Manipulator Kinematics

December 16, 2009

Kinematics

- the science of motion that treats the subject (motion) without regard to the forces that cause it.
- the position, velocity, acceleration, and all higher order derivatives of the position variables w.r.t. time or any other variables

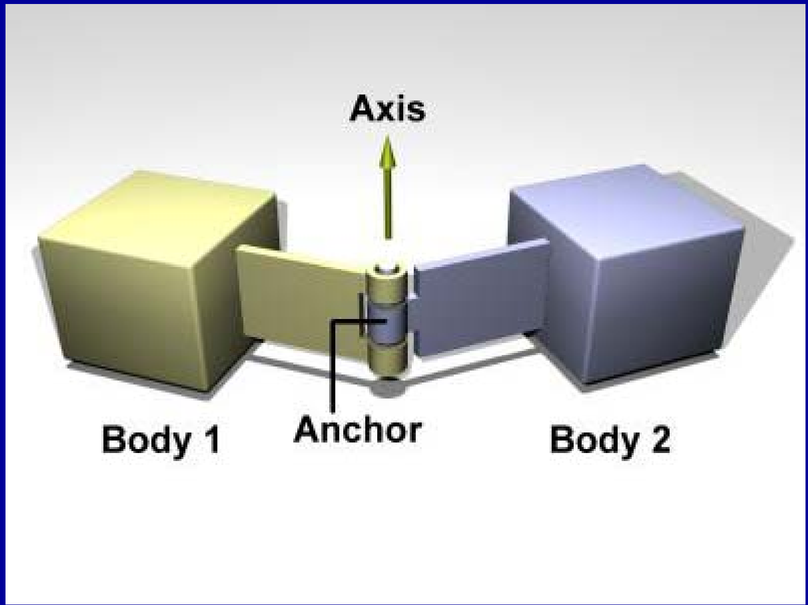
Kinematics of manipulators: static (Chapter 3)

- How the locations of the frames change as the mechanism articulates?
- A method to compute the position and orientation of the manipulator's end-effector relative to the base of the manipulator as a function of the joint variables

A Manipulator

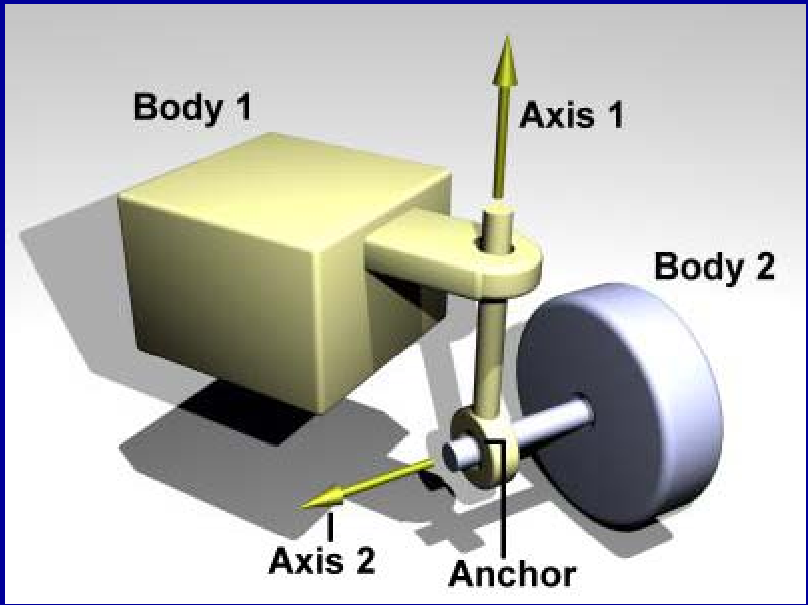
- May be though of as a set of bodies (*links*) connected in a chain by *joints*.
- Without loss of generality, we will consider only manipulators which have joints with a single degree of freedom.
- A joint having n degrees of freedom can be modeled as n joints of one degree of freedom connected with n–1 links of zero length.

Revolute (Hinge) Joint



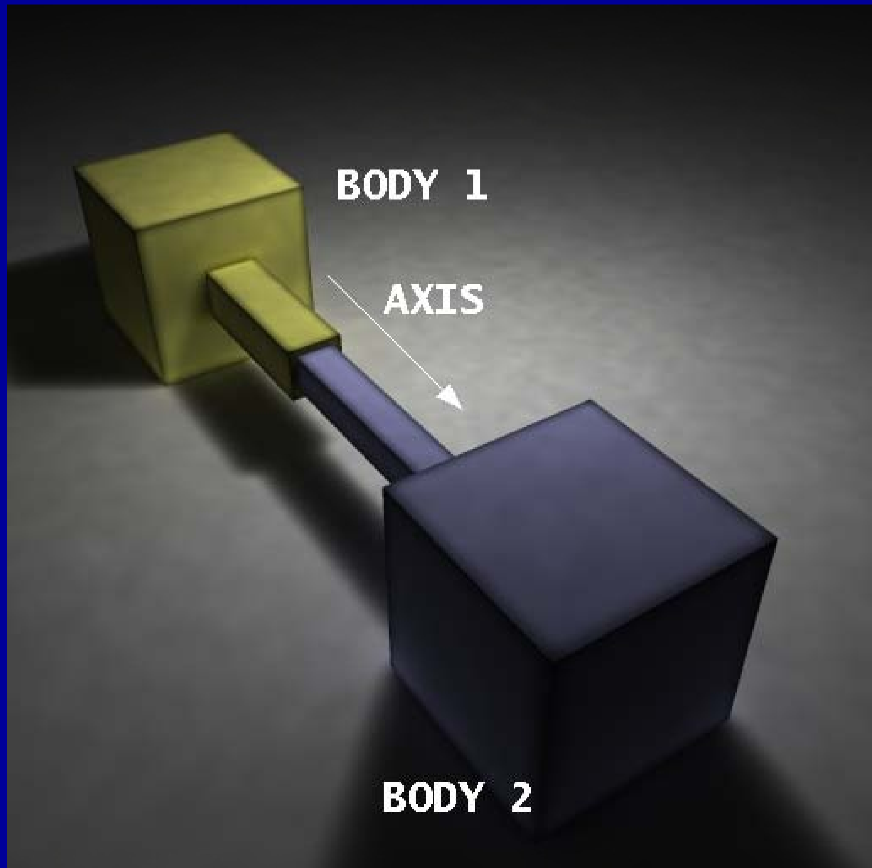
http://opende.sourceforge.net/wiki/index.php/Manual_(Joint_Types_and_Functions)

Hinge–2 Joint

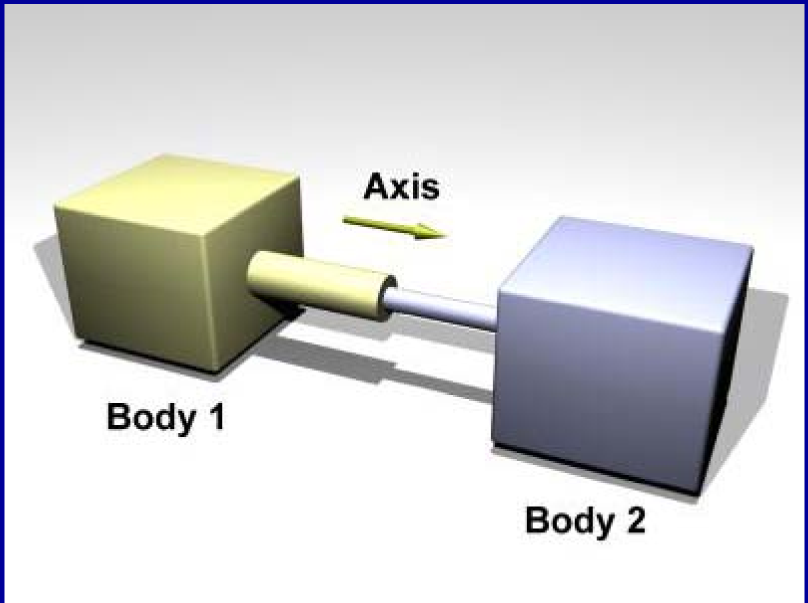


Two hinges connected in series, with different hinge axes

Prismatic (Slider) Joint



Cylindrical (Piston) Joint

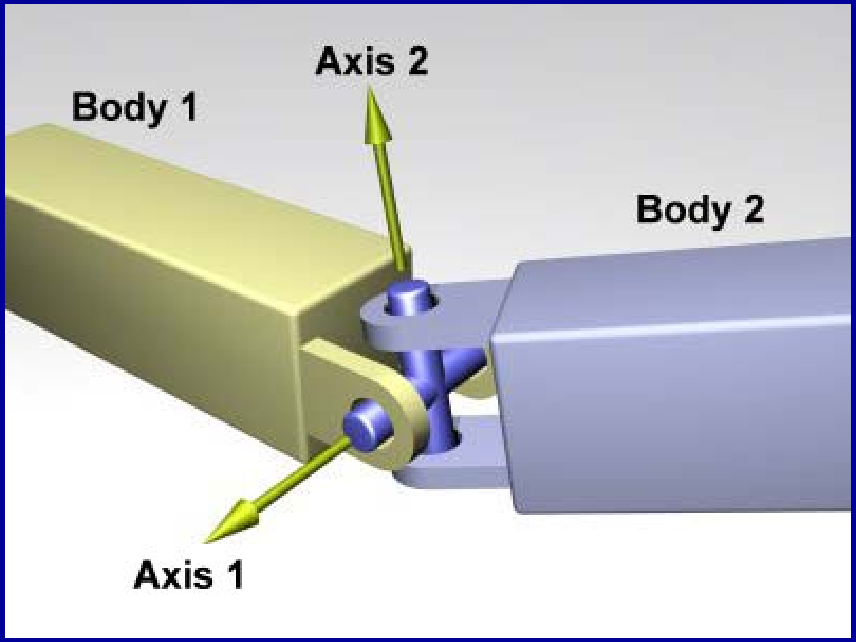


Rotation around the translation axis is possible!

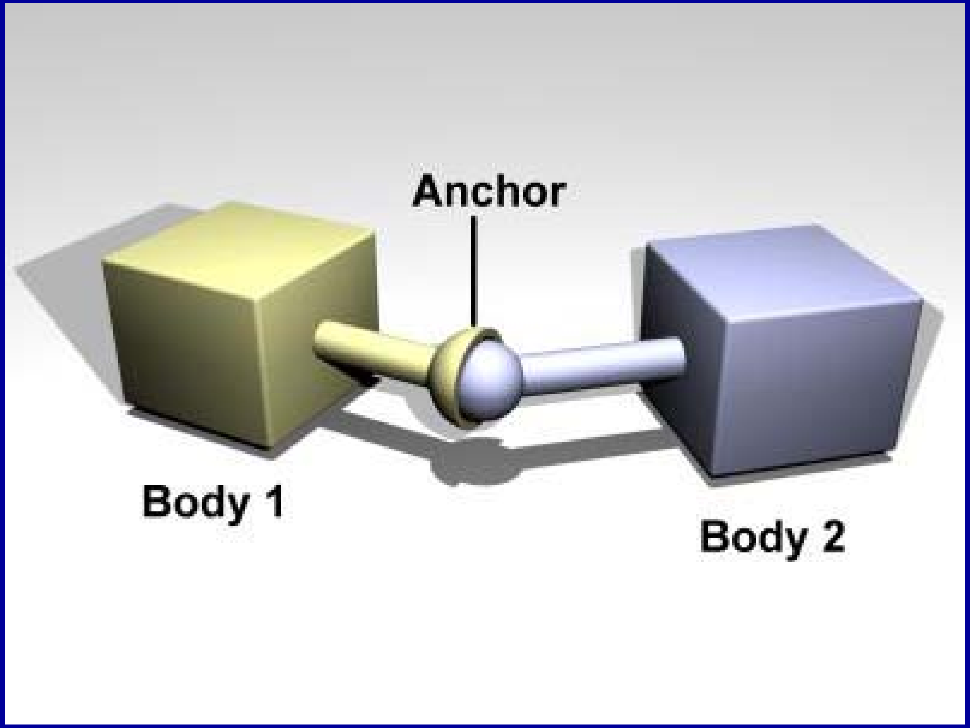
Quiz #3 – *Everyone please answer now?*

What are the degrees of freedom of a screw joint?

