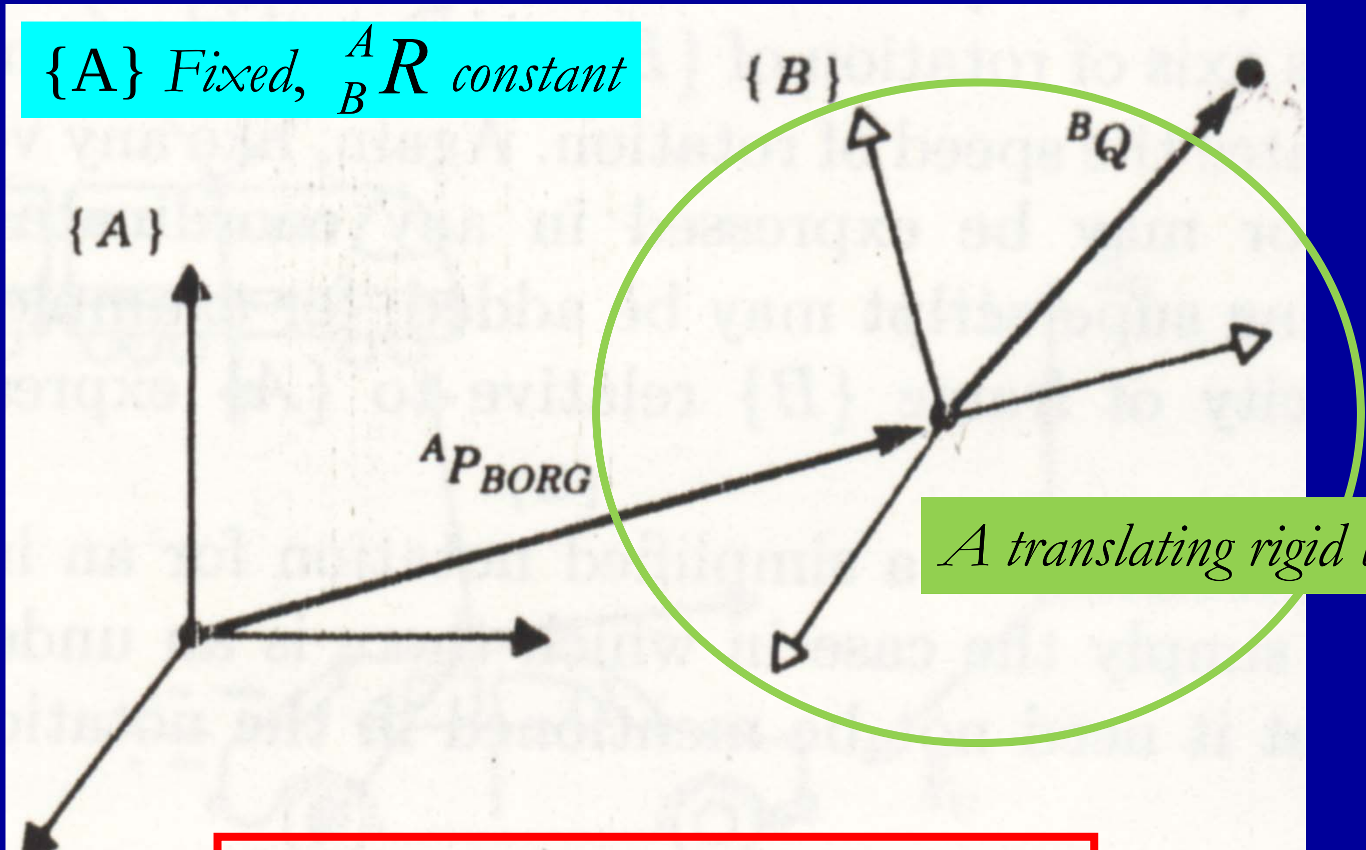
${}^{C}({}^{A}\Omega_{B})$

The angular velocity of frame {B} relative to {A} expressed in terms of frame {C}

- Whereas linear velocity describes an attribute of a point, angular velocity describes an attribute of a body.
- Since we always attach a frame to the bodies we consider, we can also think of angular velocity as describing rotational motion of a frame.

Linear velocity of a rigid body

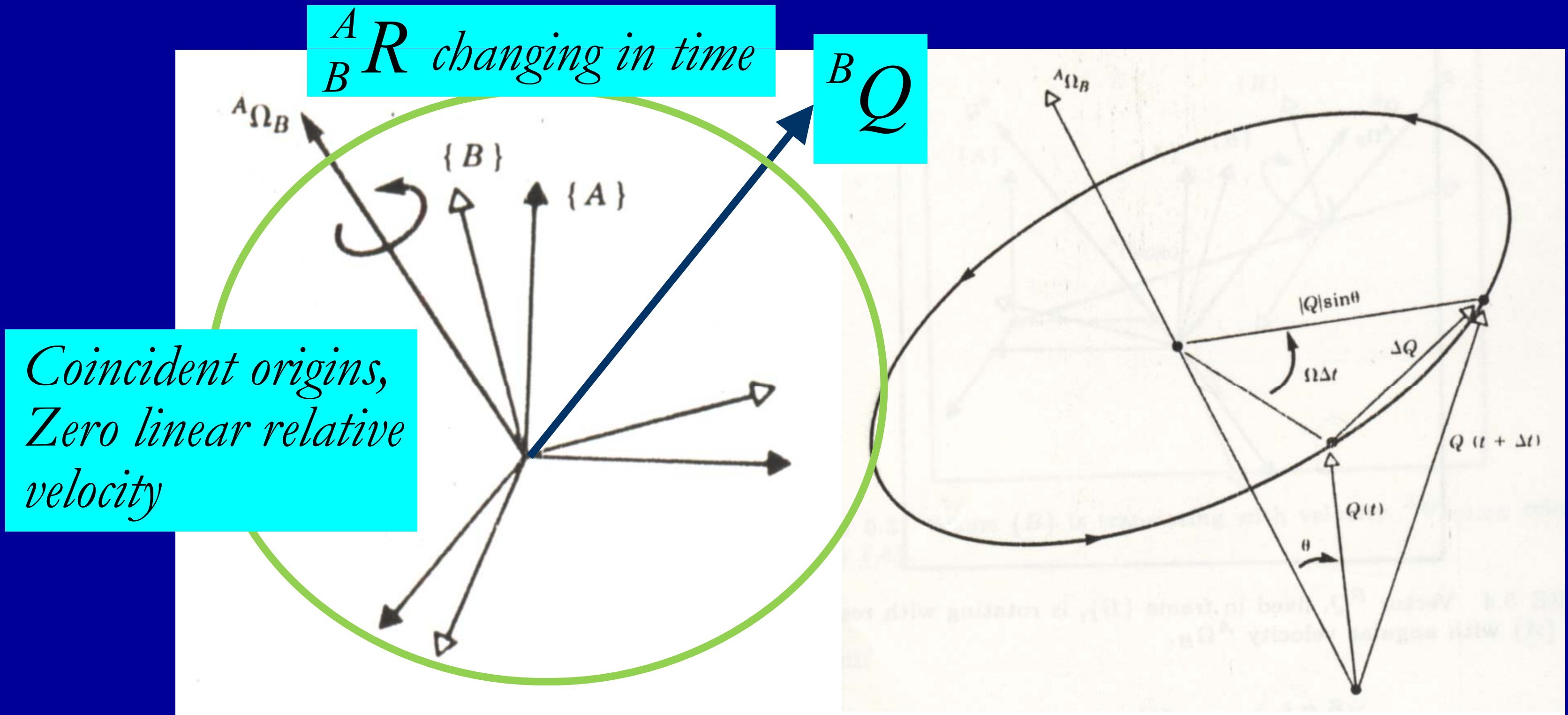
{A} *Fixed*, ${}^{A}_{B}R$ *constant*



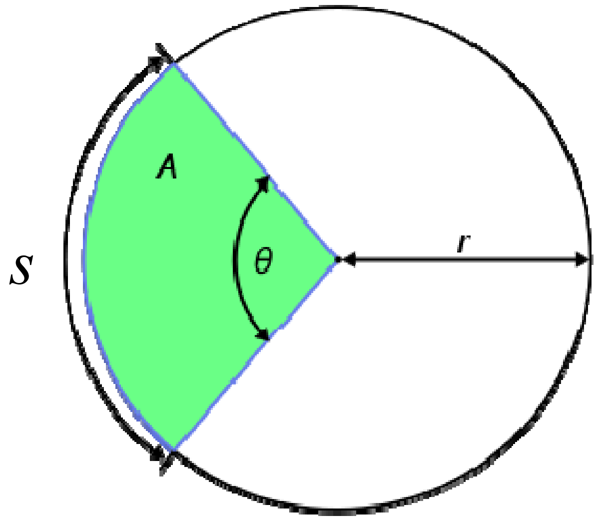
A translating rigid body

$$ {}^{A}V_{Q} = {}^{A}V_{BORG} + {}^{A}_{B}R\,{}^{B}V_{Q}. $$

Rotational velocity of a rigid body



$$ {}^{A}V_{Q} = {}^{A}_{B}R\,{}^{B}V_{Q} + {}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}Q. $$



$$\frac{\theta}{2\pi} = \frac{s}{circumference} = \frac{s}{2\pi r} \quad \Rightarrow \quad s = \theta r$$

$$\omega = \frac{d\theta}{dt} = \frac{d(s/r)}{dt} = \frac{1}{r}\frac{ds}{dt} = \frac{v}{r}$$

Simultaneous linear and rotational velocity

$$ {}^{A}V_{Q} = {}^{A}V_{BORG} + {}^{A}_{B}R\,{}^{B}V_{Q} + {}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}Q. $$

Origins are not coincident.

A Property of the derivative of an orthonormal matrix

$$RR^T = I_n$$

$$\dot{R}R^T + R\dot{R}^T = 0_n$$

$$\dot{R}R^T + (\dot{R}R^T)^T = 0_n$$

$$S = \dot{R}R^T$$

$$S + S^T = 0_n$$

S *a skew-symmetric matrix*

$$S = \dot{R}R^{-1}$$

Velocity of a point due to rotating reference frame

$$^{A}P = {}^{A}_{B}R\,{}^{B}P$$

^{B}P *a fixed vector*

$$^{A}\dot{P} = {}^{A}_{B}\dot{R}\,{}^{B}P$$

$$^{A}V_{P} = {}^{A}_{B}\dot{R}\,{}^{B}P$$

$$= {}^{A}_{B}\dot{R}\,{}^{B}_{A}R\,{}^{A}P$$

$$= {}^{A}_{B}\dot{R}\,{}^{A}_{B}R^{-1}\,{}^{A}P$$

$$= {}^{A}_{B}S\,{}^{A}P$$

Angular-velocity matrix

Skew-symmetric matrices and the vector cross-product

$$S = \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix}, \quad \Omega = \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix}$$

