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FORWARD AND INVERSE KINEMATICS OF IRb-6 MANIPULATOR

TADEUSZ SZKODNY

Institute of Automation of Silesian Technical University, Gliwice, Poland

Abstract—The paper presents equations of links and actuator kinematics of the IRb-6 manipulator in matrix form. Also solution of equations of link kinematics as well as formulae joining link and actuator natural coordinates of the manipulator have been presented.

NOMENCLATURE

$\mathbb{A}_i$—homogeneous transformation describing the relation between $i-1$st link and ith link
$\alpha_i, l_i, \lambda_i, \Theta_i$—Hartenberg–Denavit parameters
$\mathbb{E}$—homogeneous transformation describing the relation between task and (5th) working link
Φ, Θ, Ψ—external coordinates of orientation (Euler angles)
h_2, h_3, k_1, k_4, k_5—kinematic parameters of driving units of IRb-6 manipulator
$\mathbb{T}_5$—homogeneous transformation describing the relation between 5th link and base line
$\mathbb{T}_{ai}$—homogeneous transformation describing the relation between ith actuator effector and ith link
Θ'_i—natural coordinate of ith link
Θ_{ai}—natural coordinate of ith actuator
x, y, z—external coordinates of position
$x_{ai}y_{ai}z_{ai}$—coordinate system associated with ith actuator effector
$x_{bi}y_{bi}z_{bi}$—coordinate system associated with the body of the ith actuator

1. INTRODUCTION

To describe the manipulator kinematics, dekstrorotary coordinate system associated with particular components of the manipulator will be used. To describe the position and orientation of the systems homogeneous transform [1–4] will be used. Due to this transform a joint description of the position and orientation is possible, which is essential while describing kinematics, and particularly dynamics of manipulators.

Natural coordinates of links will be called those describing relative motion of adjacent links [4–6].

External space of the manipulator is described by its external coordinates of position x, y, z and orientation Φ, Θ, Ψ (z–y–z Euler angles). These coordinates describe the manipulator effector in relation to a selected relative system regardless of the manipulator kinematic structure. The manipulator internal space is described by its internal coordinates. These are natural coordinates of links and actuators [4].

Industrial robots are most often equipped with manipulators with V class kinematic pairs and only such are regarded in the paper.

The fundamental problem in industrial robots control concerns algorithms generating reference trajectories.

Papers [7–10] suggest generating algorithms of a reference trajectory, which are based on an arbitrary discretization of the manipulators internal coordinates. A discretized description of the manipulator external space results from the discretized description of its internal space. A reference trajectory in the external space is approximated using a discretized description of the manipulator external space. Each point of discretization in the external space approximating a reference trajectory is corresponded by a known discretized internal coordinate of the manipulator. The disadvantage of these algorithms is that they demand large memory, big sets, being a discretized description of the internal space, have to be searched, and there is no possibility to reduce the approximation error, resulting from arbitrary discretization of the manipulator internal space.

In papers [11–13] iteration methods of determining the internal coordinates corresponding to external coordinates of the reference trajectory point have been suggested. In this method of internal coordinates determining the point of the reference trajectory is being approached in successive steps of iterative computation. Discretization step of the internal coordinates in successive steps of iterative computation depends on the error of external coordinates in the preceding iteration step. Large memories are not demanded in iteration methods as computation is carried on only for reference trajectory approximating points. In these methods a reference trajectory approximation error may be reduced though a number of iterative computation steps is thus increased. In paper [11] a modified iterative method of a reference trajectory straight segment generating has been presented. The modification consists in arbitrary accepting of error distribution in the external space, thus reducing the number of iterative computation steps. The accepted error distribution is right with only short segments, though.

A disadvantage of iterative methods is the necessity of multiple iterative computation. Whereas the advantage of methods based on arbitrary discretization of internal spaces as well as iterative methods is the simplicity of computation which lies in using only equations of the manipulator forward kinematics. This advantage however may be a catch for those computer programmers who have not considered kinematic singularities of the manipulator [4].