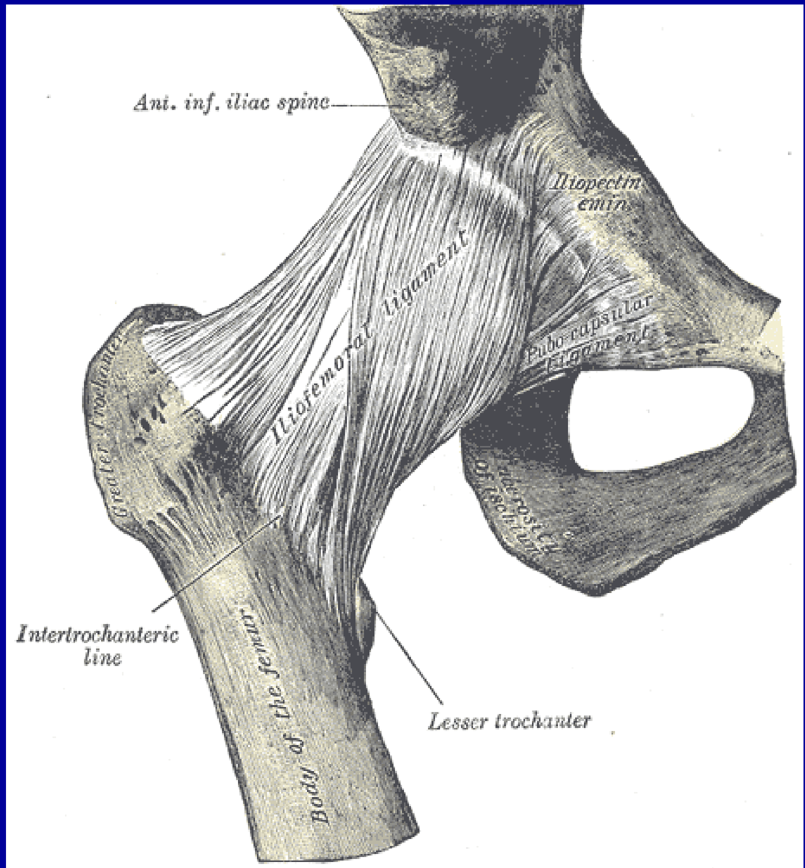
Universal Joint

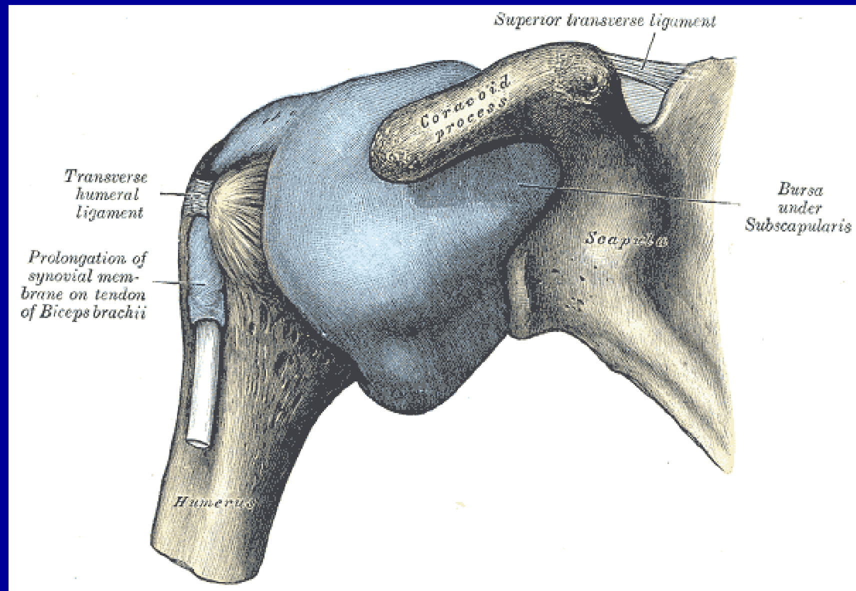


Spherical (Ball-and-Socket) Joint

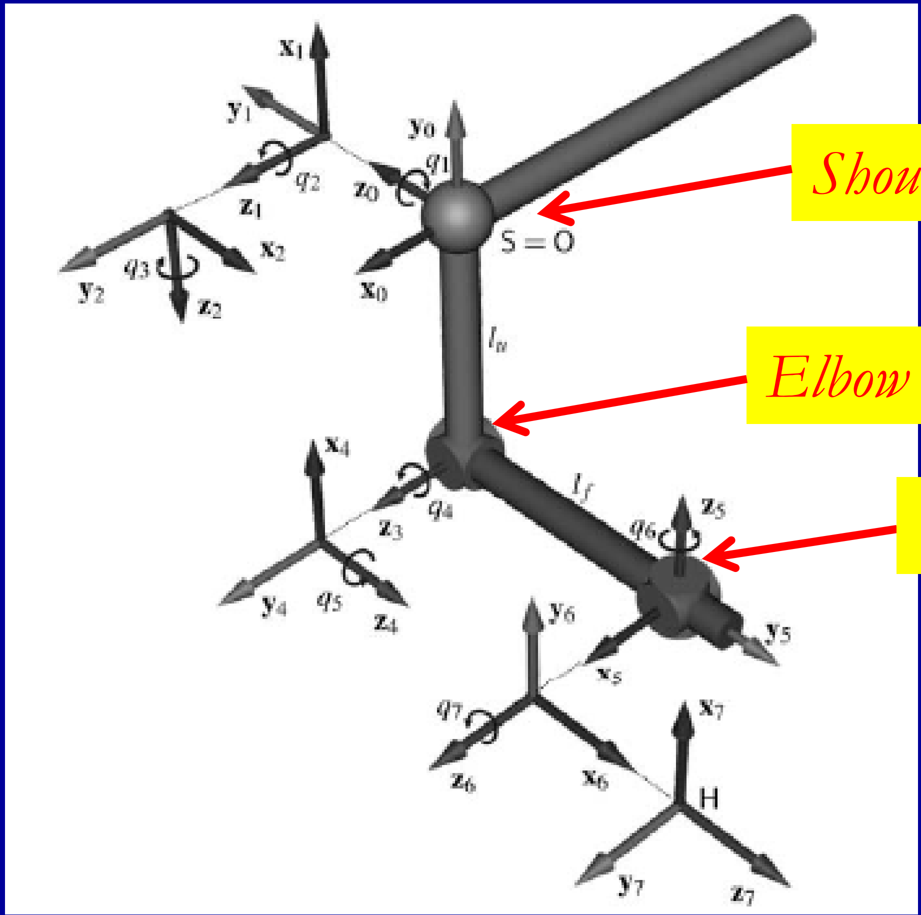


Examples: Hip, Shoulder





Simplified Human Arm Kinematics

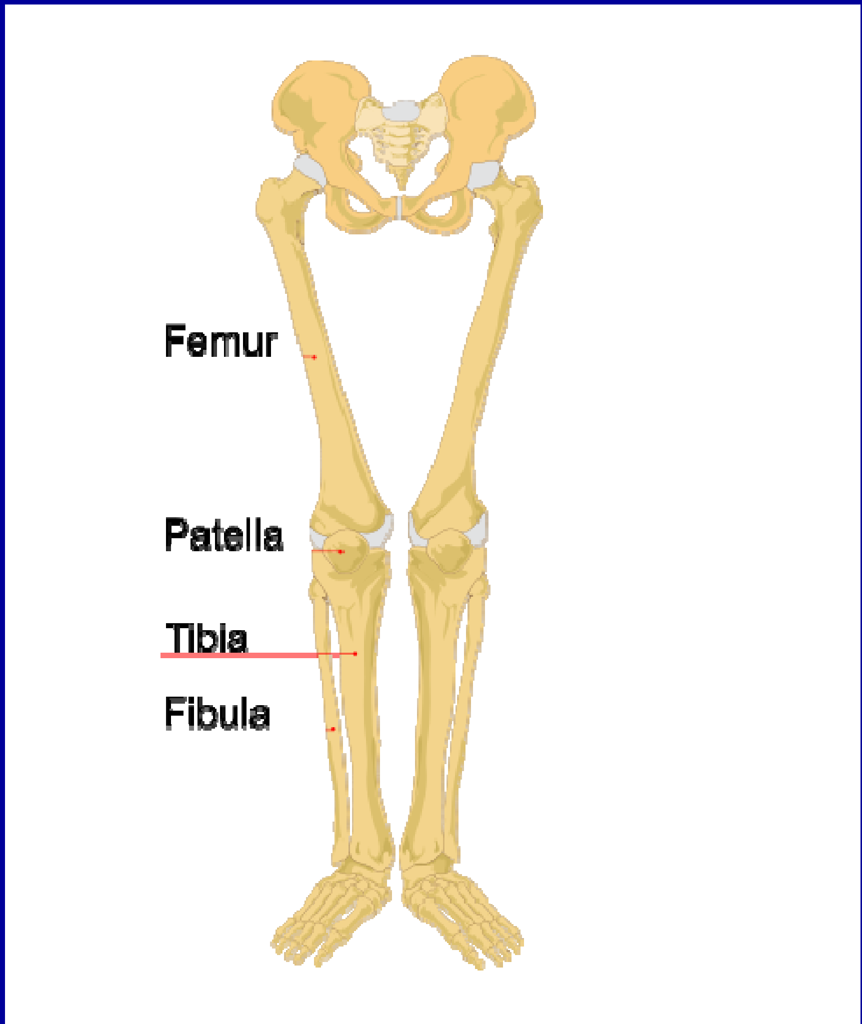


Shoulder (ball-and-socket) joint

Elbow (double hinge) joint

Wrist (double hinge) joint

Homework #8 (1 pt.) – Due Jan. 13



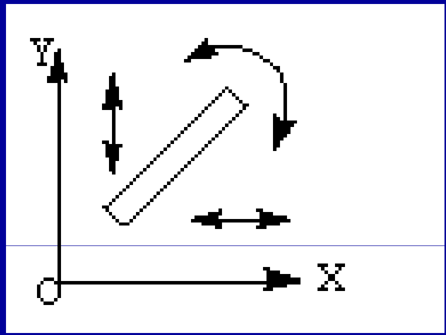
Obtain a kinematic model of the lower limb.
(Hint: 3 DOF hip joint,
1 DOF knee joint,
2 DOF ankle joint)

Degrees of Freedom (DOF)

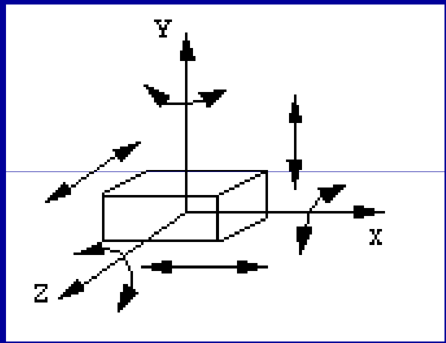
- The degrees of freedom of a rigid body is defined as the number of *independent* movements it has.

DOF of a Rigid Body

In a plane



In space



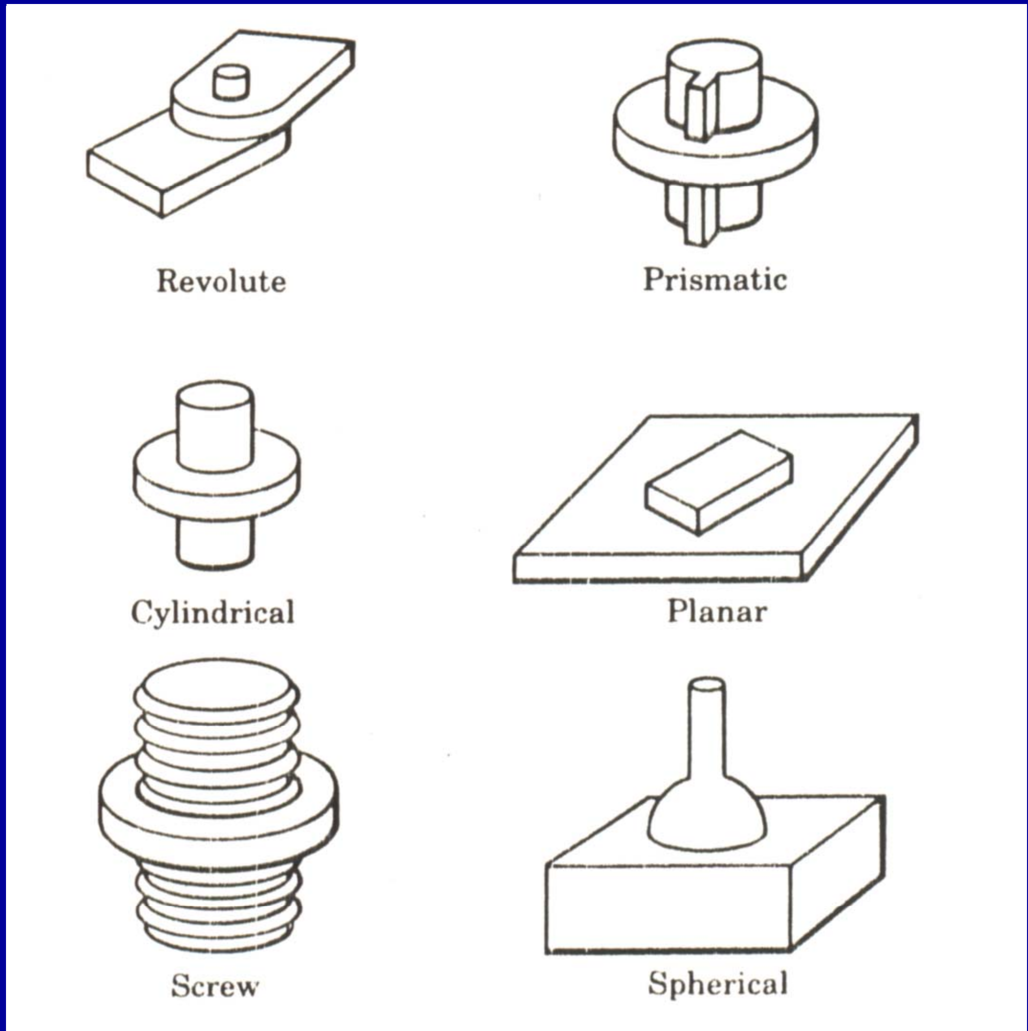
Gruebler's Equation

- Degrees of freedom of planar mechanism
- $F = 3(n - 1) - 2l - h$
- F: degrees of freedom
- n: number of links
- l: number of lower pairs (one DOF)
- h: number of higher pairs (two DOF)

Lower pair (Surface-contact)

- The connection between a pair of bodies when the relative motion is characterized by two surfaces sliding over one another

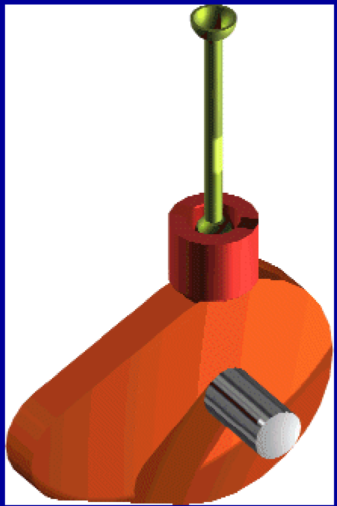
The six possible lower pair joints

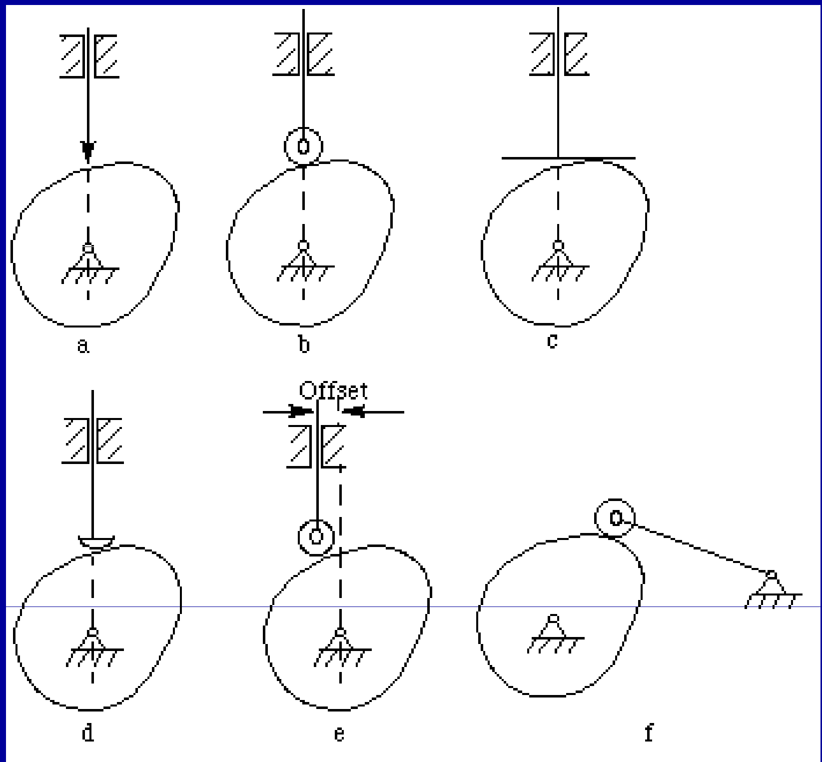


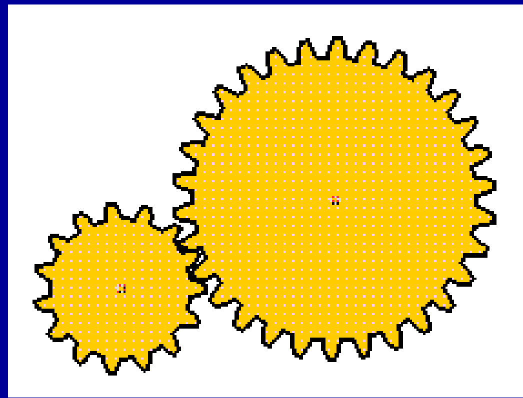
Higher pair (Point-, line-, curve-contact)