Angular-velocity vector

Any arbitrary vector

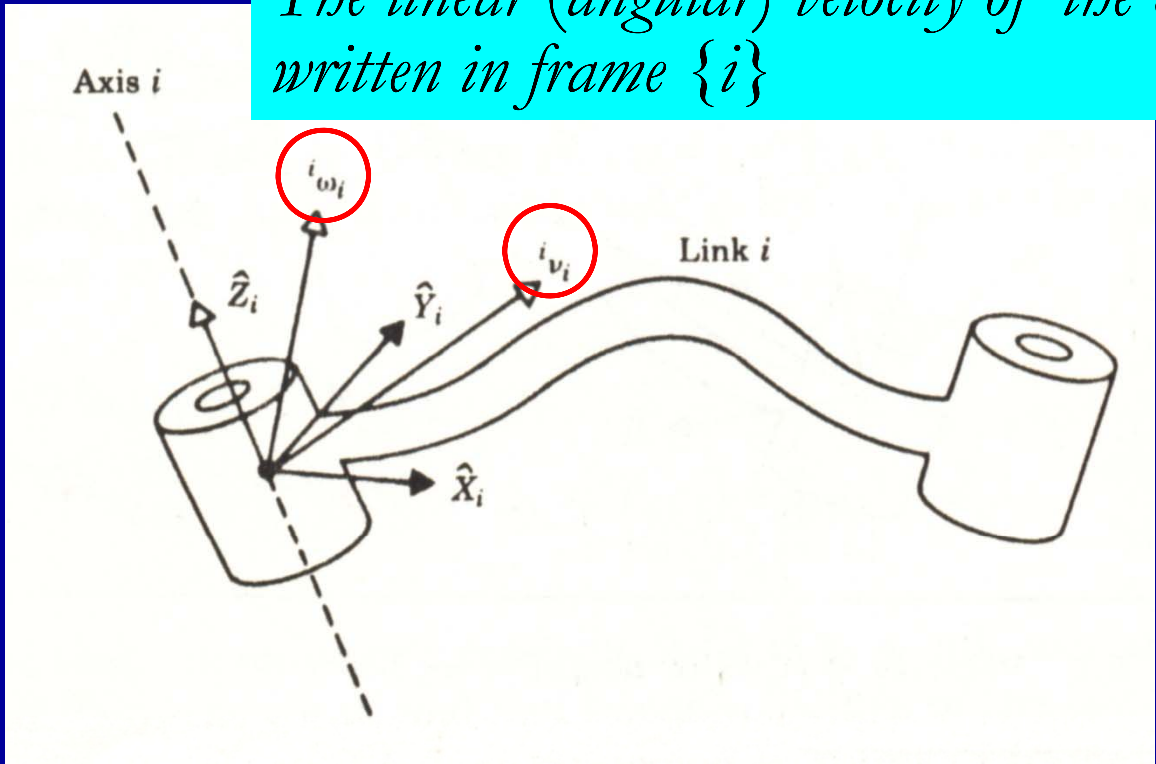
$$SP = \Omega \times P$$

The vector cross product

$${}^{A}V_{P} = {}^{A}\Omega_{B} \times {}^{A}P$$

Motion of the links of a robot

The linear (angular) velocity of the origin of link frame {i} written in frame {i}



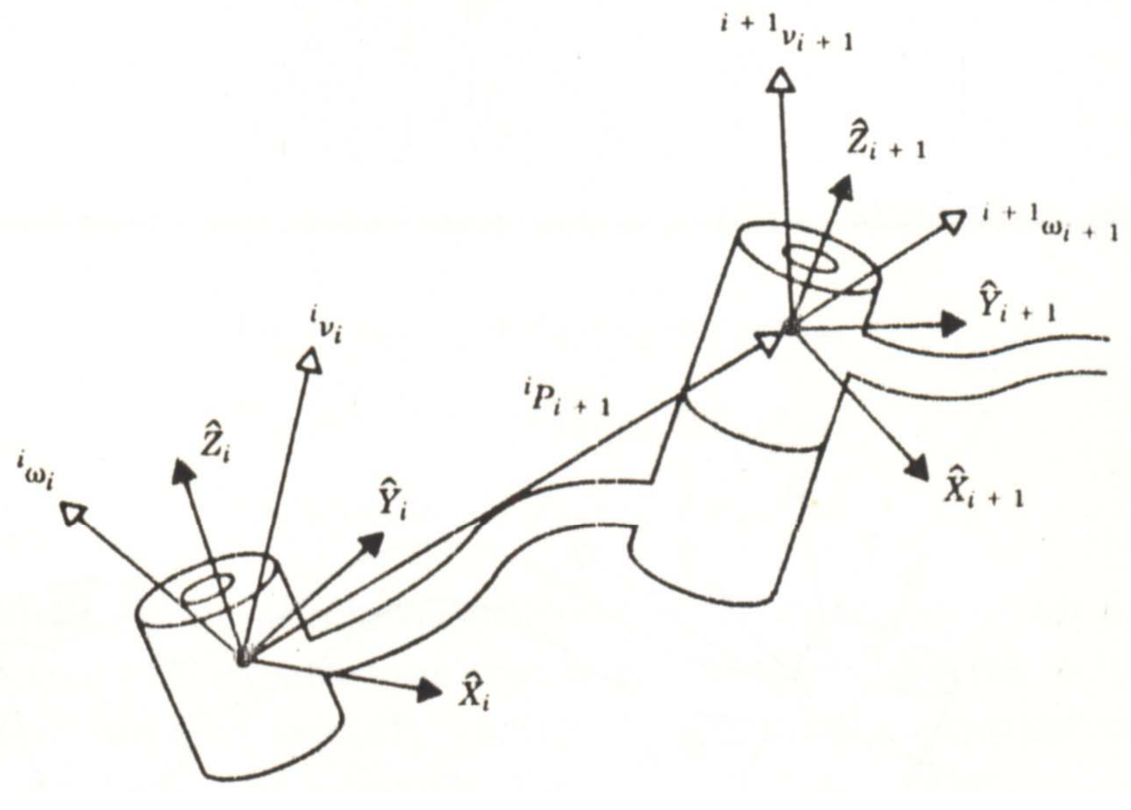
At any instant, each link of a robot in motion has some linear and angular velocity.

Velocity propagation from link to link

- We can compute the velocities of each link in order, starting from the base.
- The velocity of link $i+1$ will be that of link i, plus whatever new velocity component were added by joint $i+1$.

- Remember that linear velocity is associated with a point (*the origin of the link frame*), and angular velocity is associated with a body (*the link*).
- The angular velocity of link i+1 is the same as that of link i plus a new component caused by rotational velocity at joint i+1.

Velocity vectors of neighboring links



$$^{i}\omega_{i+1} = {}^{i}\omega_{i} + {}^{i}_{i+1}R\,\dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}.$$

$$ {}^{i+1}_{i}R\,{}^{i}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_{i} + {}^{i+1}_{i}R\,{}^{i}_{i+1}R\,\dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}. $$

$$ {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix} $$

$$ {}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_{i} + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}. $$

- The linear velocity of the origin of frame {i+1} is the same as that of the origin of frame {i} plus a new component caused by rotational velocity of link i.

$$ {}^{i}v_{i+1} = {}^{i}v_{i} + {}^{i}\omega_{i} \times {}^{i}P_{i+1}. $$

$$ {}^{i+1}_{i}R\,{}^{i}v_{i+1} = {}^{i+1}_{i}R({}^{i}v_{i} + {}^{i}\omega_{i} \times {}^{i}P_{i+1}). $$

$$ \boxed{{}^{i+1}v_{i+1} = {}^{i+1}_{i}R({}^{i}v_{i} + {}^{i}\omega_{i} \times {}^{i}P_{i+1}).} $$

For the case that joint i+1 is prismatic;

$$^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_{i},$$

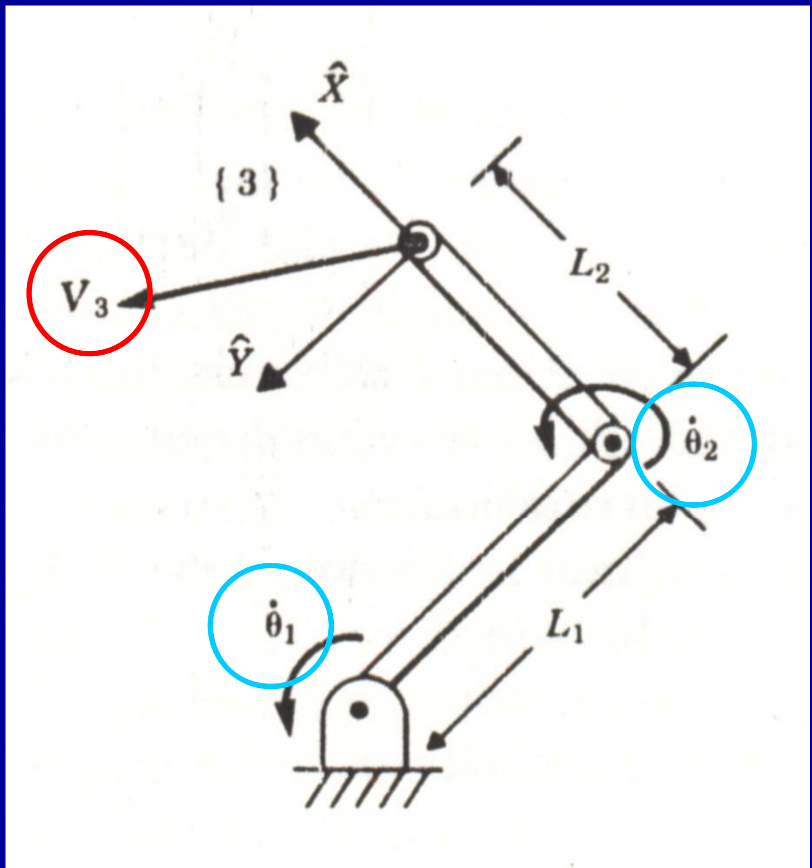
$$^{i+1}v_{i+1} = {}^{i+1}_{i}R({}^{i}v_{i} + {}^{i}\omega_{i} \times {}^{i}P_{i+1}) + \dot{d}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}.$$

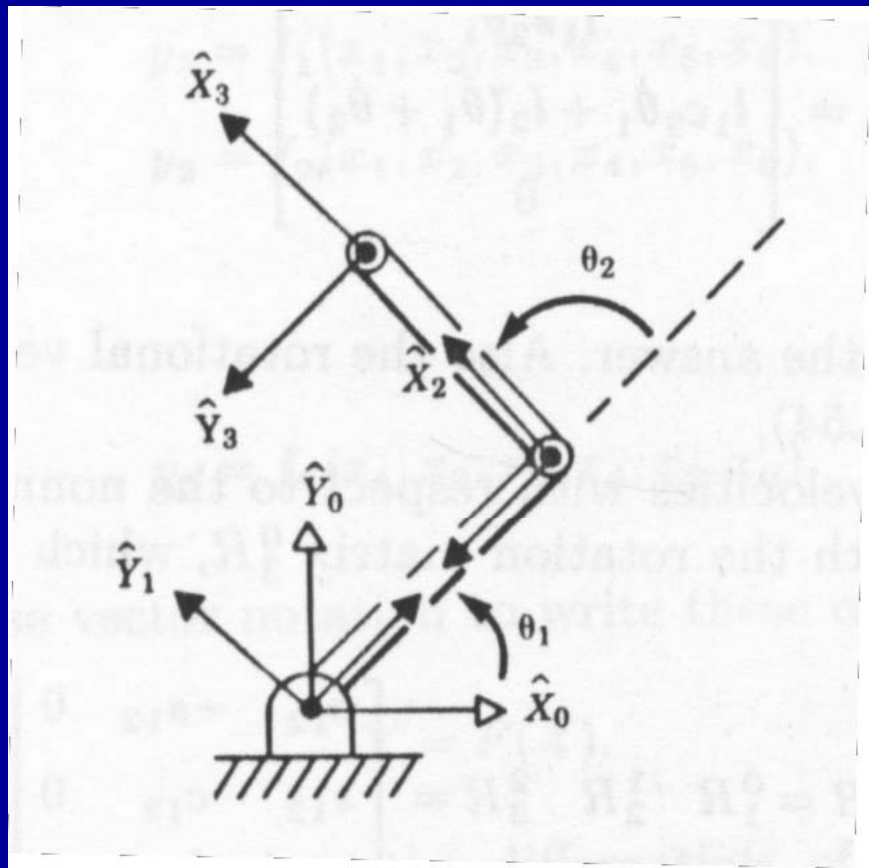
- Applying those previous equations successively *from link to link*, we can compute the rotational and linear velocities of the last link.