Analytic formulae which are the solution of an inverse problem of manipulator kinematics enable designing of trajectory generating algorithms which compute in one step only internal coordinates of points lying exactly on the reference trajectory, with the accuracy resulting from the computer register length. These formulae make the programmers forsee alternative solutions for manipulator kinematic singularities. Analytic formulae as solutions of an inverse problem of kinematics of the 6 degrees of freedom manipulators have been presented in papers [1, 2, 14, 15]. The same formulae for the $N < 6$ degrees of freedom manipulators have been presented in papers [1, 16]. However a contrains equation of an effector link [4] has not been presented in the paper [16]. It suggests that the link is able to realize the reference trajectories with 6 degrees of freedom, which is not possible.

What follows from the foregoing review is that the kinematics models as presented in papers [7–13, 16] do not allow to design accurate and at the same time fast reference trajectory generating algorithms with defined kinematics for the under 6 degrees of freedom manipulators.

In the second paragraph equations of forward kinematics of IRb-6 manipulator links have been presented. The third paragraph contains formulae being the solution of inverse problem of links kinematics. Then follow equations of actuator kinematics which are vital for dynamic analysis of the IRb-6 manipulator. The fifth paragraph presents the example illustrating the usage of the formulae presented in Section 3. The sixth section contains the conclusion.

2. EQUATIONS OF LINKS KINEMATICS

IRb-6 manipulator (Fig. 1) has 5 links joined by rotational kinematic pairs. Figure 2 shows a homogeneous transform graph describing the manipulator kinematics. Number of links in Fig. 1 have been circled. Coordinate systems have been associated with links after Hartenberg–Denavit notation. Hartenberg–Denavit parameters describing this manipulator are shown in Table 1 [4].

The following modification of angles will be introduced to facilitate solution of the inverse problem of kinematics [4]:

$$\Theta_1' = \Theta_1 - 90^\circ, \quad \Theta_2' = \Theta_2 - 90^\circ, \quad \Theta_3' = \Theta_3 + 90^\circ, \quad \Theta_4' = \Theta_4 - 90^\circ, \quad \Theta_5' = \Theta_5. \tag{1}$$

Table 1.

Link number	α_i [°]	l_i [m]	λ_i [m]	Θ_i [°]
1	90	0	0.70	90–430
2	0	0.45	0	50–130
3	0	0.67	0	−130–−50
4	90	0	0	−25–−220
5	0	0	0.095	$\Delta\Theta_5 = 360$