- A higher pair joint is one which contact occurs only at isolated points or along a line segments

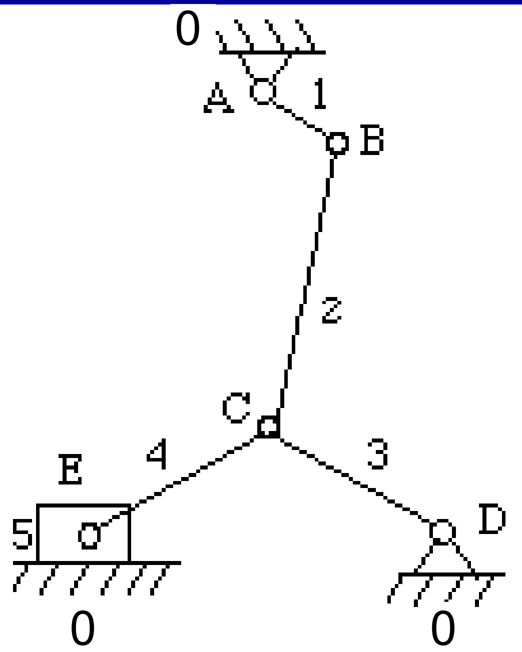
Higher Pair Examples







Example



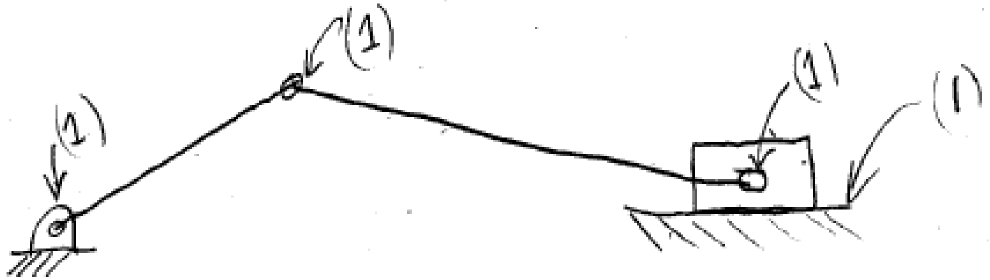
$$F = 3(6-1) - 2(7) - 0 = 1$$

including ground

Kutzbach Criterion

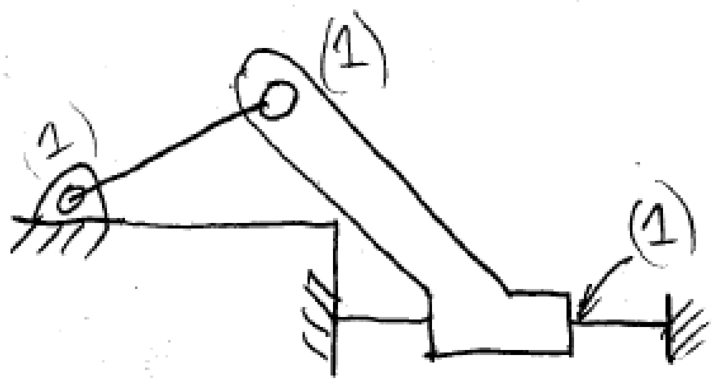
- Calculates the mobility (the number of degrees of freedom of a mechanism)
- The mobility is the number of input parameters (usually pair variables) that must be independently controlled to bring the device into a particular position.
- In order to control a mechanism, the number of independent input motions must equal the number of degrees of freedom of the mechanism.

Ex.



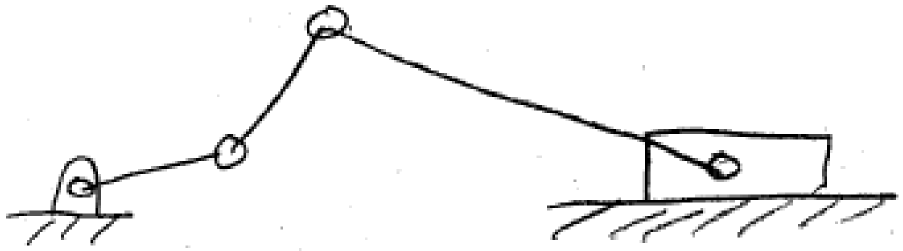
$$M = 3(4-1)-2(4)-0=1$$

Mobile



$$M = 3(3-1)-2(3)-0=0$$

Rigid

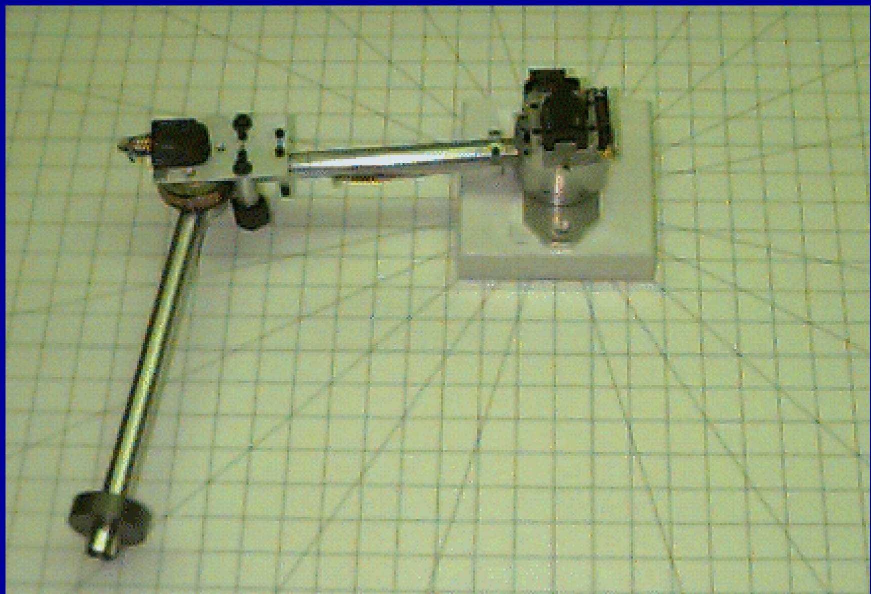


Mobile, 2 input motors

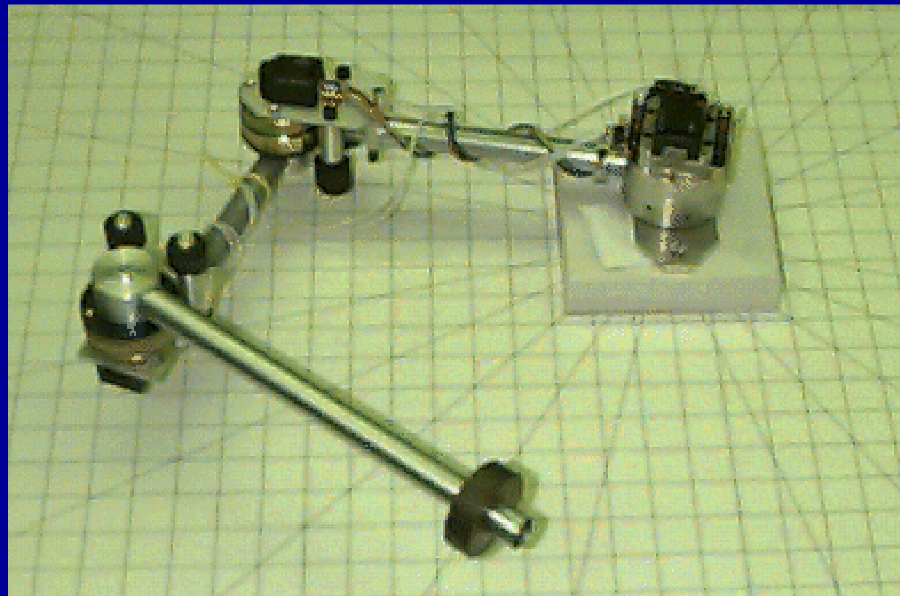
$$M = 3(5-1)-2(5)-0=2$$

CMU Underactuated Manipulators

Less actuators than degrees of freedom



2-link underactuated manipulator with one passive joint



3-link underactuated manipulator with two passive joints

Redundant Manipulator

- Called *kinematically redundant* if the manipulator possesses more degrees of freedom than is necessary for performing a specified task.
- In the three-dimensional space, a manipulator with *seven or more* joints is redundant since six degrees of freedom are sufficient to position and orient the end-effector in any desired configuration.

Link description

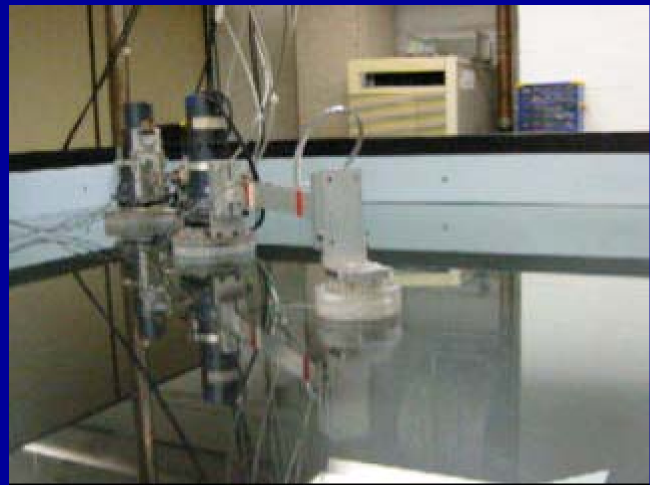
- The links are numbered starting from the immobile base of the arm, which is link 0.
- The first moving body is link 1, and so on, out to the free end of the arm, which is link *n*.
- In order to position an end-effector generally in 3-D space, a minimum of six joints is required.

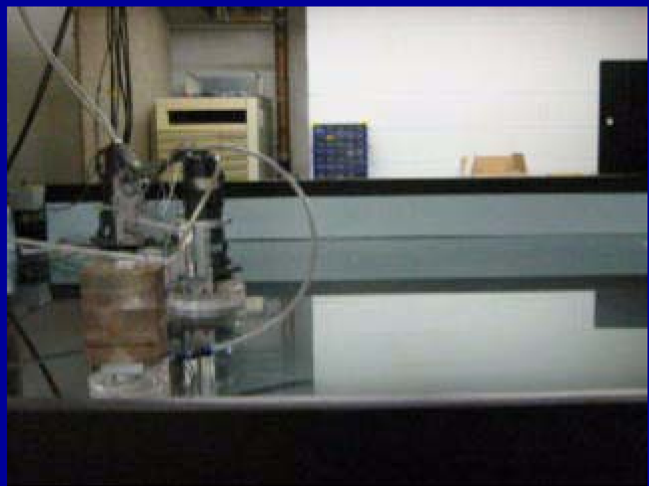
Defining the relationship between two axes in space

- Link length
- Link twist

A link is considered only as a rigid body that defines the relationship between two neighboring joint axes of a manipulator.

Flexible link (Video)





Long Reach Space Manipulators

Lightweight, flexible, limited speed and dexterity

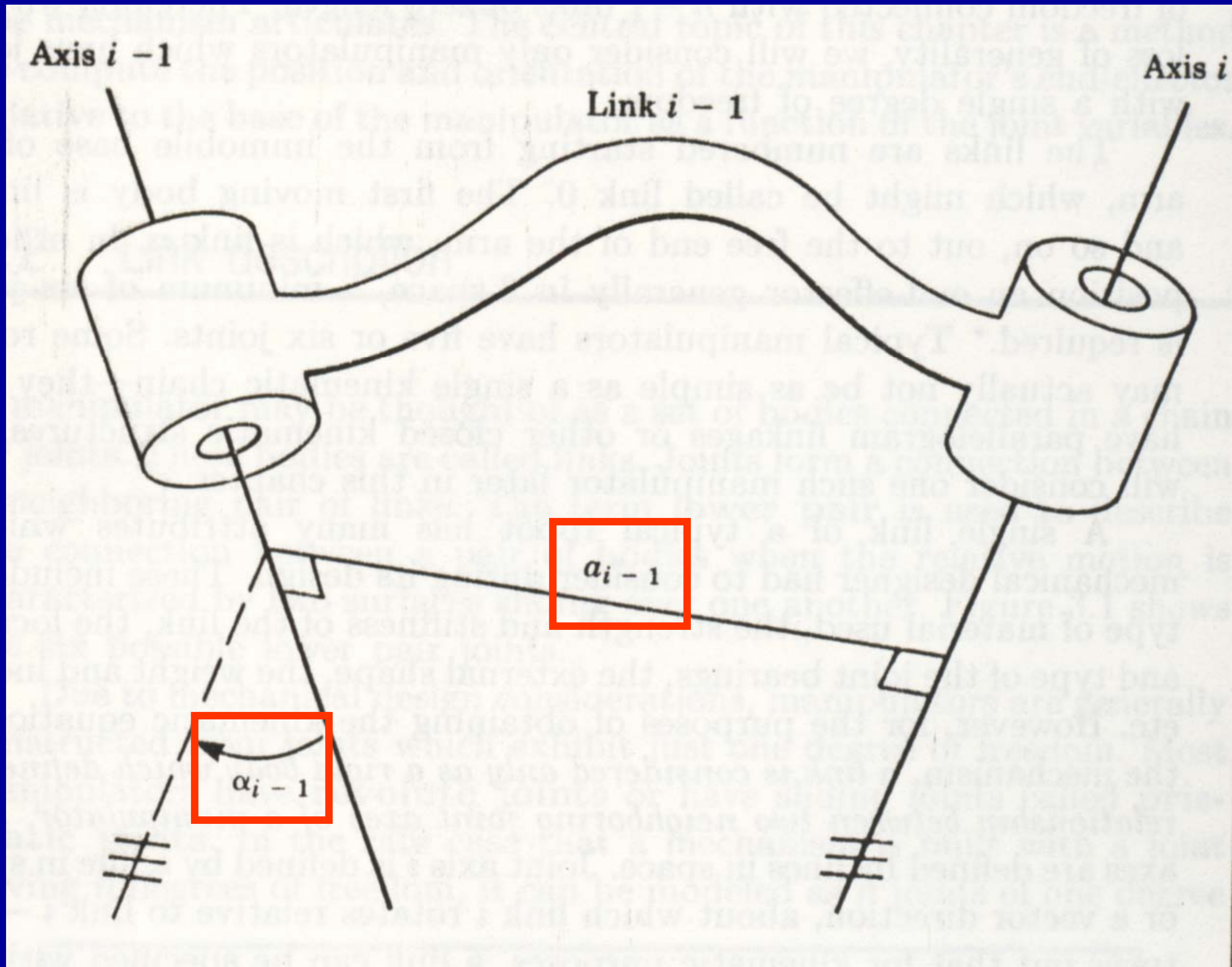
Link length

- Measured along a line which is mutually perpendicular to both axes.
- The mutually perpendicular always exists and is unique except when both axes are parallel.