$$ {}^{N}\omega_{N}, {}^{N}v_{N} $$

$$ {}^{0}\omega_{N} = {}^{0}_{N}R\,{}^{N}\omega_{N}, \; {}^{0}v_{N} = {}^{0}_{N}R\,{}^{N}v_{N} $$

Example 5.3





A 2-link manipulator with rotational joints

$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$
$$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

$$\dot{x} = -L_1 \sin\theta_1 \cdot \dot{\theta}_1 - L_2 \sin(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$$
$$\dot{y} = L_1 \cos\theta_1 \cdot \dot{\theta}_1 + L_2 \cos(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$$

$$\dot{x} = \begin{bmatrix} -L_1 \sin\theta_1 - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \\ L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \dot{\theta}$$
$$= J\dot{\theta}$$

$$ {}^{1}\omega_1 = {}^{1}_{0}R\,{}^{0}\omega_0 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}, \quad {}^{1}v_1 = {}^{1}_{0}R\left({}^{0}v_0 + {}^{0}\omega_0 \times {}^{0}P_1\right) $$

$$ {}^{2}\omega_2 = {}^{2}_{1}R\,{}^{1}\omega_1 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}, \quad {}^{2}v_2 = {}^{2}_{1}R\left({}^{1}v_1 + {}^{1}\omega_1 \times {}^{1}P_2\right) $$

$$ {}^{3}\omega_3 = {}^{3}_{2}R\,{}^{2}\omega_2 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix}, \quad {}^{3}v_3 = {}^{3}_{2}R\left({}^{2}v_2 + {}^{2}\omega_2 \times {}^{2}P_3\right) $$

$$
{}^{0}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad
{}^{1}_{2}T = \begin{bmatrix} c_2 & -s_2 & 0 & l_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad
{}^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

$$
{}^{1}\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}, \quad {}^{1}v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},
$$

$$
{}^{2}\omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}, \quad
{}^{2}v_2 = \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_1\dot{\theta}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} l_1 s_2 \dot{\theta}_1 \\ l_1 c_2 \dot{\theta}_1 \\ 0 \end{bmatrix},
$$

$$
{}^{3}J(\Theta)
$$

$$
{}^{3}\omega_3 = {}^{2}\omega_2, \quad
{}^{3}v_3 = \begin{bmatrix} l_1 s_2 \dot{\theta}_1 \\ l_1 c_2 \dot{\theta}_1 + l_2(\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix} = \begin{bmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}.
$$

$$ {}^0_3R = {}^0_1R \quad {}^1_2R \quad {}^2_3R = \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}. $$

$$ {}^0v_3 = {}^0_3R\,{}^3v_3 = \begin{bmatrix} -l_1 s_1 \dot{\theta}_1 - l_2 s_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ l_1 c_1 \dot{\theta}_1 + l_2 c_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix} $$

$$ = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}. $$

$$ {}^0J(\Theta) $$

Jacobians

$$y_1 = f_1(x_1, x_2, x_3, x_4, x_5, x_6),$$

$$y_2 = f_2(x_1, x_2, x_3, x_4, x_5, x_6),$$

$$\vdots$$

$$y_6 = f_6(x_1, x_2, x_3, x_4, x_5, x_6),$$

Nonlinear

$$Y = F(X).$$

$$\delta y_1 = \frac{\partial f_1}{\partial x_1}\delta x_1 + \frac{\partial f_1}{\partial x_2}\delta x_2 + \cdots + \frac{\partial f_1}{\partial x_6}\delta x_6,$$

$$\delta y_2 = \frac{\partial f_2}{\partial x_1}\delta x_1 + \frac{\partial f_2}{\partial x_2}\delta x_2 + \cdots + \frac{\partial f_2}{\partial x_6}\delta x_6,$$

Chain rule

$$\vdots$$

$$\delta y_6 = \frac{\partial f_6}{\partial x_1}\delta x_1 + \frac{\partial f_6}{\partial x_2}\delta x_2 + \cdots + \frac{\partial f_6}{\partial x_6}\delta x_6,$$

$$\delta Y = \frac{\partial F}{\partial X}\delta X.$$

J(X)

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_6} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_6}{\partial x_1} & \cdots & \frac{\partial f_6}{\partial x_6} \end{bmatrix}$$

All first-order partial derivatives of a vector-valued function

$f : R^n \rightarrow R^m$, *If n=m,* J *is a square matrix.*

J *need not be a square matrix!*

$$\begin{cases} y_1 = x_1 \\ y_2 = 5x_3 \\ y_3 = 4x_2^2 - 2x_3 \\ y_4 = x_3 \sin x_1 \end{cases} \qquad f : R^3 \to R^4$$

$$J = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \frac{\partial y_1}{\partial x_3} \\ \frac{\partial y_2}{\partial x_1} & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \frac{\partial y_4}{\partial x_1} & \cdots & \frac{\partial y_4}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 5 \\ 0 & 8x_2 & -2 \\ x_3 \cos x_1 & 0 & \sin x_1 \end{bmatrix}$$

Linear transformations

$$\dot{Y} = J(X)\dot{X}.$$

Maps velocities in X to those in Y

In the field of robotics, we generally speak of Jacobians that relate joint velocities to Cartesian velocities of the tip of the arm.

$$^{0}V = \begin{bmatrix} ^{0}v \\ ^{0}\omega \end{bmatrix} = {}^{0}J(\Theta)\dot{\Theta}.$$

$$^{0}\dot{V} = {}^{0}\dot{J}(\Theta)\dot{\Theta} + {}^{0}J(\Theta)\ddot{\Theta}.$$

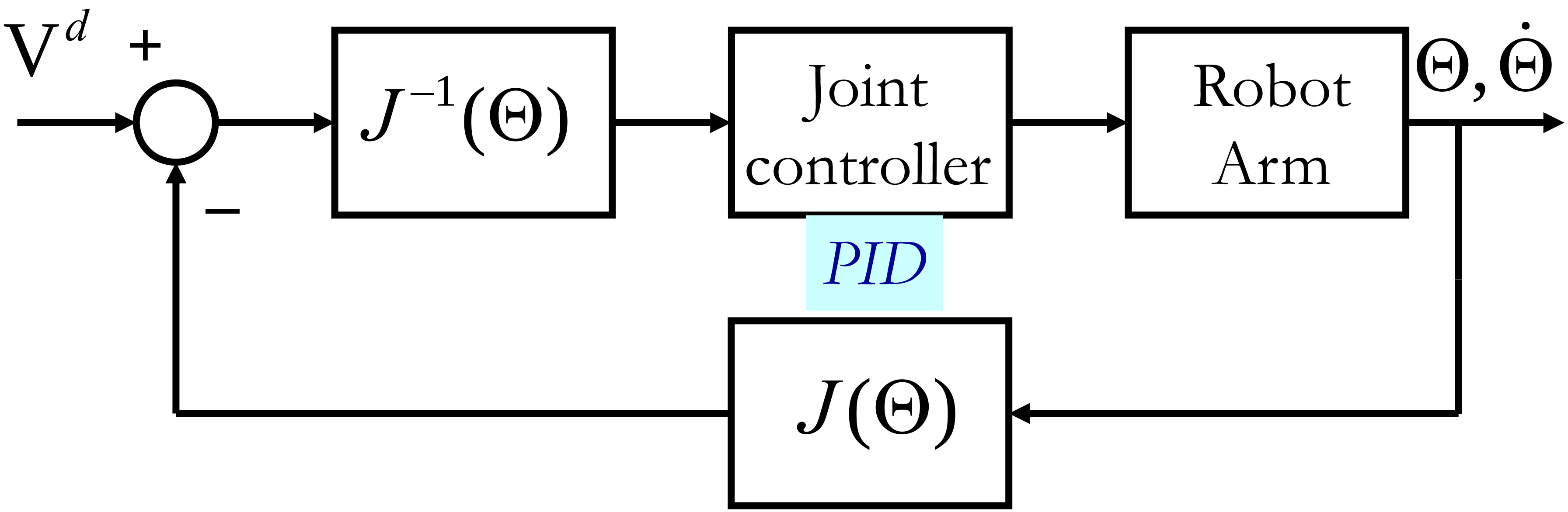
Resolved Motion Rate Control (RMRC)

- Proposed by D. E. Whitney (1969)
- The motions of the various joint motors are combined and run simultaneously at different time-varying rates in order to achieve steady end-effector motion along any Cartesian coordinate axis.

$$\dot{\Theta} = J^{-1}\mathrm{V}$$

The desired rate along the world coordinates

The RMRC block diagram



Joint rates approach infinity as the singularity is approached.

Resolved Motion Acceleration Control (RMAC)

- Proposed by J. Y. S. Luh (1980)
- Extends the concept of RMRC to include acceleration control.
- Presents an alternative position control which deals directly with the position and orientation of the end-effector of a manipulator.
- Assumes that the desired accelerations of a preplanned end-effector motion are specified by the user.

$$\ddot{\Theta} = J^{-1}(\dot{V} - \dot{J}\dot{\Theta})$$

Singular Value Decomposition

$$A \in R^{m \times n}, \qquad rank(A) = k$$

$$A = U \Sigma V^T, \qquad U \in R^{m \times m}, V \in R^{n \times n}, \Sigma \equiv diag\left(\underbrace{\sigma_1, \cdots, \sigma_p}_{p=\min(m,n)} \right) \in R^{m \times n}$$

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_k > 0, \qquad \sigma_{k+1} = \cdots = \sigma_p = 0$$

$$A^+ = V \Sigma^+ U^T, \qquad \Sigma^+ \equiv diag\left(\underbrace{\frac{1}{\sigma_1}, \frac{1}{\sigma_2}, \cdots \frac{1}{\sigma_k}, 0, \cdots, 0}_{p=\min(n,m)} \right) \in R^{n \times m}$$