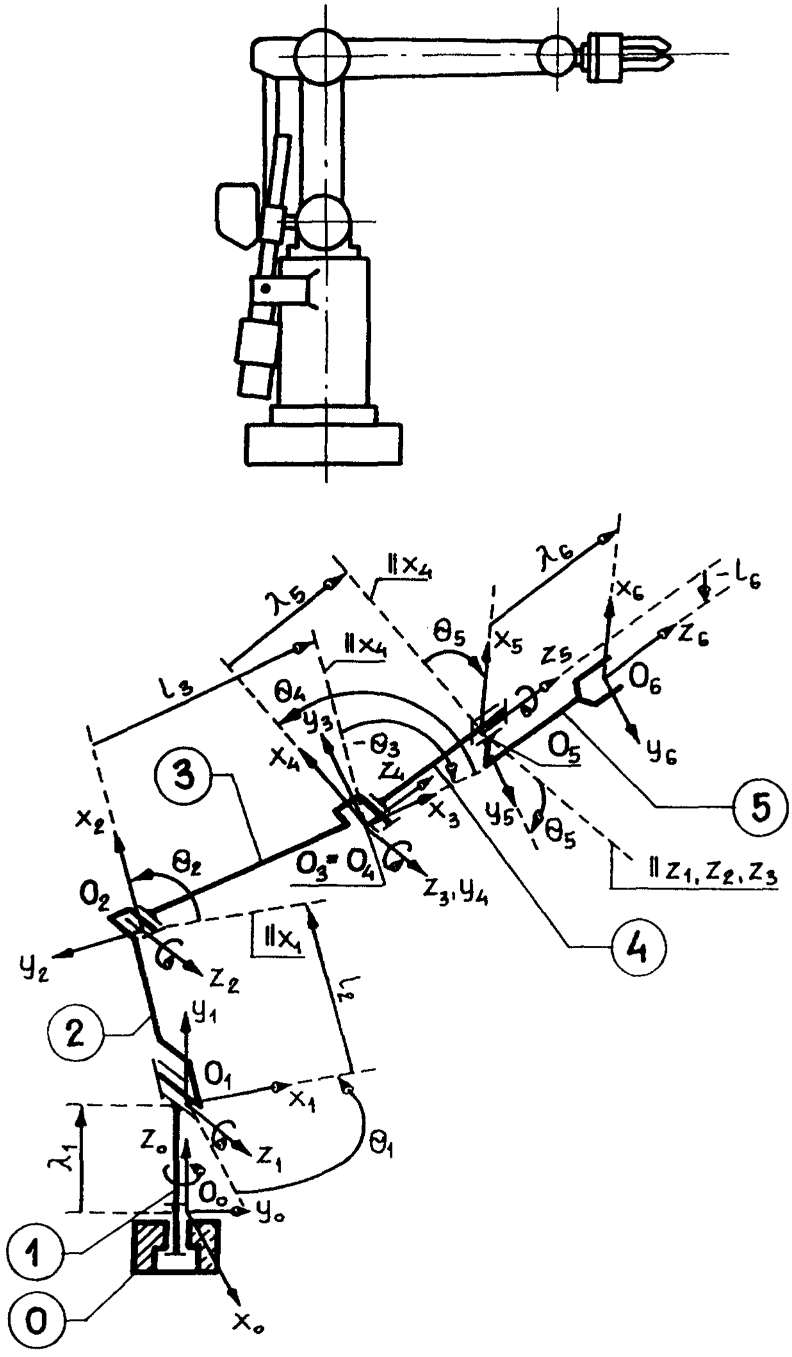

Fig. 1. IRb-6 robot manipulator.

Ranges of change of these angles are as follows [4]:

$$0^\circ \leqslant \Theta_1' \leqslant 340^\circ,$$

$$-40^\circ \leqslant \Theta_2' \leqslant 40^\circ,$$

$$-40^\circ - \Theta_2' \leqslant \Theta_3' \leqslant 40^\circ \quad \text{for } -40^\circ \leqslant \Theta_2' \leqslant 15^\circ,$$

$$-40^\circ - \Theta_2' \leqslant \Theta_3' \leqslant 25^\circ - \Theta_2' \quad \text{for } -15^\circ \leqslant \Theta_2' \leqslant 0^\circ,$$

$$-40^\circ \leqslant \Theta_3' \leqslant 25^\circ - \Theta_2' \quad \text{for } 0^\circ \leqslant \Theta_2' \leqslant 40^\circ,$$

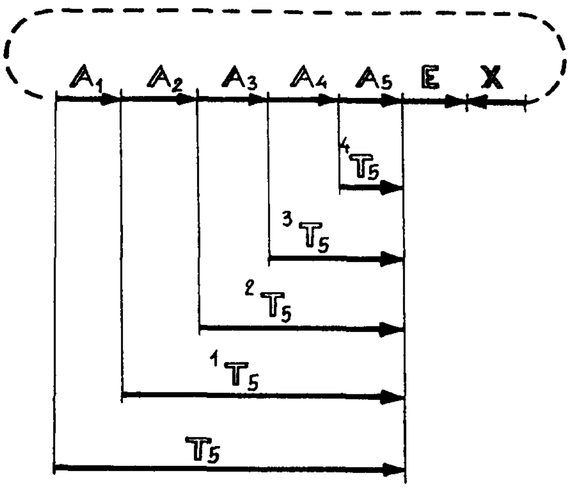

Fig. 2. Homogeneous transform graph of IRb-6 manipulator.

$$-90^\circ - \Theta_2' - \Theta_3' \leqslant \Theta_4' \leqslant 90^\circ - \Theta_2' - \Theta_3',$$

$$-270^\circ + k_5^{-1}(\Theta_2' + \Theta_3' + \Theta_4') \leqslant \Theta_5' \leqslant 90^\circ + k_5^{-1}(\Theta_2' + \Theta_3' + \Theta_4'), \quad k_5^{-1} = 32/19. \tag{2}$$

To simplify the notation, the following designation will be used:

$$\sin \Theta_i' = S_i, \quad \cos \Theta_i' = C_i, \quad \sin(\Theta_i' + \Theta_j') = S_{ij}, \quad \cos(\Theta_i' + \Theta_j') = C_{ij} \text{ etc.}$$

In further consideration it will be assumed that the angles Θ_1'–Θ_5' are natural coordinates of links.