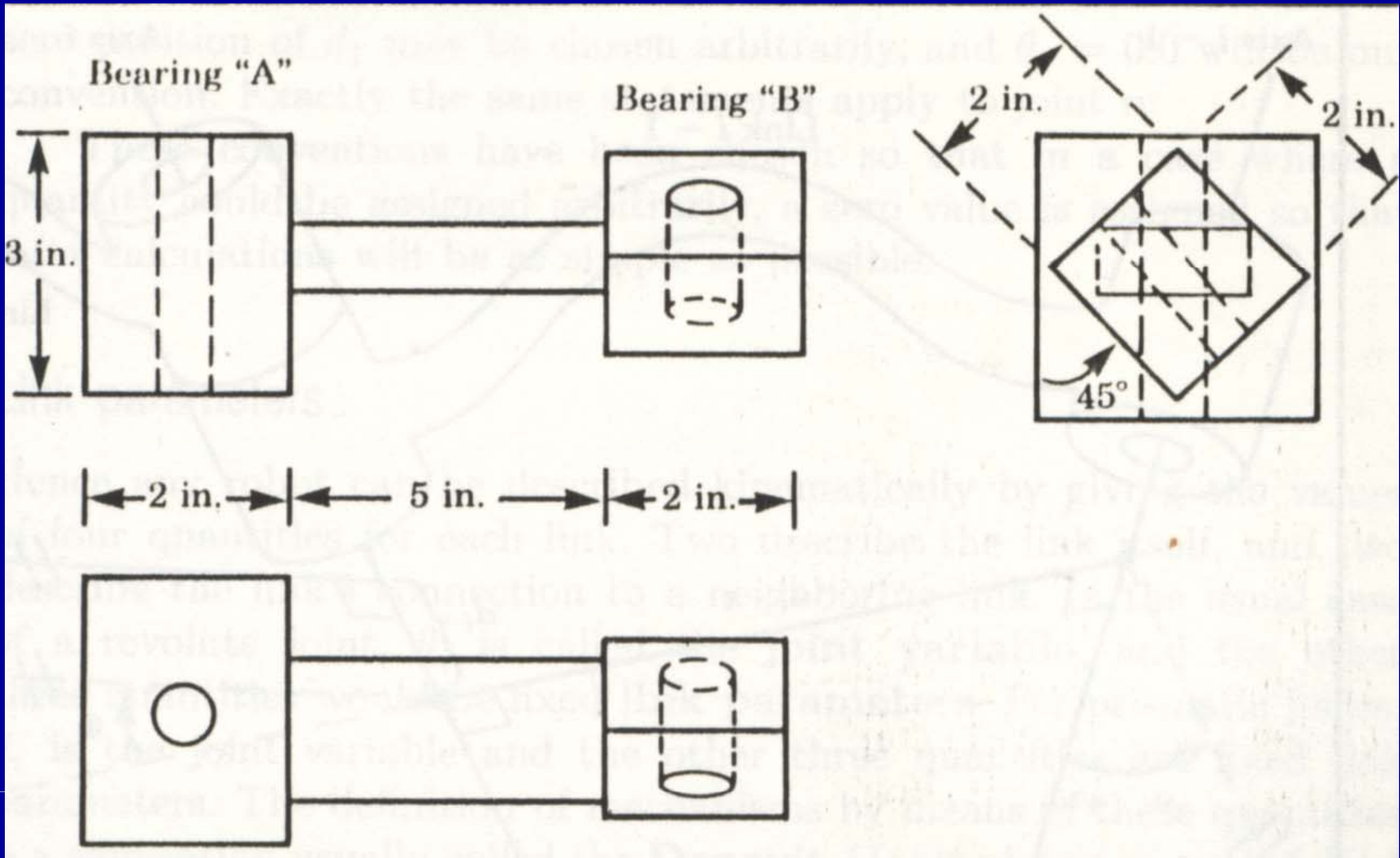
Link twist

- Project both axes *i*–1 and *i* onto the plane whose normal is the mutually perpendicular line, and measure the angle between them
- Right-hand sense

Link length and link twist



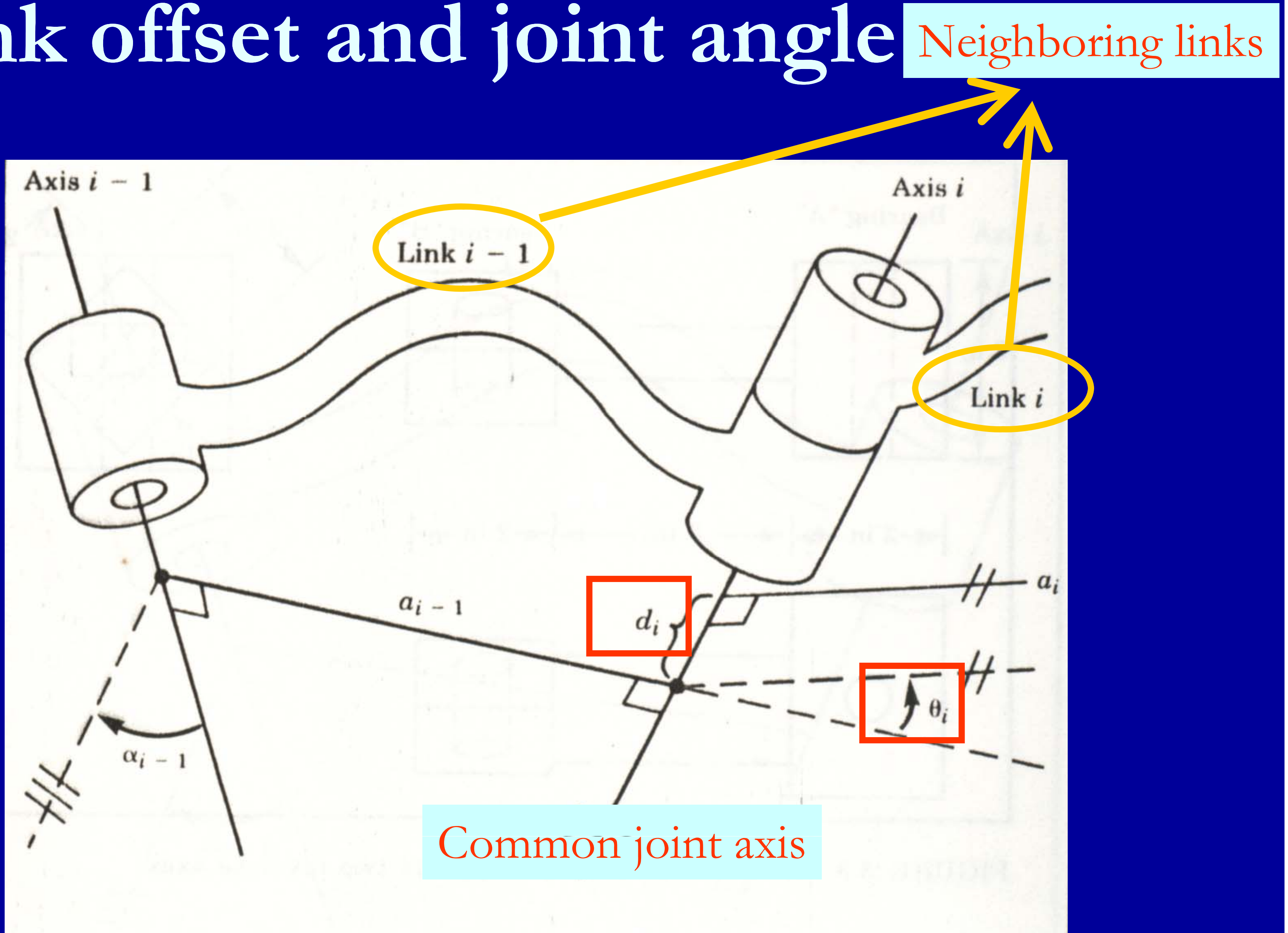
Example 3.1: length and twist



Link connection

- Link offset: the distance along the common axis from one link to the next
- Link angle: the amount of rotation about the common axis

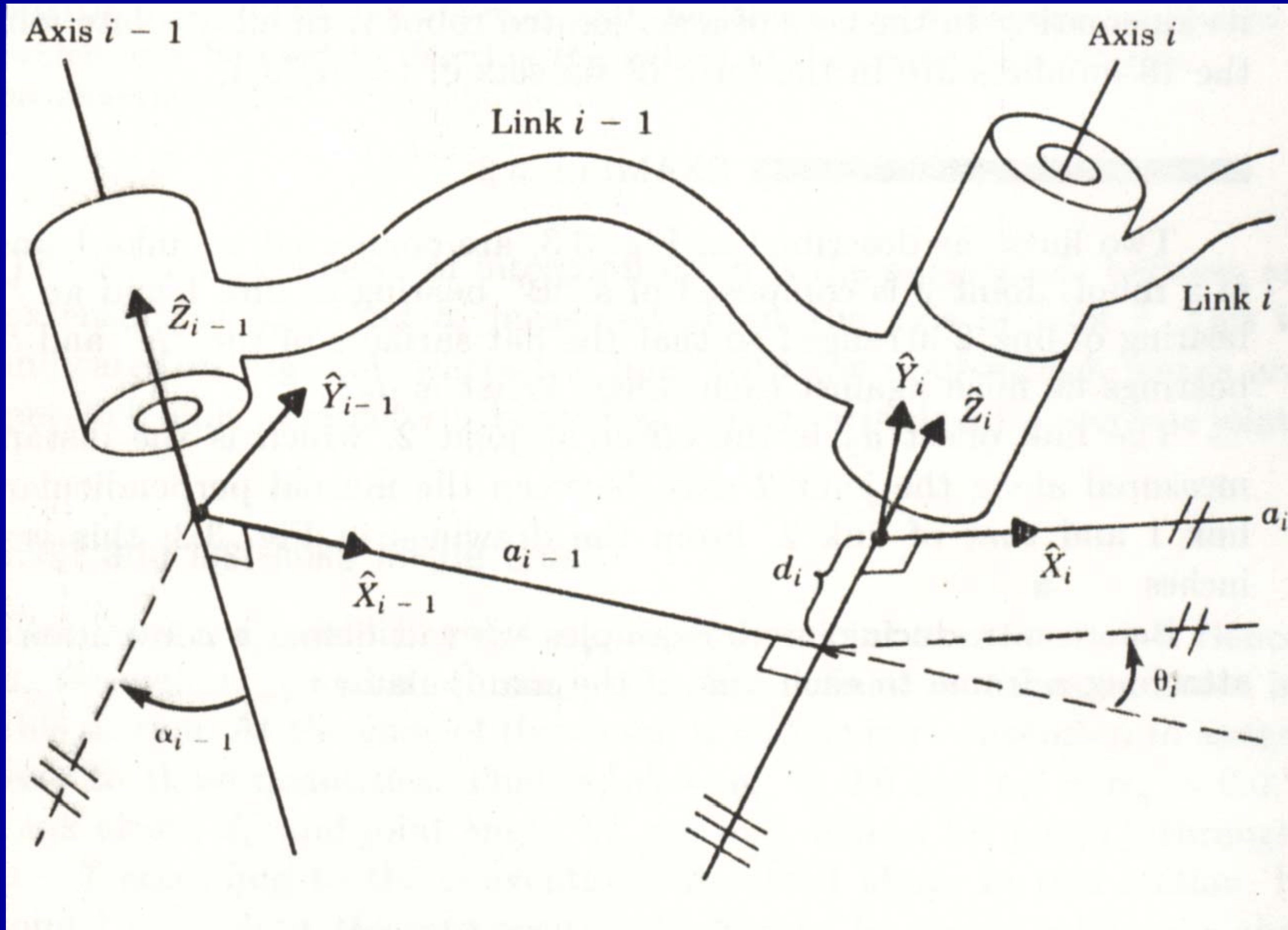
Link offset and joint angle



Denavit-Hartenberg Notation

- Any robot can be described kinematically by giving the values of four quantities for each link.
- One joint variable, three fixed link parameters

Affixing frames to links



Intermediate links in the chain

$\hat{Z}_i$ is coincident with the joint axis ***i.***

The origin of frame {i} is located where the a_i ***perpendicular intersects the joint i axis.***

$\hat{X}_i$ ***points along*** a_i ***in the direction from joint i to i+1.***

In the case of $a_i = 0$, $\hat{X}_i$ ***is normal to the plane of*** $\hat{Z}_i$ ***and*** $\hat{Z}_{i+1}$.

First and last links in the chain

Frame {0}, the base of the robot is arbitrary.