MATLAB Syntax

$$X = \begin{matrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{matrix}$$

$$[U, S, V] = svd(X)$$

$$U = \begin{matrix} 0.1525 & 0.8226 & -0.3945 & -0.3800 \\ 0.3499 & 0.4214 & 0.2428 & 0.8007 \\ 0.5474 & 0.0201 & 0.6979 & -0.4614 \\ 0.7448 & -0.3812 & -0.5462 & 0.0407 \end{matrix}$$

$$S = \begin{matrix} 14.2691 & 0 \\ 0 & 0.6268 \\ 0 & 0 \\ 0 & 0 \end{matrix}$$

$$V = \begin{matrix} 0.6414 & -0.7672 \\ 0.7672 & 0.6414 \end{matrix}$$

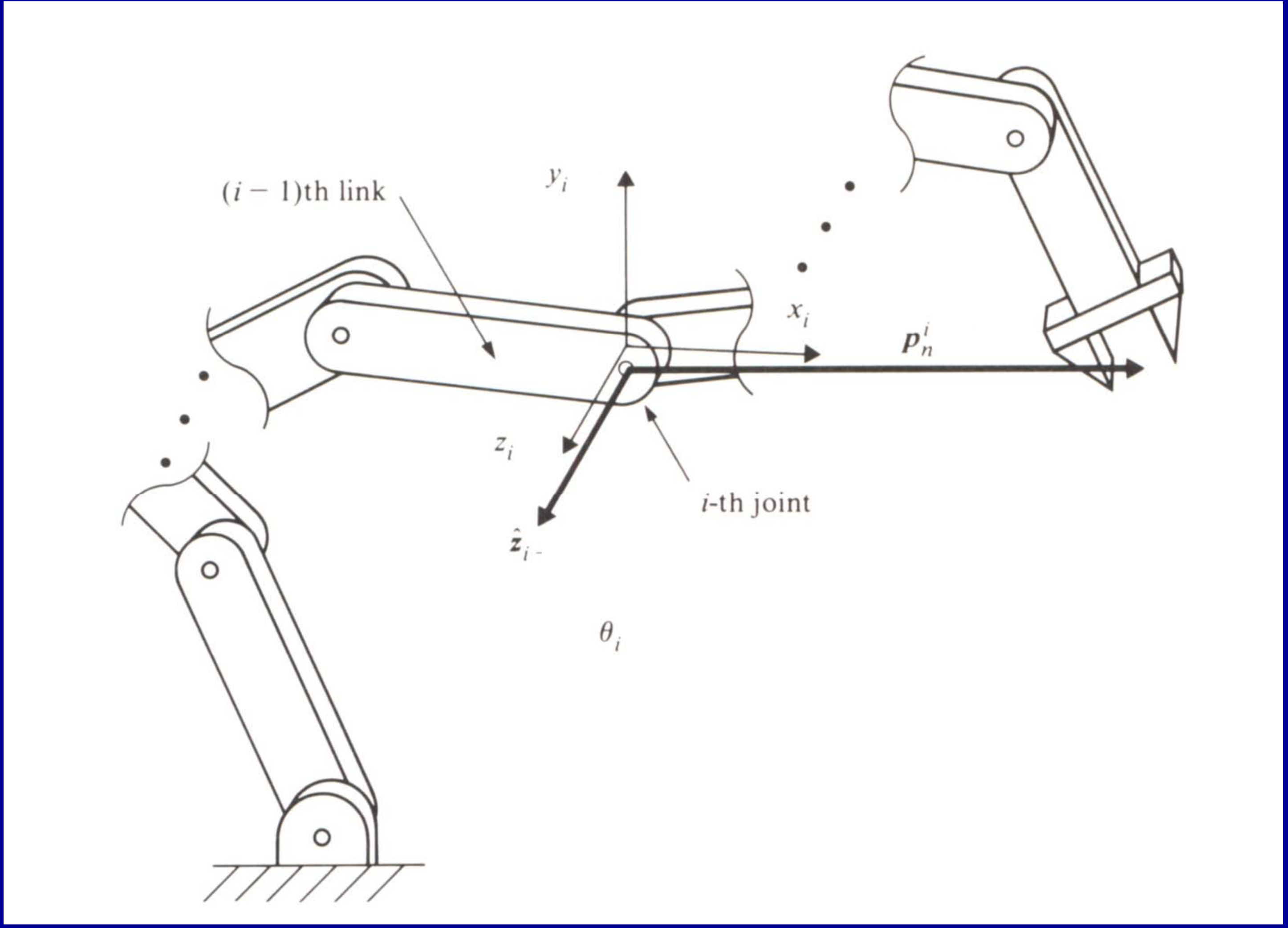
Changing a Jacobian's frame of reference

$$\begin{bmatrix} {}^{A}v \\ {}^{A}\omega \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & 0 \\ 0 & {}^{A}_{B}R \end{bmatrix} \begin{bmatrix} {}^{B}v \\ {}^{B}\omega \end{bmatrix},$$

$$\begin{bmatrix} {}^{A}v \\ {}^{A}\omega \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & 0 \\ 0 & {}^{A}_{B}R \end{bmatrix} {}^{B}J(\Theta)\dot{\Theta},$$

$$\boxed{{}^{A}J(\Theta) = \begin{bmatrix} {}^{A}_{B}R & 0 \\ 0 & {}^{A}_{B}R \end{bmatrix} {}^{B}J(\Theta).}$$

Computation of the Jacobian Matrix



$$J = [J_1 \quad J_2 \quad \cdots \quad J_n], \qquad J_i \in R^6$$

$$J_i = \begin{cases} \begin{pmatrix} \hat{z}_i \times {}^i p_n \\ \hat{z}_i \end{pmatrix} & \text{(Revolute joint)} \\ \begin{pmatrix} \hat{z}_i \\ 0 \end{pmatrix} & \text{(Prismatic joint)} \end{cases}, \qquad \hat{z}_i \in R^3$$

Linear Velocity

$$ {}^0v_n = \sum_{i=1}^{n} \frac{\partial\, {}^0v_n}{\partial \theta_i} \dot{\theta}_i $$

We see that *i*-th column of J_v, which we denote as J_{vi} is given by

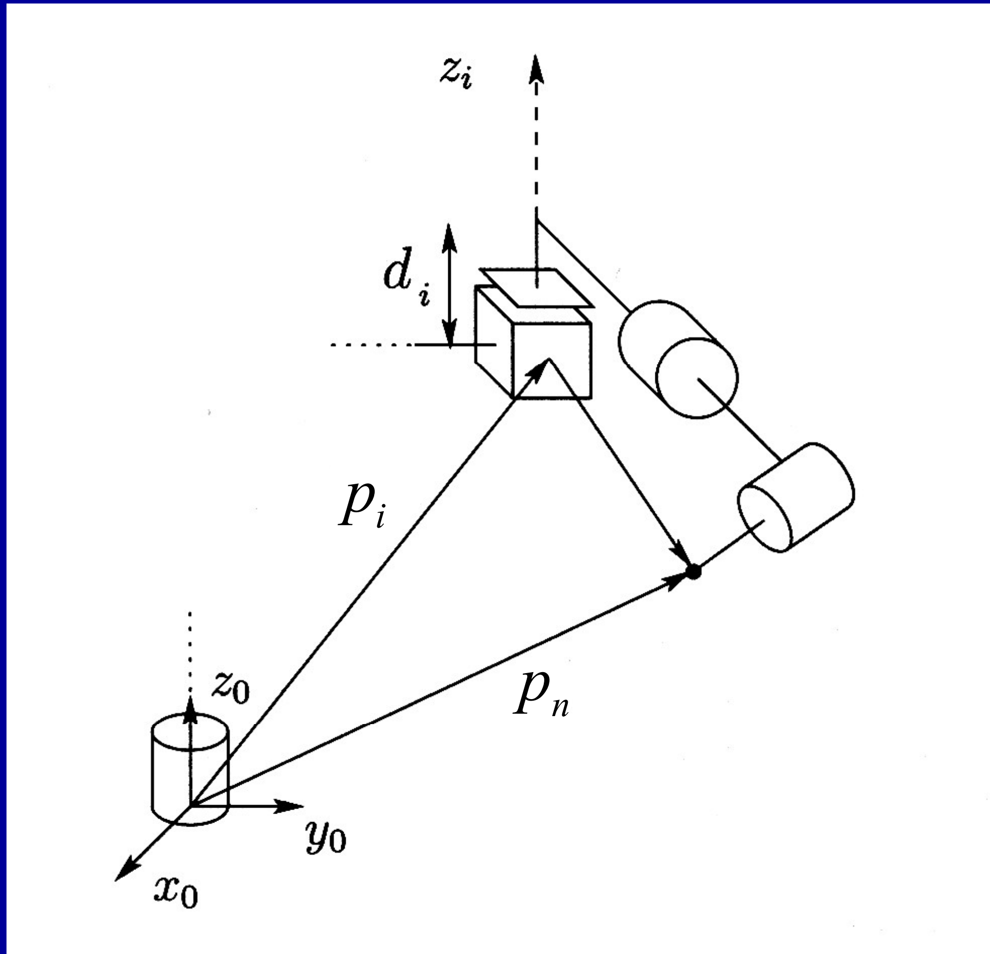
$$ J_{vi} = \frac{\partial\, {}^0v_n}{\partial \theta_i} $$

The linear velocity of the end-effector that would result if $\dot{\theta}_i$ *were equal to one and the other* $\dot{\theta}_j$ *were zero*

Case 1: Prismatic Joints

$$^{0}v_{n} = \dot{d}_{i}\,{}^{0}_{i}R\begin{bmatrix}0\\0\\1\end{bmatrix} = \dot{d}_{i}\,{}^{0}\hat{z}_{i}$$

$$J_{vi} = \hat{z}_{i}$$



Case 2: Revolute Joints

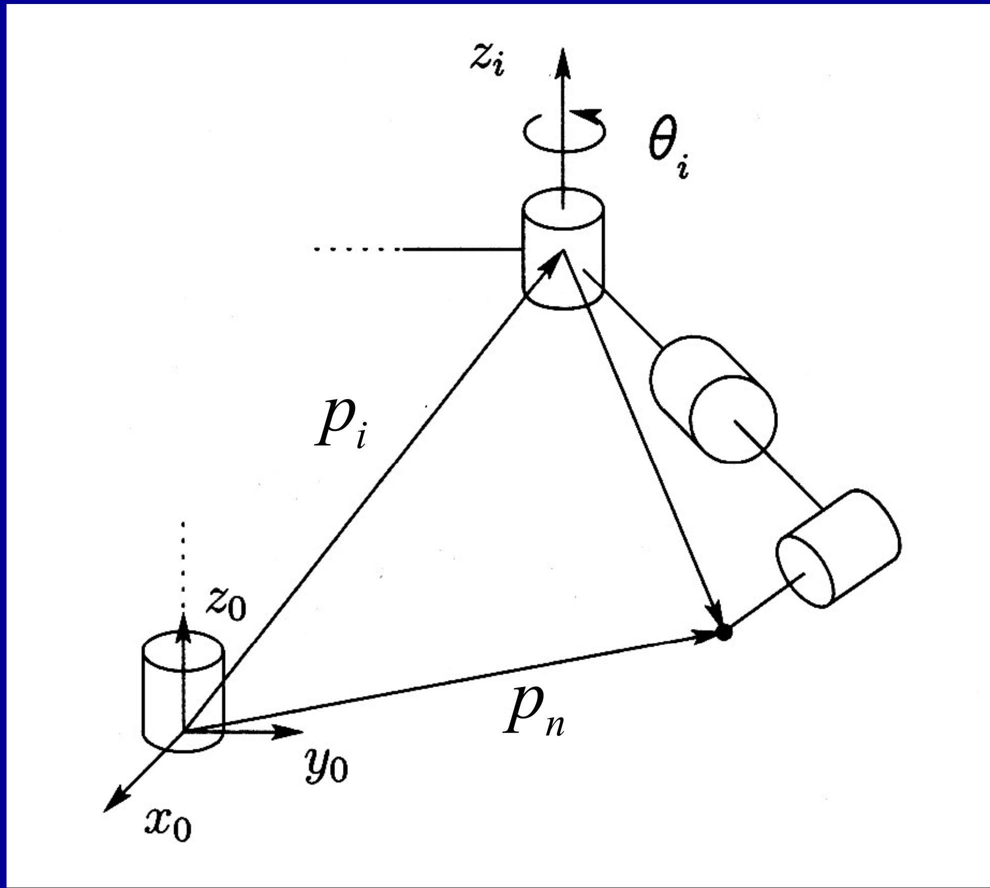
The linear velocity of the end-effector is simply the form of $\omega \times r$, where

$$\omega = \dot{\theta}_i \hat{z}_i$$

and

$$r = p_n - p_i$$

$$J_{vi} = \hat{z}_i \times (p_n - p_i)$$



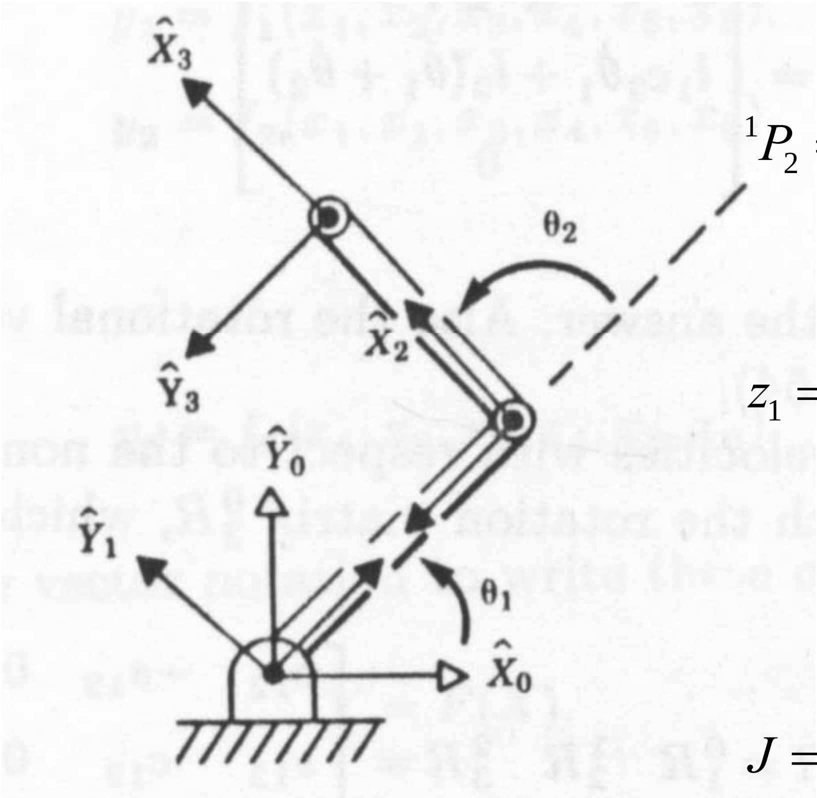
Angular Velocity

$$^{0}\omega_n = \rho_1\dot{\theta}_1 k + \rho_2\dot{\theta}_2\,{}^{0}_{2}Rk + \cdots + \rho_n\dot{\theta}_n\,{}^{0}_{n}Rk = \sum_{i}^{n} \rho_i\dot{\theta}_i\,{}^{0}\hat{z}_i$$

ρ_i *is equal to* 1 *if joint i is revolute and* 0 *if joint i is prismatic.*

$$J_\omega = [\rho_1 z_1, \rho_1 z_2, \cdots, \rho_n z_n]$$

We *revisit* Example 5.3



$$J(\theta) = \begin{bmatrix} z_1 \times {}^1P_3 & z_2 \times {}^2P_3 \\ z_1 & z_2 \end{bmatrix}$$

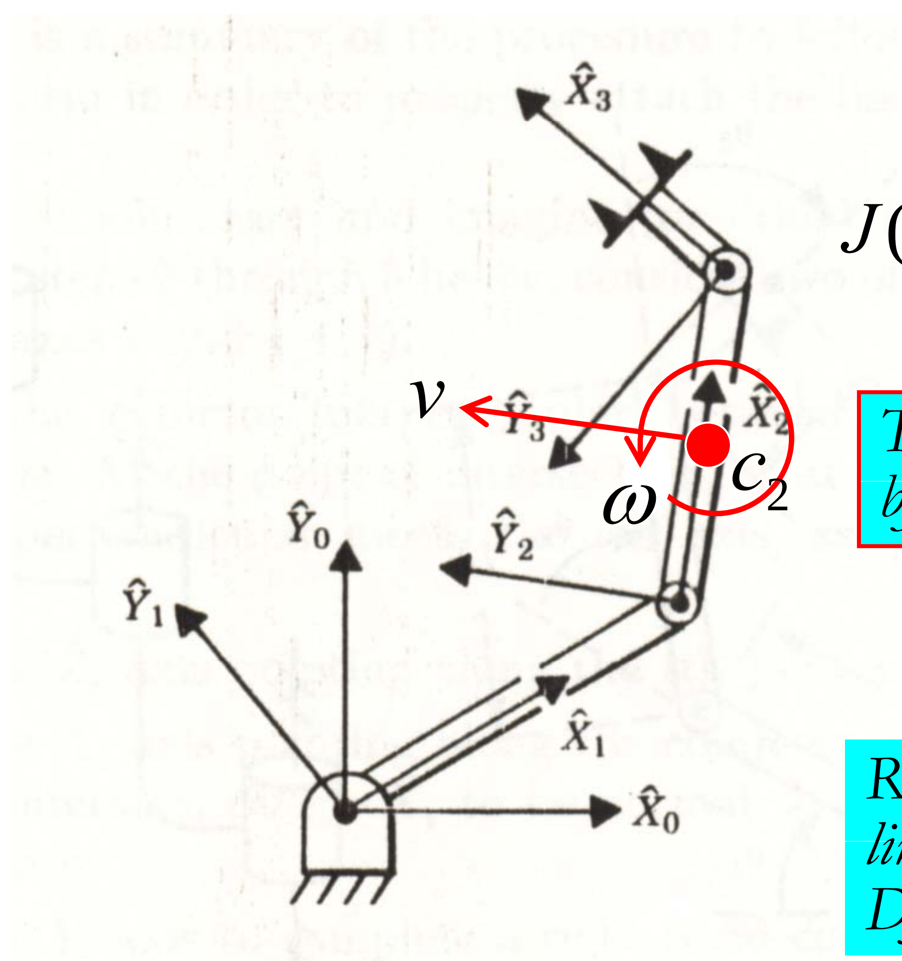
$${}^1P_2 = \begin{bmatrix} l_1c_1 \\ l_1s_1 \\ 0 \end{bmatrix}, \quad {}^1P_3 = \begin{bmatrix} l_1c_1 + l_2c_{12} \\ l_1s_1 + l_2s_{12} \\ 0 \end{bmatrix}, \quad {}^2P_3 = {}^1P_3 - {}^1P_2 = \begin{bmatrix} l_2c_{12} \\ l_2s_{12} \\ \end{bmatrix}$$

$$z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1s_1 - l_2s_{12} & -l_2s_{12} \\ l_1c_1 + l_2c_{12} & l_2c_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

Jacobian for an Arbitrary Point on a Link

Wish to compute the linear velocity v *and the angular velocity* ω *of the center of link 2*



$$J(\theta) = \begin{bmatrix} z_1 \times {}^1P_{c_2} & z_2 \times {}^2P_{c_2} & 0 \\ z_1 & z_2 & 0 \end{bmatrix}$$

The velocity of the second link is unaffected by motion of the third link: Kinematic effects

Reaction forces on link 2due to the motion of link 3 will influence the motion of link 2: Dynamic effects

Lab. #2 (2 pt.) – Due Jan. 20

Using the Robotics Toolbox for MATLAB,
make the PUMA 560 arm move in a straight line
from x=0.02m, y=-0.15m, z=0.86m
to x=0.02m, y=-0.15m, z=-0.86m.

Display a stick figure animation of the robot moving along the path and submit a printed copy of your MATLAB code.

Singularities

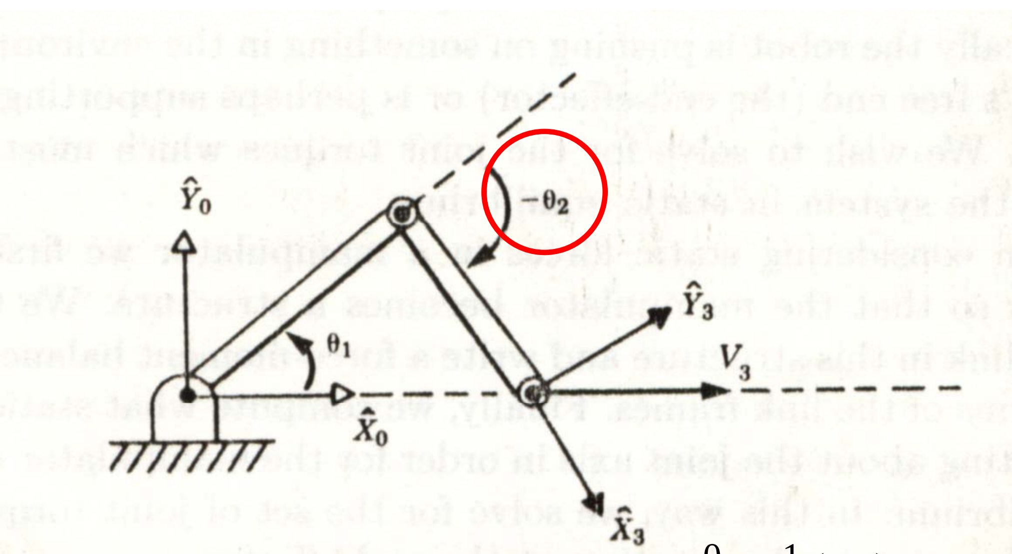
- Workspace boundary singularities: fully stretched out or folded back on itself
- Workspace interior singularities: two or more joint axes are lined up

Example 5.4

$$DET[J(\Theta)] = |J(\Theta)| = \begin{vmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{vmatrix} = l_1 l_2 s_2 = 0.$$