It always simplifies matters to choose $\hat{Z}_0$ ***along axis 1 and to locate frame {0} so that it coincides with frame {1} when joint variable 1 is zero.***

For joint n revolute, the direction of $\hat{X}_N$ ***is chosen so that it aligns with*** $\hat{X}_{N-1}$ ***when*** $\theta_n = 0.0$, ***and the origin of frame {N} is chosen so that*** $d_n = 0.0$.

Assign link frames so as to cause as many link parameters as possible to become zero!

The link parameters in terms of the link frames

$a_i =$ ***the distance from*** $\hat{Z}_i$ ***to*** $\hat{Z}_{i+1}$ ***measured along*** $\hat{X}_i$

$\alpha_i =$ ***the angle between*** $\hat{Z}_i$ ***and*** $\hat{Z}_{i+1}$ ***measured about*** $\hat{X}_i$

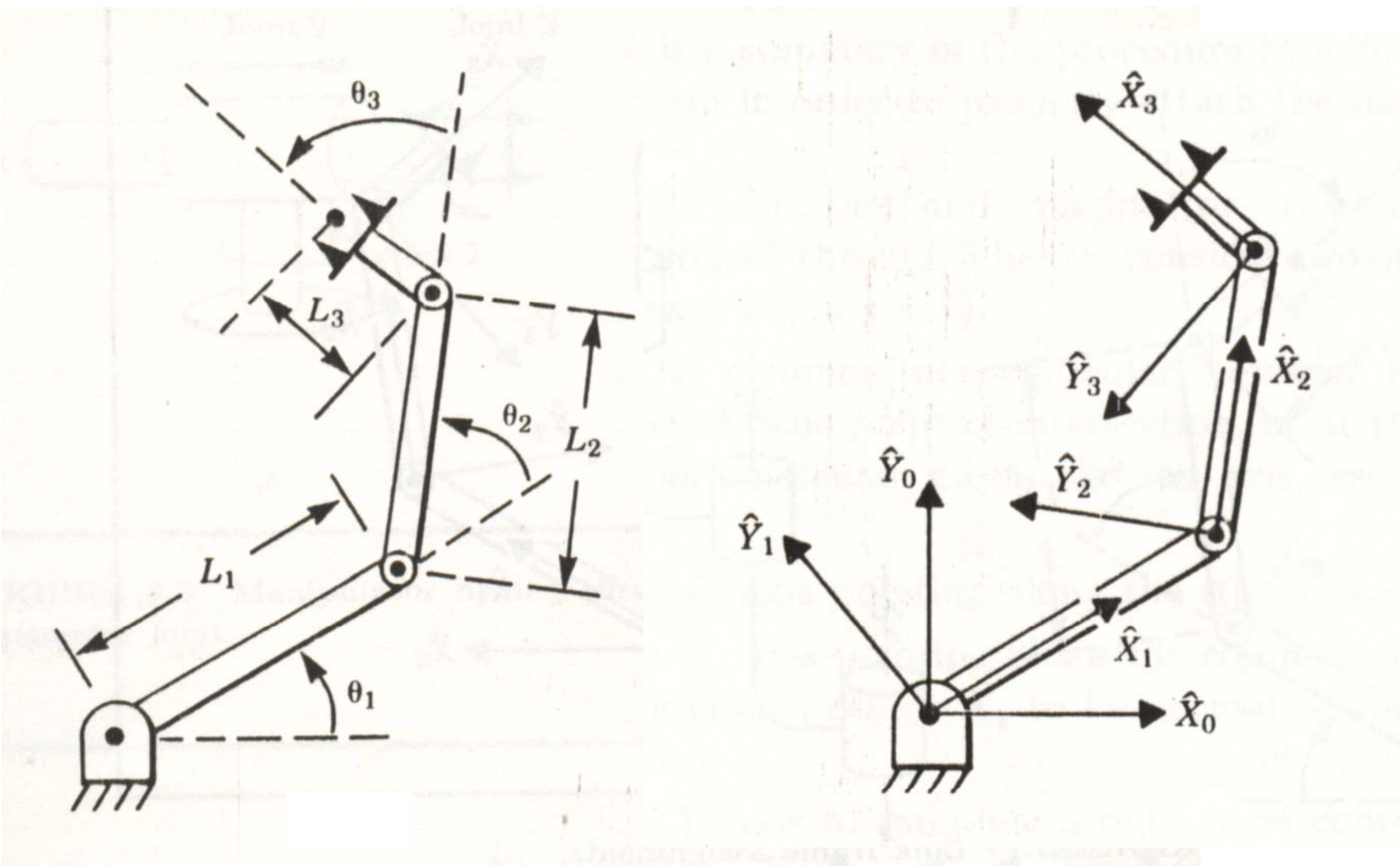
$d_i =$ ***the distance from*** $\hat{X}_{i-1}$ ***to*** $\hat{X}_i$ ***measured along*** $\hat{Z}_i$

$\theta_i =$ ***the angle between*** $\hat{X}_{i-1}$ ***to*** $\hat{X}_i$ ***measured about*** $\hat{Z}_i$

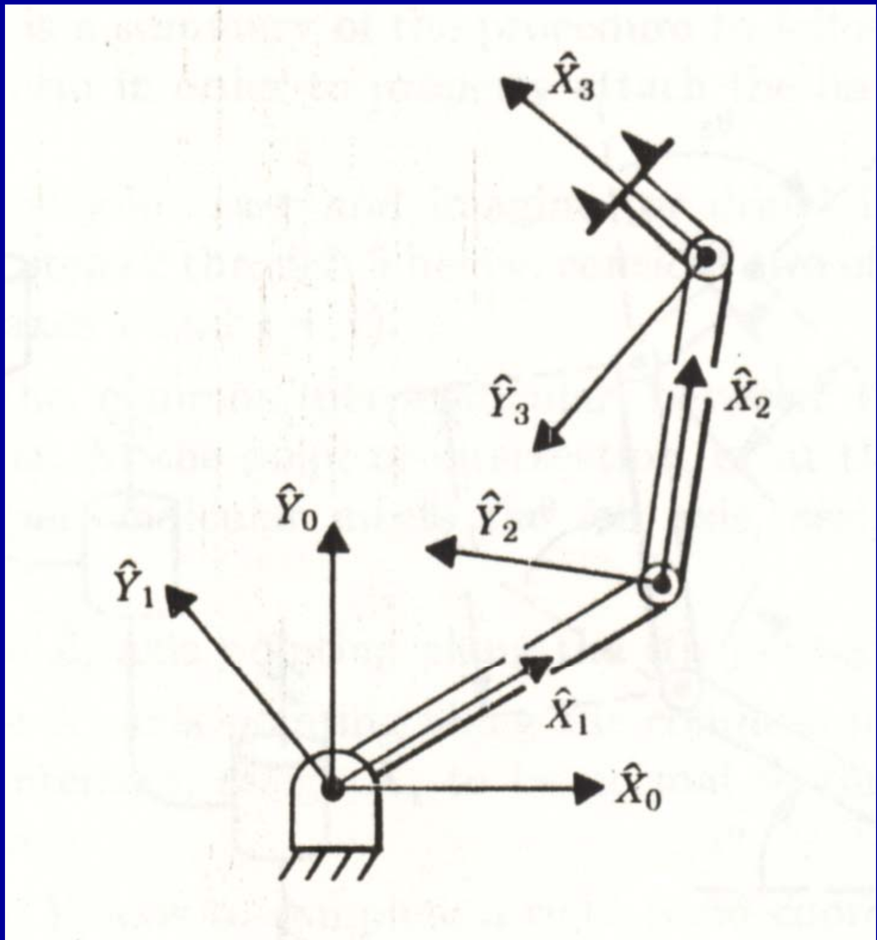
$a_i > 0$

$\alpha_i, d_i, \theta_i :$ ***signed quantities***

Example 3.3 (RRR): link-frame assignments

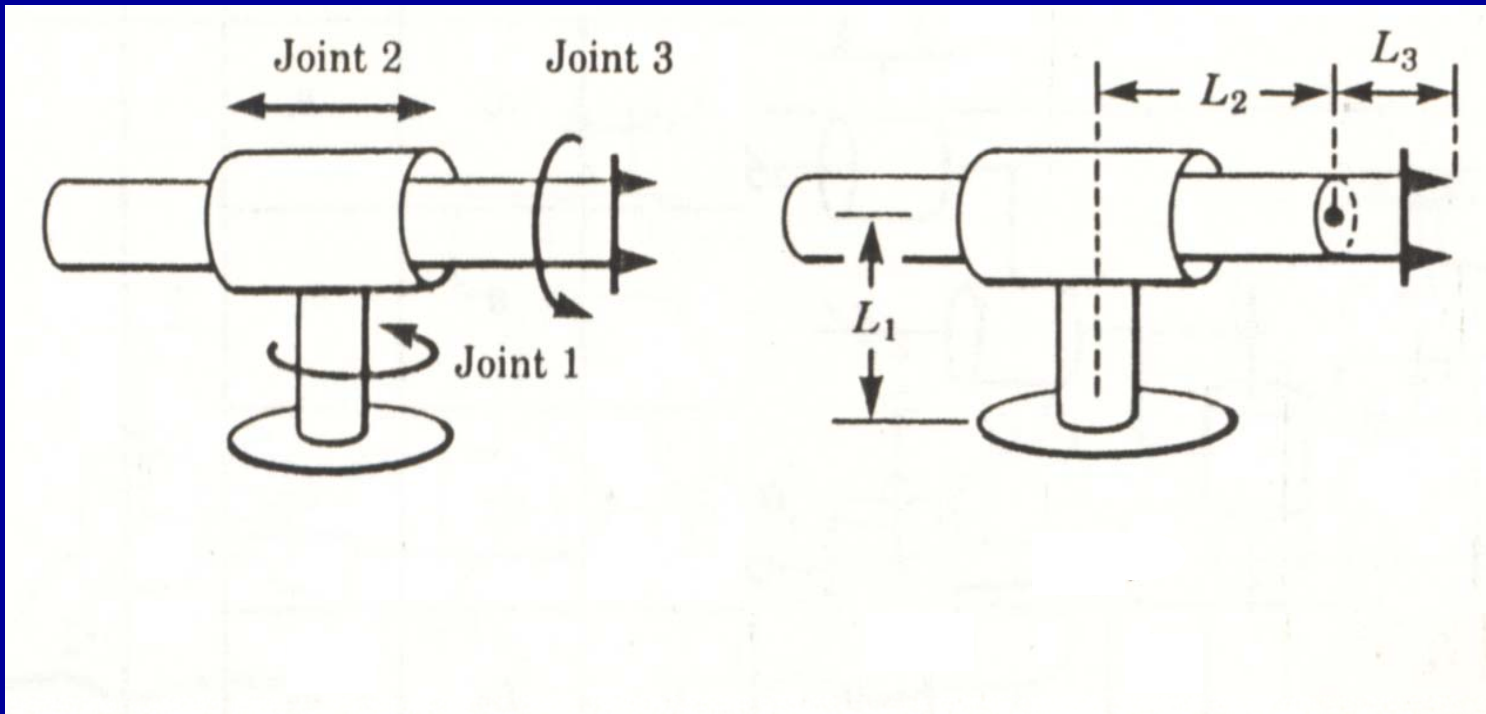


Example 3.3: link parameters



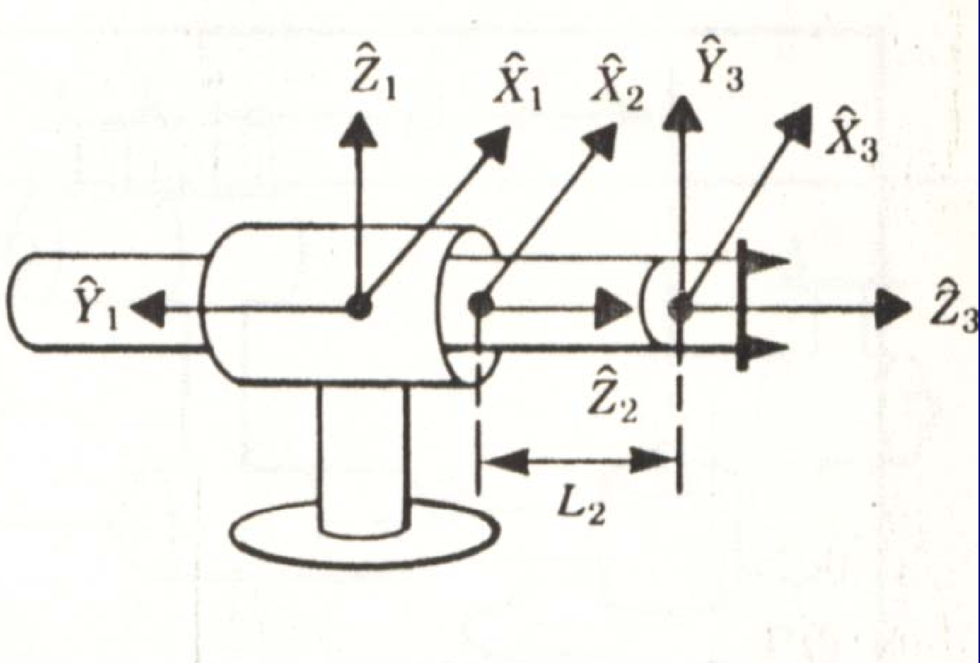
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

Example 3.4 (RPR)



Example 3.4: link frame assignments

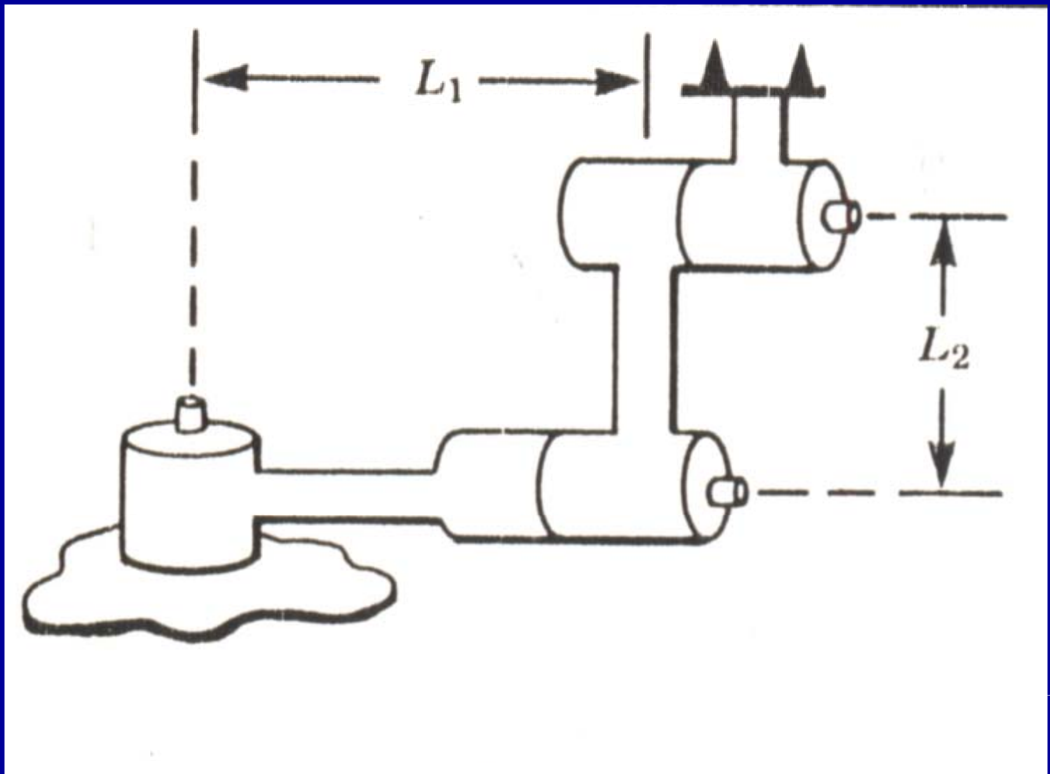
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3



(b)

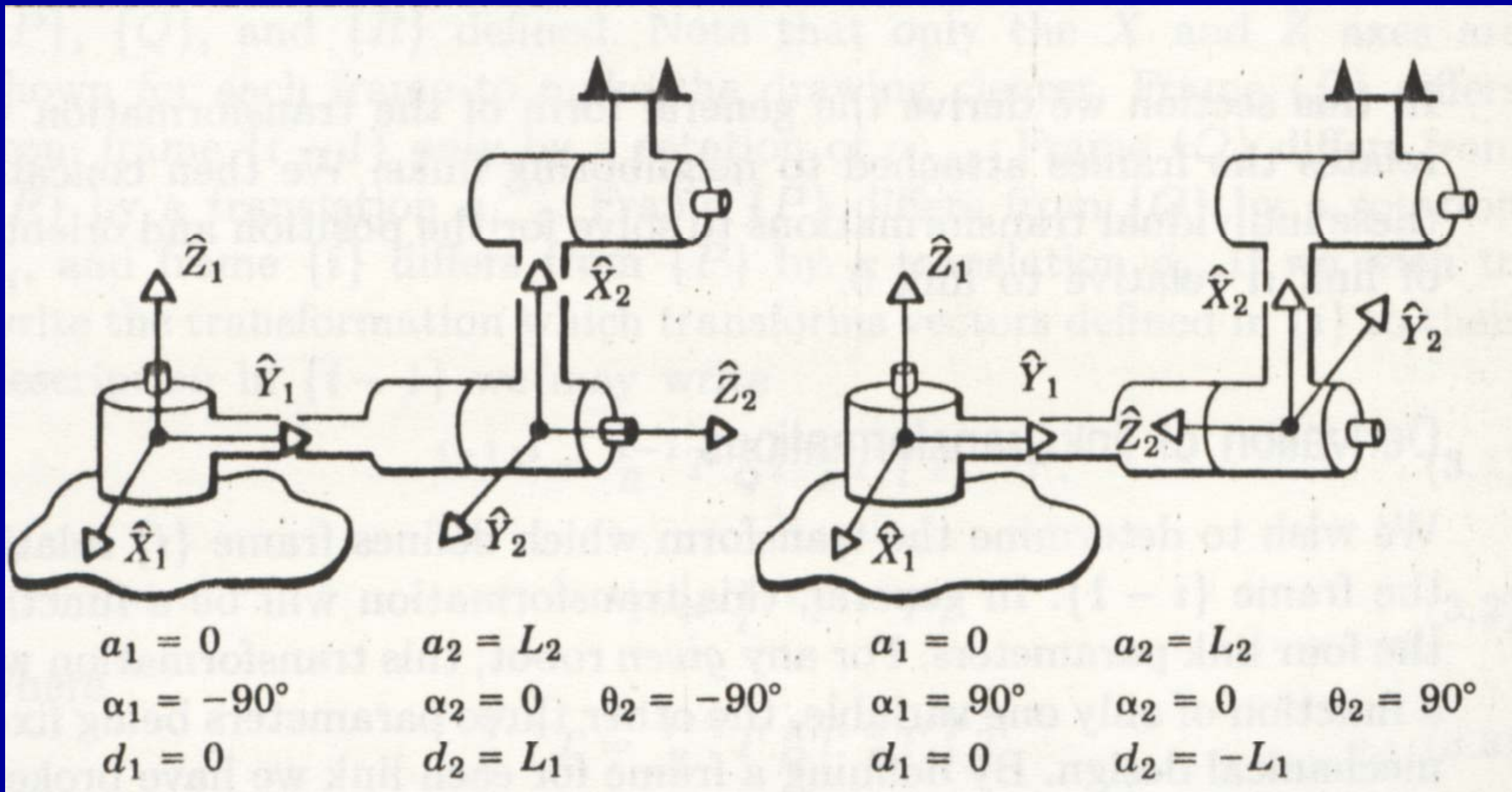
Example 3.5: non-planar manipulator

The nonuniqueness of frame assignment and of the D-H *parameters*

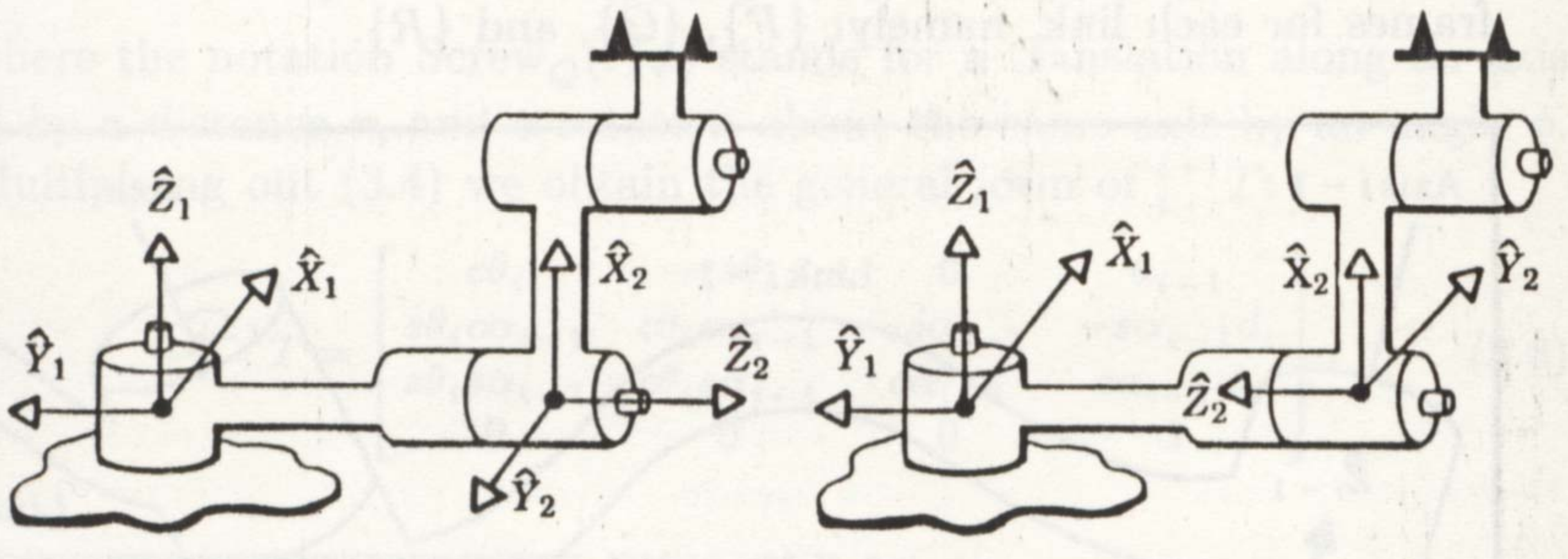


Axes 1 and 2 intersect.
Axes 2 and 3 are parallel.

Example 3.5: link frame assignments



Example 3.5: link frame assignments



$a_1 = 0$ $\quad a_2 = L_2$

$\alpha_1 = 90°$ $\quad \alpha_2 = 0$ $\quad \theta_2 = 90°$

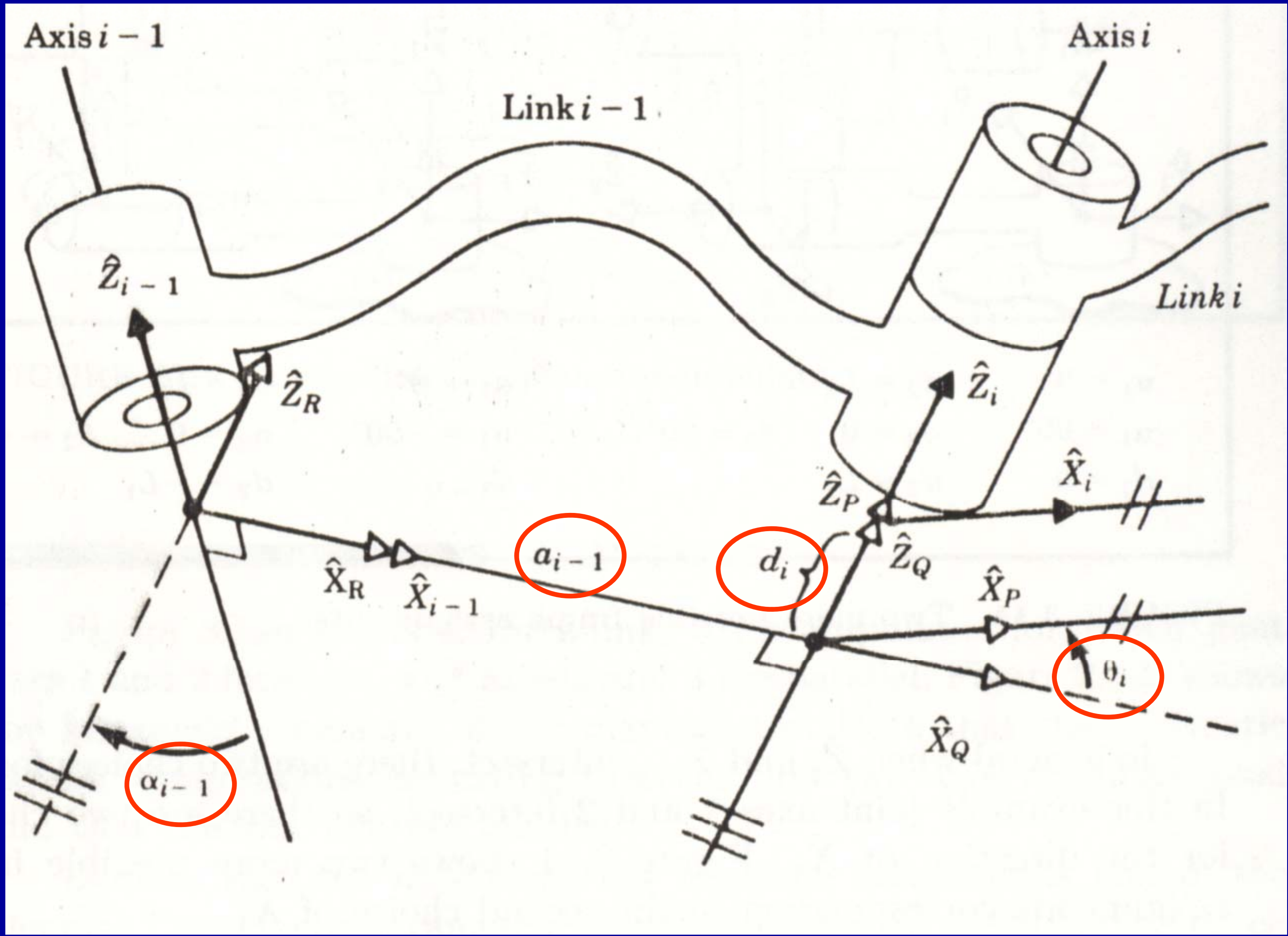
$d_1 = 0$ $\quad d_2 = L_1$

$a_1 = 0$ $\quad a_2 = L_2$

$\alpha_1 = -90°$ $\quad \alpha_2 = 0$ $\quad \theta_2 = -90°$

$d_1 = 0$ $\quad d_2 = -L_1$

Link transformations



Transform that defines frame {*i*} relative to the frame {*i*-1}

$$^{i-1}P = {}^{i-1}_{R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T\,{}^{i}P$$

$$^{i-1}_{i}T = R_X(\alpha_{i-1})D_X(a_{i-1})R_Z(\theta_i)D_Z(d_i)$$

$$= Screw_X(a_{i-1},\alpha_{i-1})Screw_Z(d_i,\theta_i)$$

$$
{}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

Example 3.6

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

$$
{}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^{1}_{2}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