$\theta_2 = 0^\circ, 180^\circ \rightarrow$ *Workspace boundary singularities*

Example 5.5

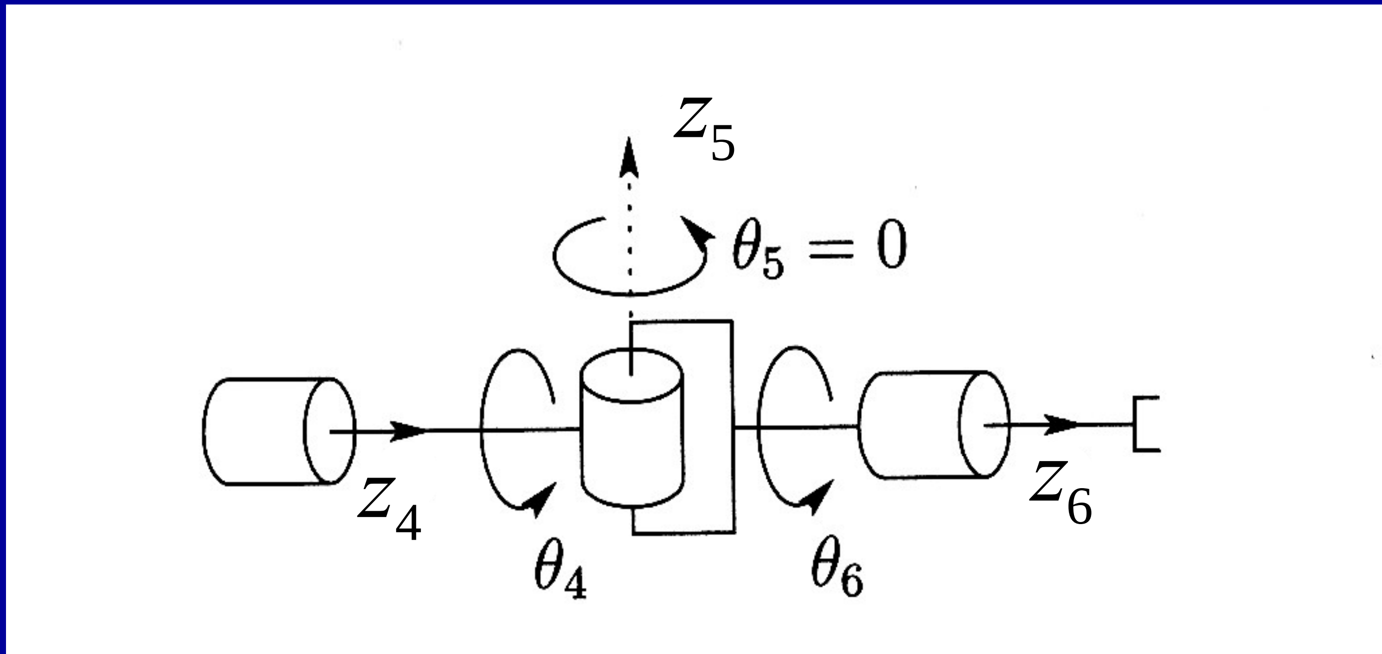


$$^0J^{-1}(\Theta) = \frac{1}{l_1 l_2 s_2}\begin{bmatrix} l_2 c_{12} & l_2 s_{12} \\ -l_1 c_1 - l_2 c_{12} & -l_1 s_1 - l_2 s_{12} \end{bmatrix}.$$

$$\dot{\theta}_1 = \frac{c_{12}}{l_1 s_2},$$

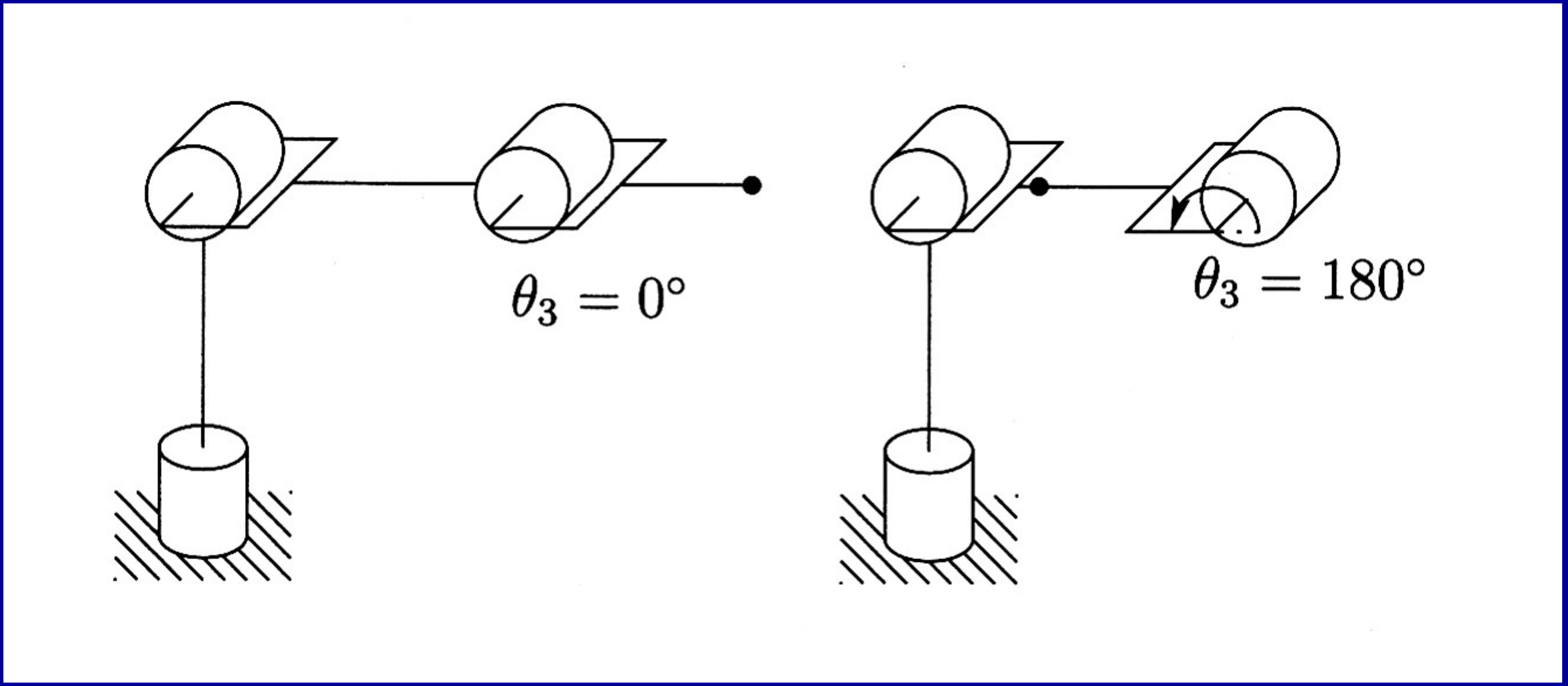
$$\dot{\theta}_2 = -\frac{c_1}{l_2 s_2} - \frac{c_{12}}{l_1 s_2}.$$

Spherical Wrist Singularity



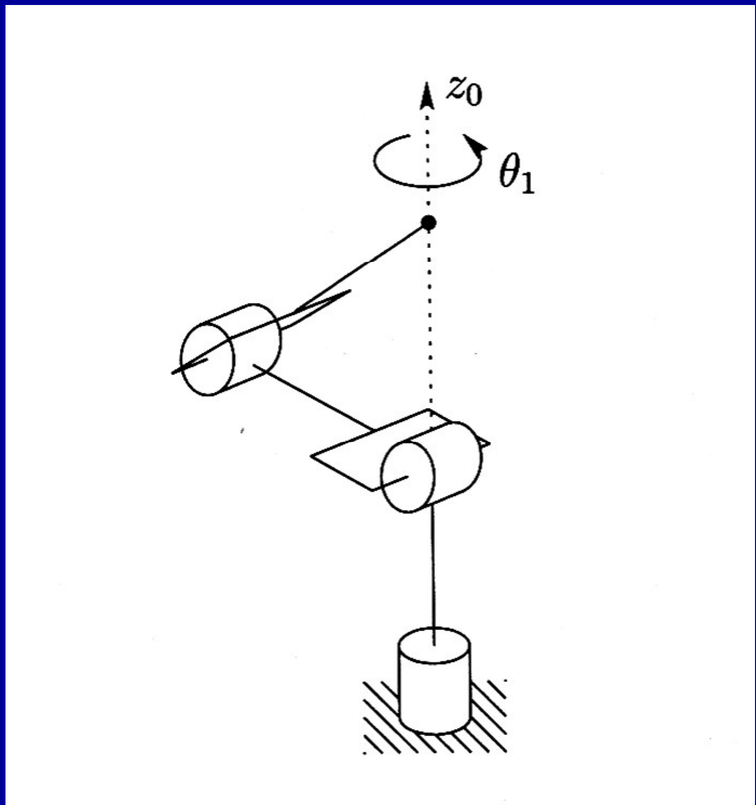
The joint axes z_4 *and* z_6 *are collinear.*

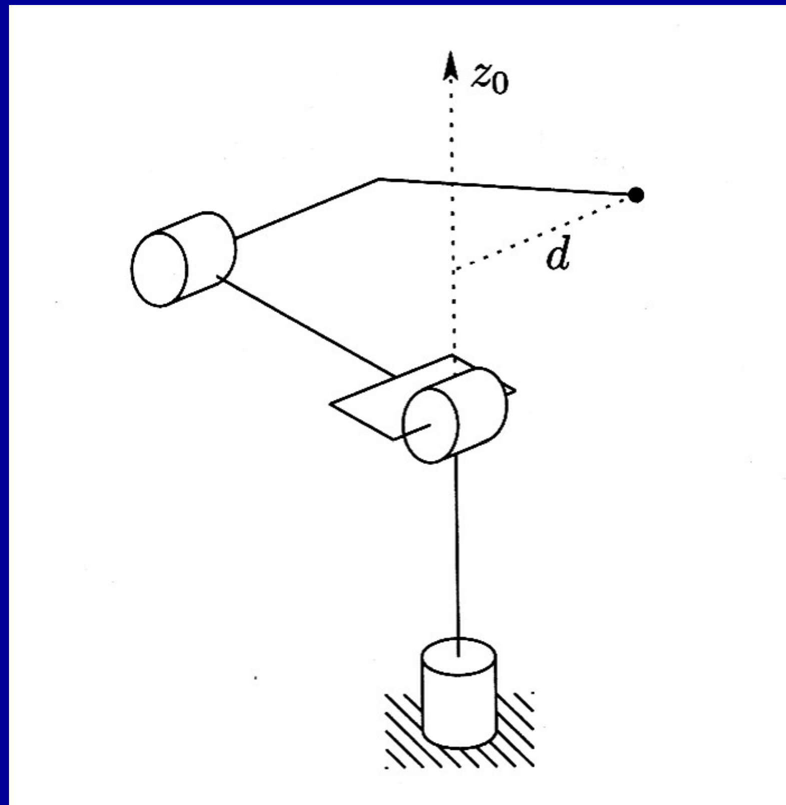
Elbow Singularities of the Elbow Manipulator



Fully extended or fully retracted

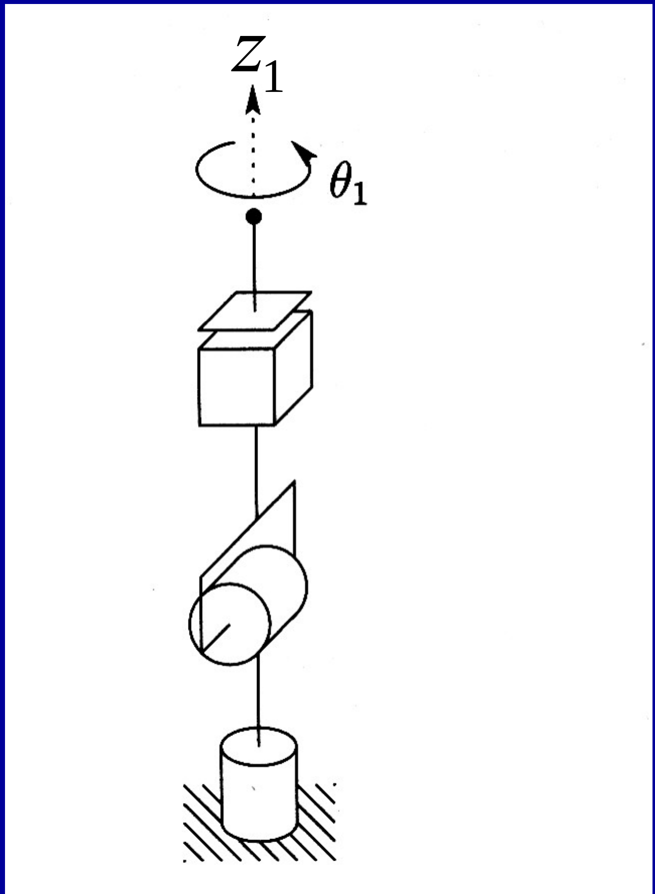
Elbow Manipulator with an Offset at the Elbow