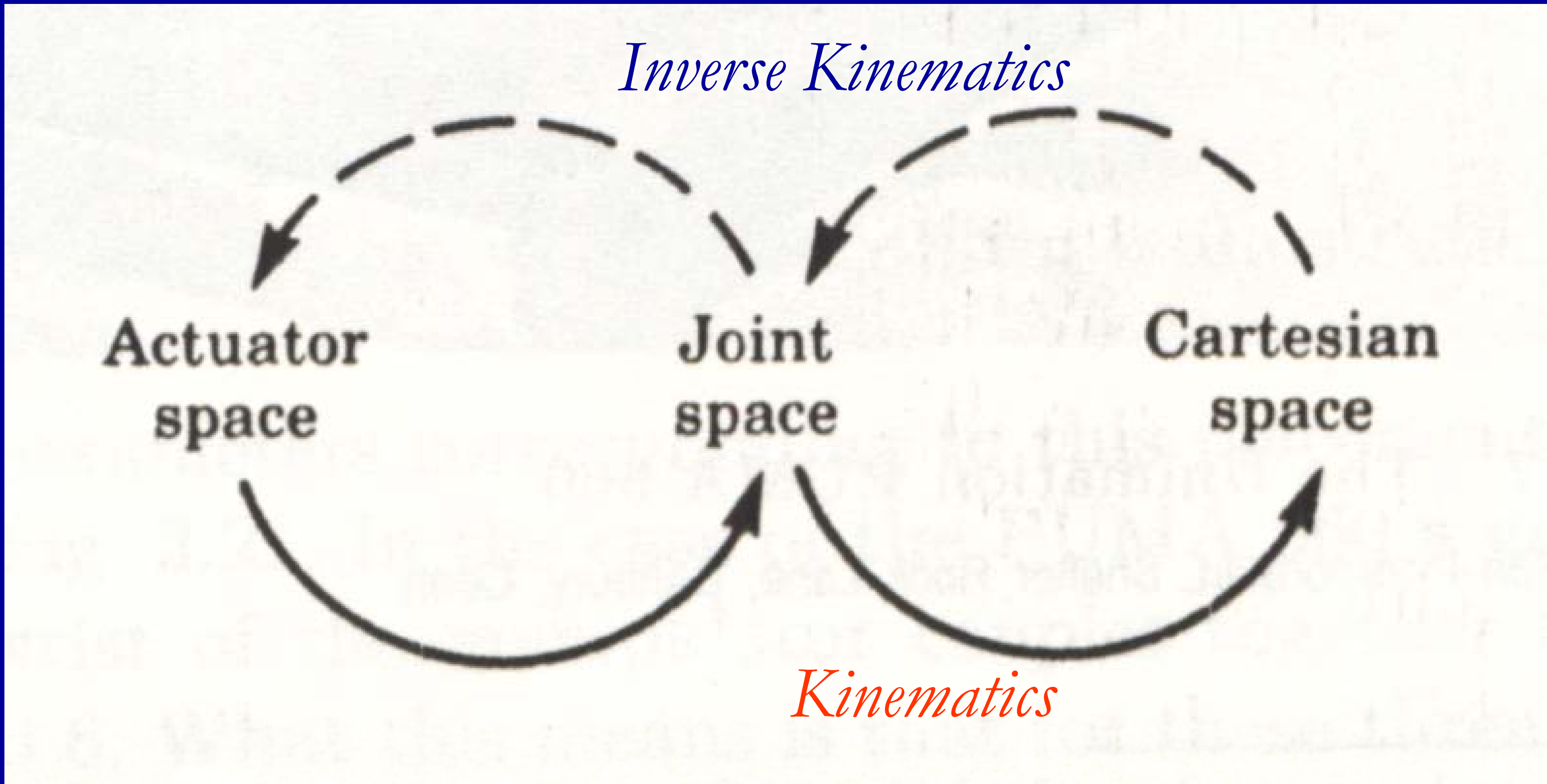
$$
{}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

Concatenating link transformations

Find the single transformation that relates frame $\{N\}$ to frame $\{0\}$

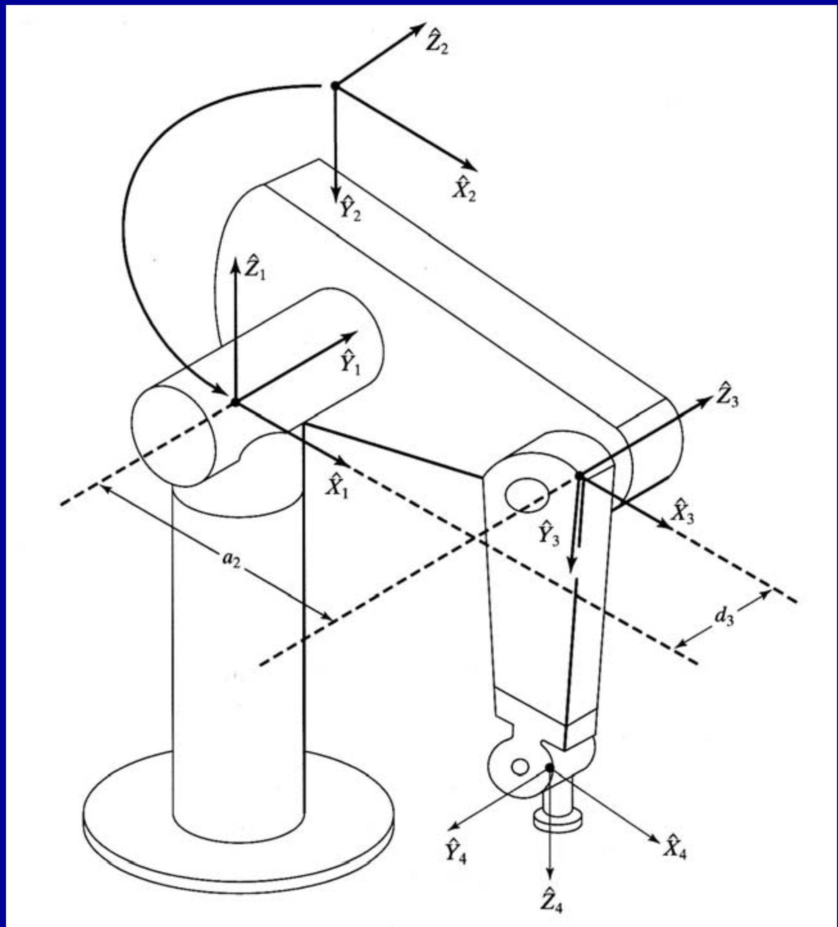
$$ {}^{0}_{N}T = {}^{0}_{1}T \quad {}^{1}_{2}T \quad {}^{2}_{3}T \quad \cdots \quad {}^{N-1}_{N}T. $$

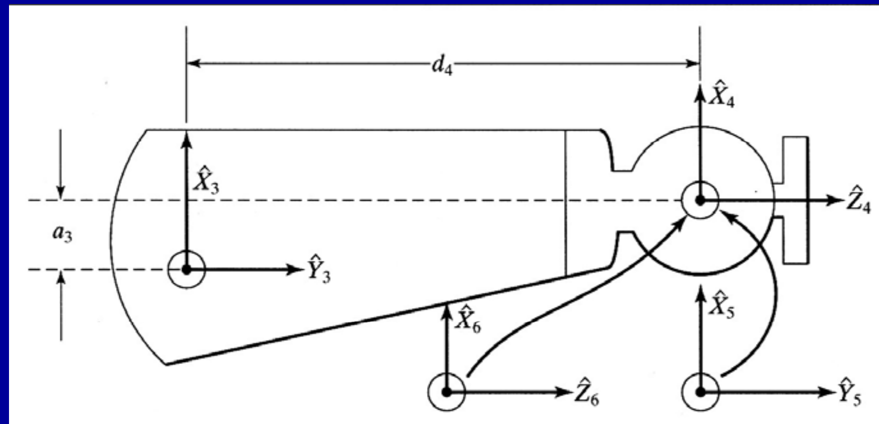
Mapping between kinematic descriptions

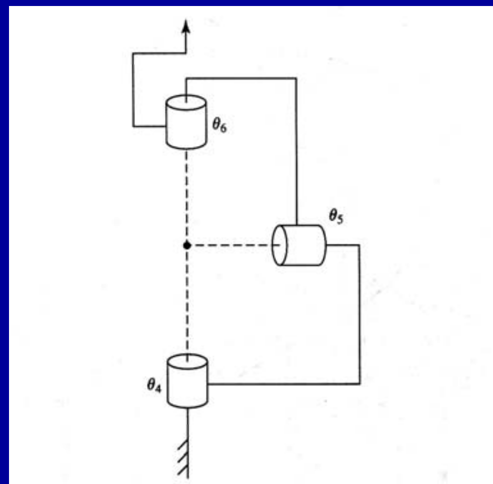


- Joint space: the spaces of all joint vectors is referred
- Cartesian space: position is measured along orthogonal axes; orientation is measured according to any of the conventions in Ch. 2
- Actuator space: the space of all actuator vectors is referred

Example (6R): The PUMA 560



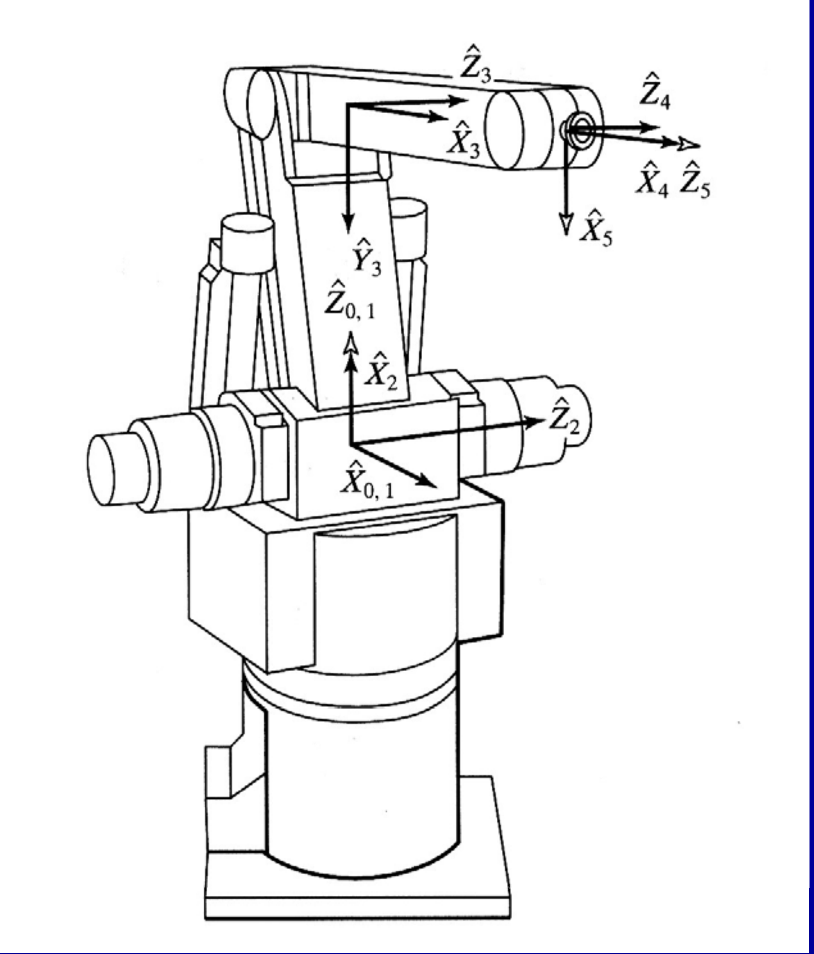




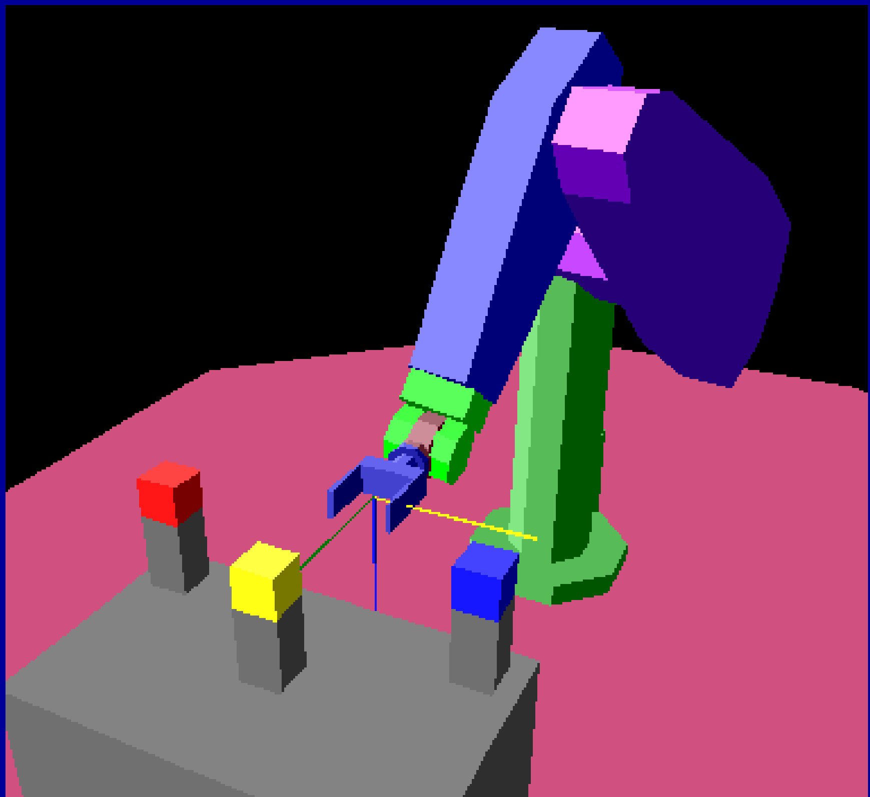
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90°	0	0	θ_2
3	0	a_2	d_3	θ_3
4	-90°	a_3	d_4	θ_4
5	90°	0	0	θ_5
6	-90°	0	0	θ_6

Example: Yasukawa Motoman L-3

(Refer to the textbook.)



Lab #1 (5 pt.) – Due Feb. 10

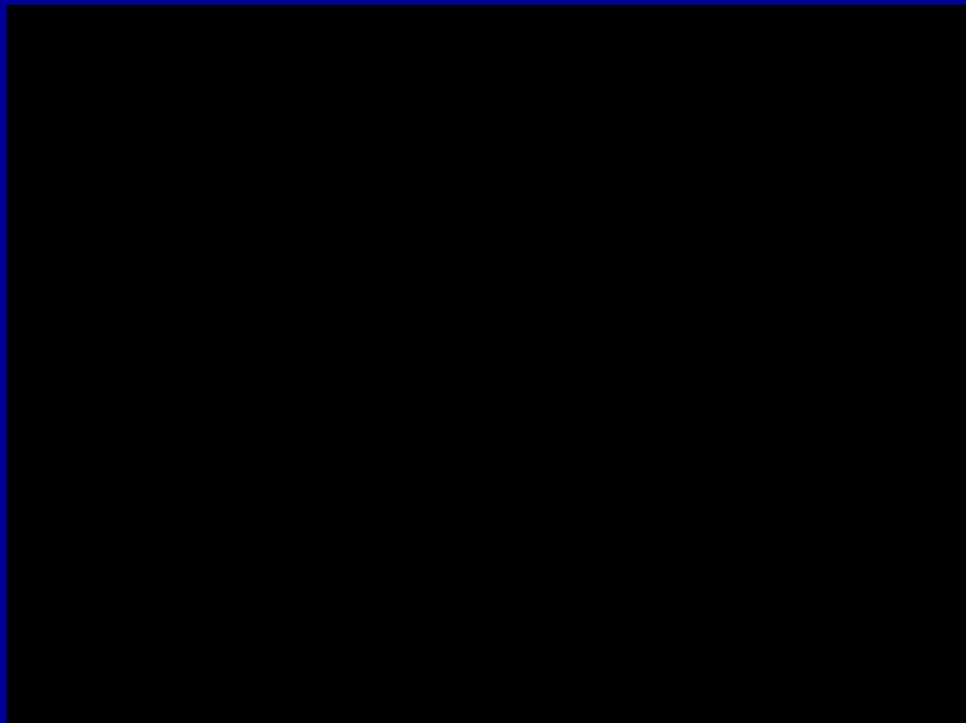


Simulate the kinematics of a manipulator (PUMA 560, PA-10) with Open Dynamics Engine (ODE).

http://opende.sourceforge.net/wiki/index.php/Main_Page ODE Wiki

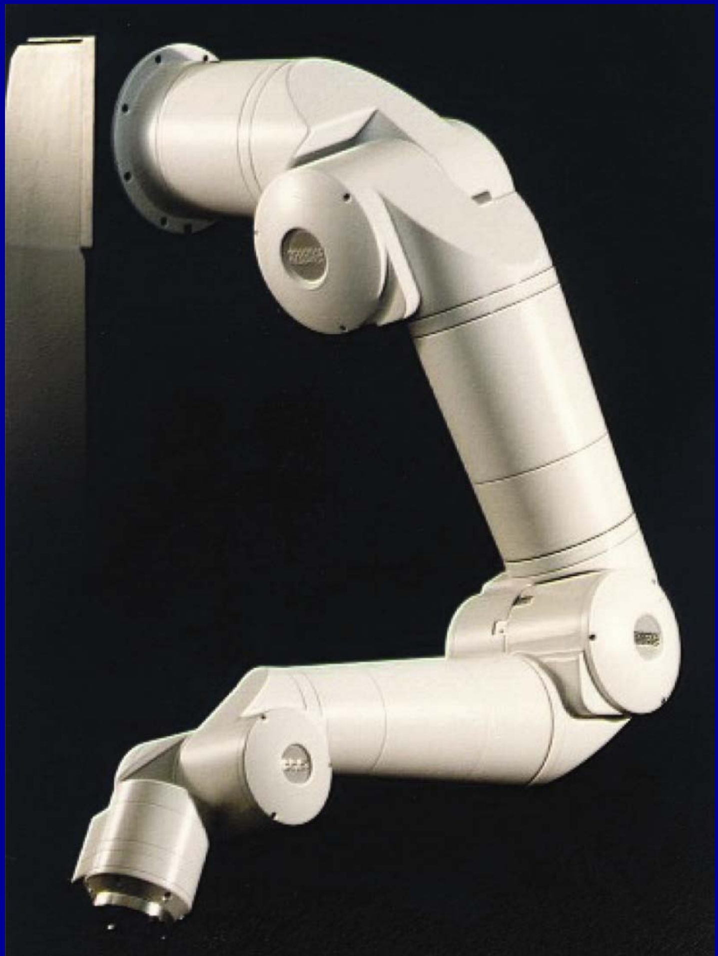
Book: Robot Simulation - Robot Programming with ODE (in *Japanese*)

RRC K-1207i (Video)



Homework #9 (1 pt.) – Due Jan. 13

Find the forward kinematics using homogeneous matrices.

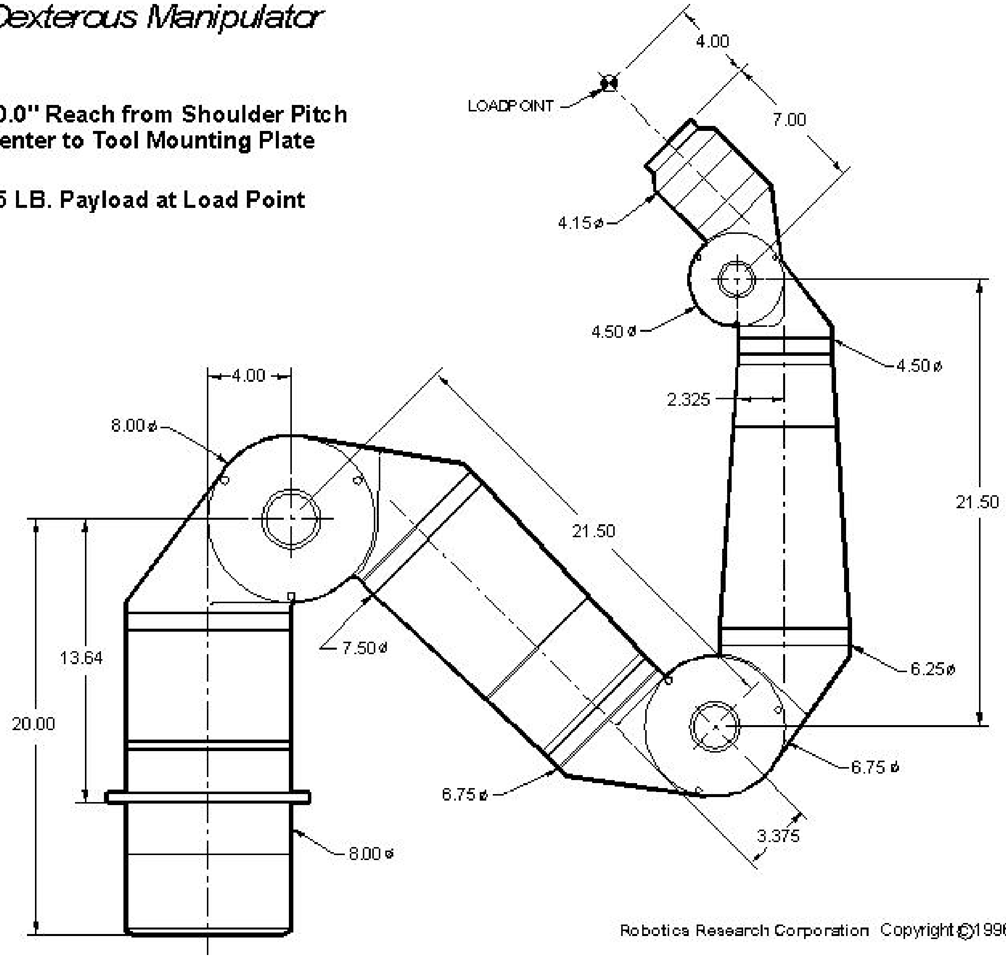


K-1207i

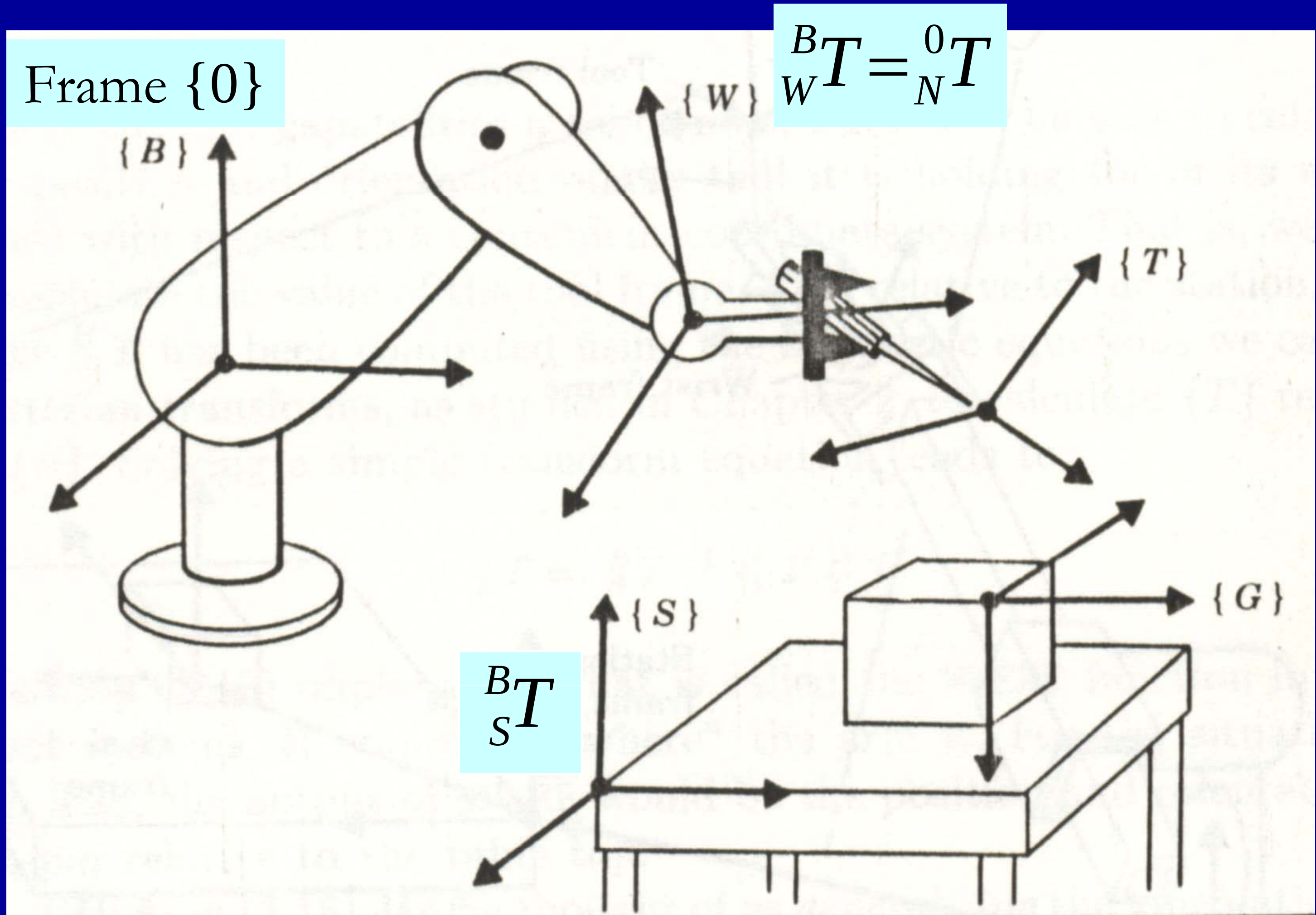
Dexterous Manipulator

50.0" Reach from Shoulder Pitch Center to Tool Mounting Plate

35 LB. Payload at Load Point



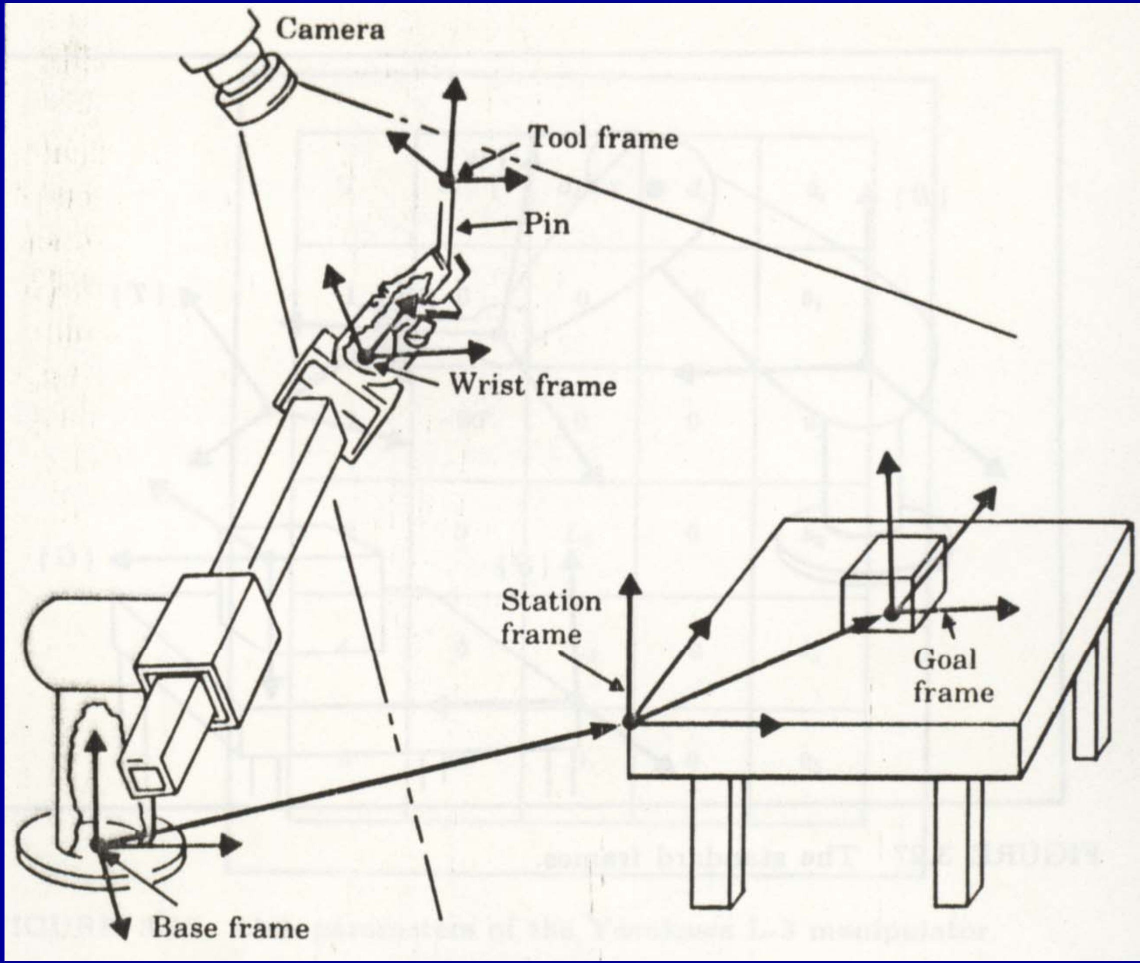
Standard Frames



Frames with Standard Names

- The base frame, {B}
- The station frame, {S}
- The wrist frame, {W}
- The tool frame, {T}
- The goal frame, {G}

Example of the assignment of standard frames



Where is the tool?

$$ {}^{S}_{T}T = {}^{B}_{S}T^{-1} \, \boxed{{}^{B}_{W}T} \, {}^{W}_{T}T $$

Manipulator kinematics

How to calculate the position and orientation of the tool w.r.t. a convenient coordinate system?

Quiz #4 – Due Dec. 21

- 3.3, 3.4, 3.8, 3.17, 3.20