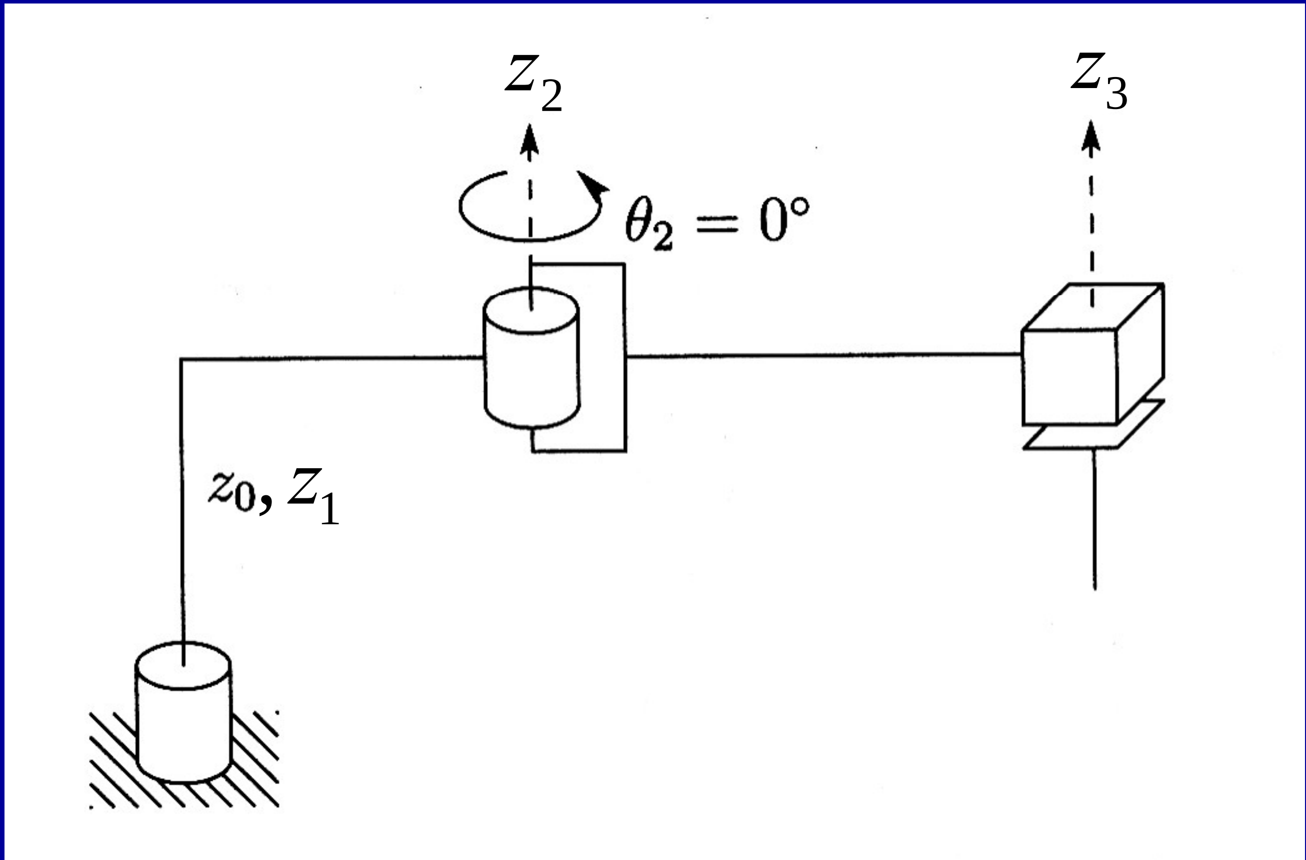
The wrist center intersects the axis of the base rotation.

Spherical Manipulator with no Offset

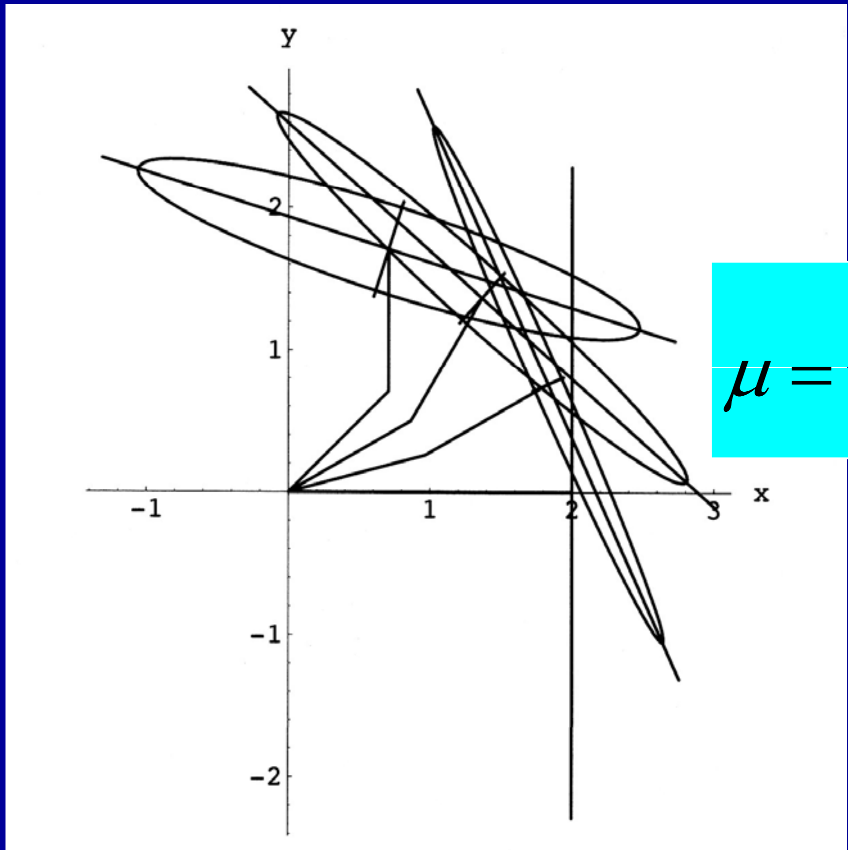


The wrist center intersects z_1.
Any rotation about the base leaves this point fixed.

SCARA Manipulator Singularity



Manipulability



$$\mu = \sqrt{\det JJ^T} = |\lambda_1 \lambda_2 \cdots \lambda_m| = |\det J|$$

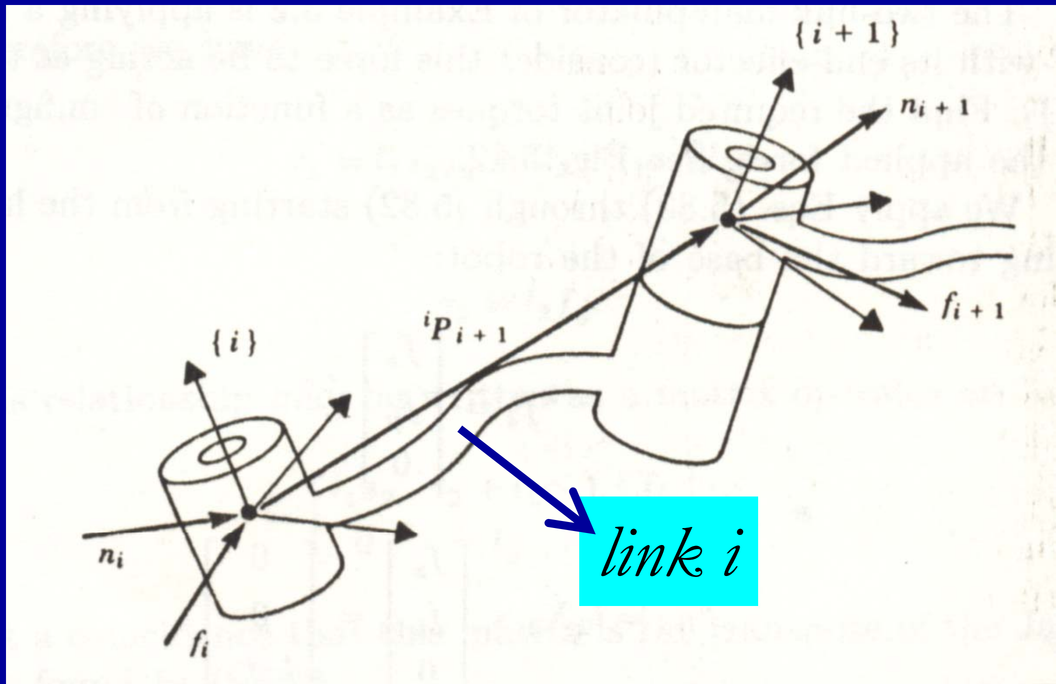
The eigenvalues of J

Manipulability ellipsoids for several configurations of a two-link arm

Static forces in manipulators

- How forces and moments propagate from one link to the next?
- The robot is pushing on something in the environment with the end-effector or supporting a load at the hand.

Static forces in manipulators



f_i = *force exerted on link i by link i-1*

n_i = *torque exerted on link i by link i-1*

Solve for the joint torques that must be acting to keep the system in static equilibrium.

$$^{i}f_i = {}^{i}f_{i+1},$$

$$^{i}n_i = {}^{i}n_{i+1} + {}^{i}P_{i+1} \times {}^{i}f_{i+1}.$$

No net forces, no net torques (moments)

$$\sum f = 0, \sum n = 0$$

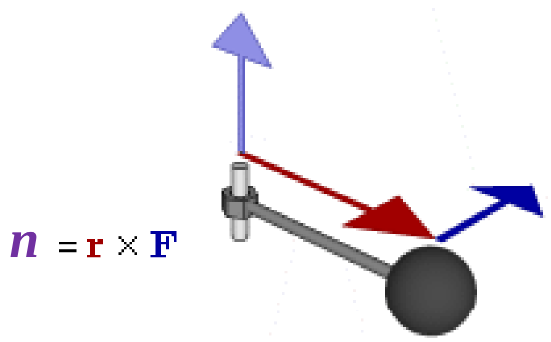
$$^{i}f_i = {}^{i}_{i+1}R \, {}^{i+1}f_{i+1},$$

$$^{i}n_i = {}^{i}_{i+1}R \, {}^{i+1}n_{i+1} + {}^{i}P_{i+1} \times {}^{i}f_i.$$

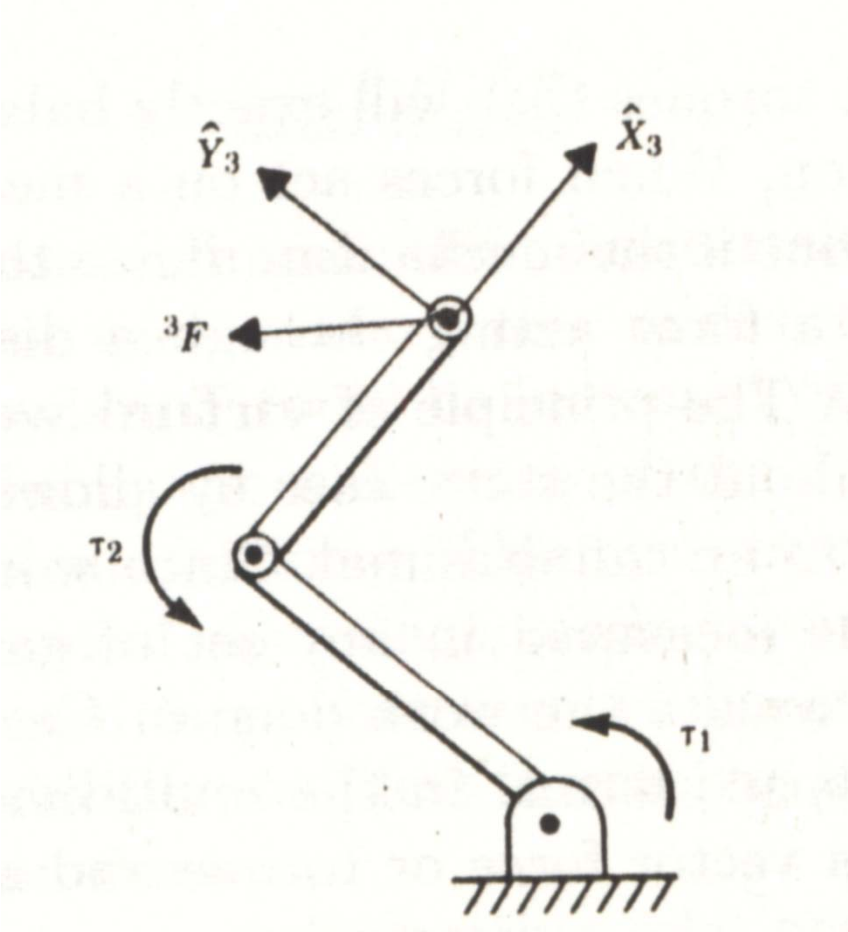
Static force propagation from link to link:

$$\tau_i = {}^{i}n_i^T \, {}^{i}\hat{Z}_i. \quad \text{Revolute joint}$$

$$\tau_i = {}^{i}f_i^T \, {}^{i}\hat{Z}_i. \quad \text{Prismatic joint}$$



Example 5.7



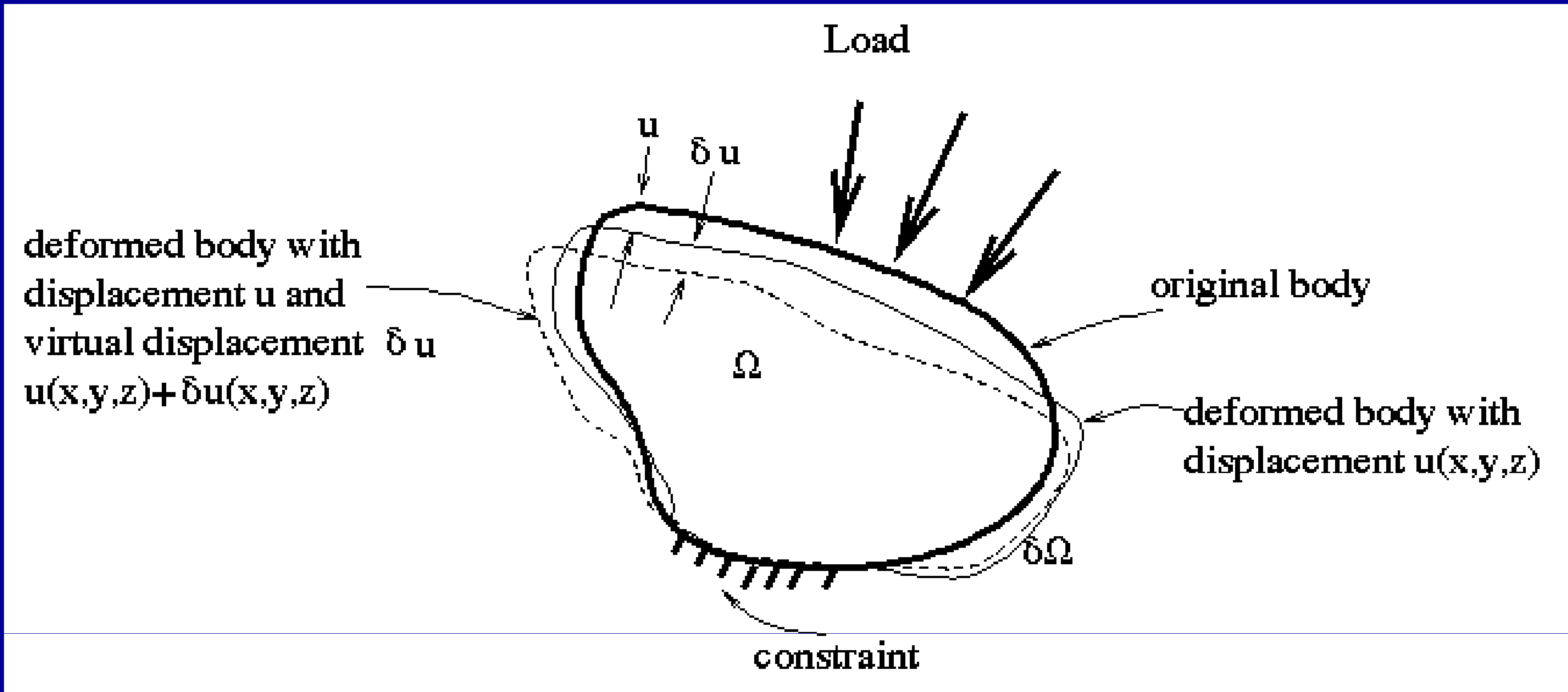
$$\tau = \begin{bmatrix} l_1 s_2 & l_2 + l_1 c_2 \\ 0 & l_2 \end{bmatrix} \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

J^T

Work-Energy Principle

- The change in the kinetic energy of an object is equal to the net work done on the object.

Principle of virtual work



External virtual work equals the internal virtual strain energy.

Jacobians in the force domain

$$F \cdot \delta X = \tau \cdot \delta\Theta,$$

$$F^T \boxed{\delta X} = \tau^T \delta\Theta,$$

$$F^T \boxed{J\delta\Theta} = \tau^T \delta\Theta \rightarrow F^T J = \tau^T.$$

$$\boxed{\tau = J^T F.}$$

$$\tau = {}^0J^T\,{}^0F.$$

Cartesian transformation of velocities and static forces

$$V = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

General velocity of a body

$$F = \begin{bmatrix} F \\ N \end{bmatrix}$$

General force of a body

6 x 6 *transformations map these quantities from one frame to another.*

A velocity transformation

$${}^{B}\mathrm{v}_{B} = {}^{B}_{A}T_{v}\,{}^{A}\mathrm{v}_{A}$$

$$\begin{bmatrix} {}^{B}v_{B} \\ {}^{B}\omega_{B} \end{bmatrix} = \begin{bmatrix} {}^{B}_{A}R & -{}^{B}_{A}R\,{}^{A}P_{BORG}\times \\ 0 & {}^{B}_{A}R \end{bmatrix} \begin{bmatrix} {}^{A}v_{A} \\ {}^{A}\omega_{A} \end{bmatrix}.$$

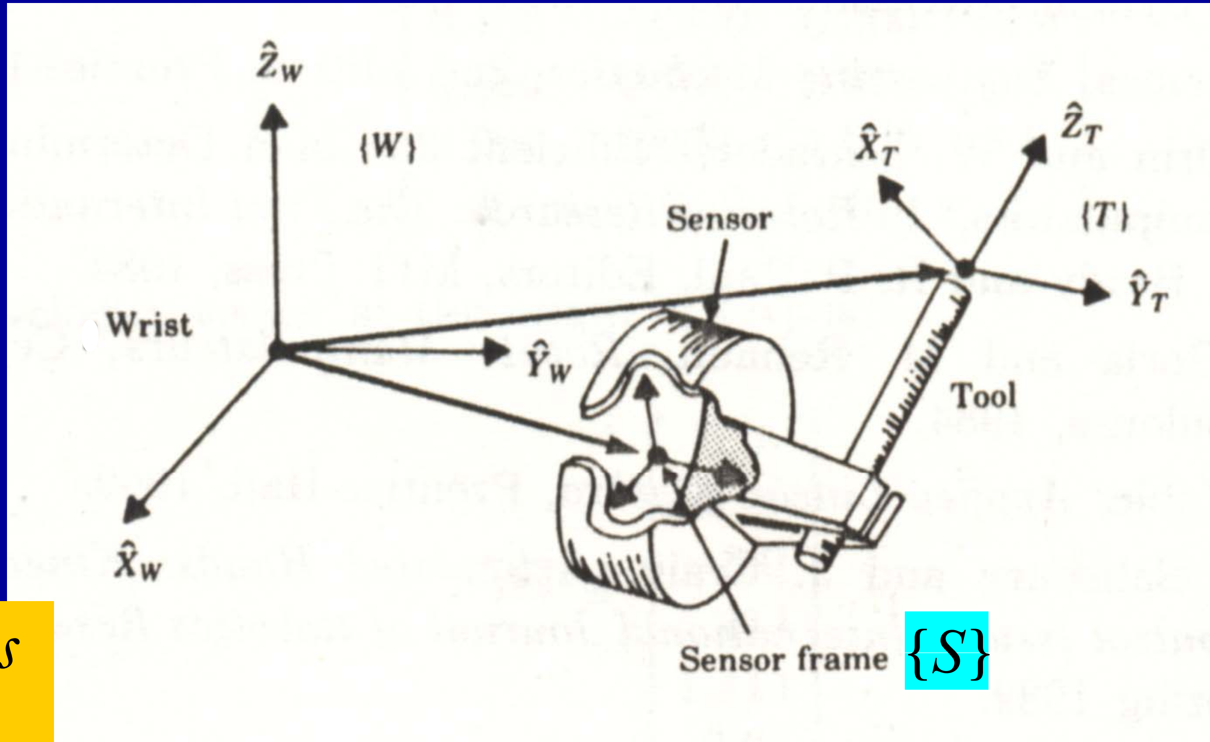
$$\begin{bmatrix} {}^{A}v_{A} \\ {}^{A}\omega_{A} \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{BORG}\times{}^{A}_{B}R \\ 0 & {}^{A}_{B}R \end{bmatrix} \begin{bmatrix} {}^{B}v_{B} \\ {}^{B}\omega_{B} \end{bmatrix}.$$

$${}^{A}_{B}T_{f} = {}^{A}_{B}T_{v}^{T}$$

$$\begin{bmatrix} {}^{A}F_{A} \\ {}^{A}N_{A} \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & 0 \\ {}^{A}P_{BORG}\times{}^{A}_{B}R & {}^{A}_{B}R \end{bmatrix} \begin{bmatrix} {}^{B}F_{B} \\ {}^{B}N_{B} \end{bmatrix}.$$

A force-moment transformation

$${}^{A}\mathrm{F}_{A} = {}^{A}_{B}T_{f}\,{}^{B}\mathrm{F}_{B}$$



The forces and torques applied at the tip of the tool

The output of the sensor

$$^{T}F_{T} = {}^{T}_{S}T_{f}\, {}^{S}F_{S},$$

$$^{T}_{S}T_{f} = \begin{bmatrix} {}^{T}_{S}R & 0 \\ {}^{T}P_{SORG} \times {}^{T}_{S}R & {}^{T}_{S}R \end{bmatrix}.$$

Homework #12 – Due Jan. 20

- 5.18, 5.19