$\mathbb{A}_1$–$\mathbb{A}_5$ and $\mathbb{E}$ transform matrices have the following forms [4]:

$$\mathbb{A}_1 = \begin{bmatrix} -S_1 & 0 & C_1 & 0 \\ C_1 & 0 & S_1 & 0 \\ 0 & 1 & 0 & \lambda_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbb{A}_2 = \begin{bmatrix} -S_2 & -C_2 & 0 & -l_2 S_2 \\ C_2 & -S_2 & 0 & l_2 C_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbb{A}_3 = \begin{bmatrix} S_3 & C_3 & 0 & l_3 S_3 \\ -C_3 & S_3 & 0 & -l_3 C_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbb{A}_4 = \begin{bmatrix} -S_4 & 0 & C_4 & 0 \\ C_4 & 0 & S_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbb{A}_5 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & \lambda_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbb{E} = \begin{bmatrix} 1 & 0 & 0 & l_6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda_6 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{3}$$

$\mathbb{T}_5$ and X matrix describing the wrist of an effector link and task as shown in Fig. 1 the following forms [4]:

$$\mathbb{T}_5 = \begin{bmatrix} S_1 S_{234} C_5 + C_1 S_5 & -S_1 S_{234} S_5 + C_1 C_5 & -S_1 C_{234} & l_2 S_1 S_2 - l_3 S_1 C_{23} - \lambda_5 S_1 C_{234} \\ -C_1 S_{234} C_5 + S_1 S_5 & C_1 S_{234} S_5 + S_1 C_5 & C_1 C_{234} & -l_2 C_1 S_2 + l_3 C_1 C_{23} + \lambda_5 C_1 C_{234} \\ C_{234} C_5 & -C_{234} S_5 & S_{234} & \lambda_1 + l_2 C_2 + l_3 S_{23} + \lambda_5 S_{234} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4a}$$

$$\mathbb{X} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{4b}$$

where n_x–n_z, o_x–o_z, a_x–a_z are identical with $\mathbb{T}_5$ matrix elements. Elements in the last column are the following:

$$p_x = l_2 S_1 S_2 - l_3 S_1 C_{23} - \lambda_5 S_1 C_{234} + l_6(S_1 S_{234} C_5 + C_1 S_5) - \lambda_6 S_1 C_{234},$$
$$p_y = -l_2 C_1 S_2 + l_3 C_1 C_{23} + \lambda_5 C_1 C_{234} + l_6(-C_1 S_{234} C_5 + S_1 S_5) + \lambda_6 C_1 C_{234},$$
$$p_z = \lambda_1 + l_2 C_2 + l_3 S_{23} + \lambda_5 S_{234} + l_6 C_{234} C_5 + \lambda_6 S_{234}.$$

$\mathbb{T}_5$ and $\mathbb{X}$ matrices enable both solving of the forward problem of kinematics of the manipulator and determining of the work space [17].

3. SOLUTION OF LINK KINEMATICS EQUATIONS

The solution of the inverse problem of IRb-6 manipulator kinematics will be expressed by means of elements of $\mathbb{T}_{5\text{ref}} = \mathbb{X}_{\text{ref}}\mathbb{E}^{-1}$ matrix whose form is:

$$\mathbb{T}_{5\text{ref}} = \mathbb{X}_{\text{ref}}\mathbb{E}^{-1} = \text{Trans}(x_{\text{ref}}, y_{\text{ref}}, z_{\text{ref}})\text{Euler}(\Phi_{\text{ref}}, \Theta_{\text{ref}}, \Psi_{\text{ref}})\mathbb{E}^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 & x_{\text{ref}} \\ 0 & 1 & 0 & y_{\text{ref}} \\ 0 & 0 & 1 & z_{\text{ref}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Phi_{\text{ref}}\cos\Theta_{\text{ref}}\cos\Psi_{\text{ref}} - \sin\Phi_{\text{ref}}\sin\Psi_{\text{ref}} & -\cos\Phi_{\text{ref}}\cos\Theta_{\text{ref}}\sin\Psi_{\text{ref}} - \sin\Phi_{\text{ref}}\cos\Psi_{\text{ref}} & \cos\Phi_{\text{ref}}\sin\Theta_{\text{ref}} & 0 \\ \sin\Phi_{\text{ref}}\cos\Theta_{\text{ref}}\cos\Psi_{\text{ref}} + \cos\Phi_{\text{ref}}\sin\Psi_{\text{ref}} & -\sin\Phi_{\text{ref}}\cos\Theta_{\text{ref}}\sin\Psi_{\text{ref}} + \cos\Phi_{\text{ref}}\cos\Psi_{\text{ref}} & \sin\Phi_{\text{ref}}\sin\Theta_{\text{ref}} & 0 \\ -\sin\Theta_{\text{ref}}\cos\Psi_{\text{ref}} & \sin\Theta_{\text{ref}}\sin\Psi_{\text{ref}} & \cos\Theta_{\text{ref}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -l_6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\lambda_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$n_x = \cos\Phi_{\text{ref}}\cos\Theta_{\text{ref}}\cos\Psi_{\text{ref}} - \sin\Phi_{\text{ref}}\sin\Psi_{\text{ref}},$$
$$n_y = -\sin\Phi_{\text{ref}}\cos\Theta_{\text{ref}}\cos\Psi_{\text{ref}} + \cos\Phi_{\text{ref}}\sin\Psi_{\text{ref}},$$
$$n_z = -\sin\Theta_{\text{ref}}\cos\Psi_{\text{ref}},$$
$$o_x = -\cos\Phi_{\text{ref}}\cos\Theta_{\text{ref}}\sin\Psi_{\text{ref}} - \sin\Phi_{\text{ref}}\cos\Psi_{\text{ref}},$$
$$o_y = -\sin\Phi_{\text{ref}}\cos\Theta_{\text{ref}}\sin\Psi_{\text{ref}} + \cos\Phi_{\text{ref}}\cos\Psi_{\text{ref}},$$
$$o_z = \sin\Theta_{\text{ref}}\sin\Psi_{\text{ref}},$$
$$a_x = \cos\Phi_{\text{ref}}\sin\Theta_{\text{ref}},$$
$$a_y = \sin\Phi_{\text{ref}}\sin\Theta_{\text{ref}}$$
$$a_z = \cos\Theta_{\text{ref}},$$
$$p_x = x_{\text{ref}} - n_x l_6 - a_x\lambda_6, \quad p_y = y_{\text{ref}} - n_y l_6 - a_y\lambda_6, \quad p_z = z_{\text{ref}} - n_z l_6 - a_z\lambda_6, \tag{5}$$

where: x_{ref}, y_{ref}, z_{ref}, Φ_{ref}, Θ_{ref}, Ψ_{ref} are the task required external coordinates and l_6, λ_6 are the task kinematic parameters.

The IRb-6 manipulator has 5 degrees of freedom and this implies one constrains equation of the effector link wrist. The kinematic structure of the manipulator hinders the rotation of effector link around the z_0 axis of the base coordinate system [4], and this is represented by the equation:

$$a_x p_y - a_y p_x = 0 \tag{6}$$

which must be satisfied by elements of $\mathbb{T}_{5ref}$ matrix in each point of the reference trajectory. It is one of the necessary conditions for the reference trajectory to be realized. The formulae being the solution of the inverse problem of kinematics for the $\mathbb{T}_{5ref}$ matrix in the form (5) are as follows [4]:

$$\Theta_1' = \begin{cases} \Theta_1^* & \text{for } p_x \leqslant p_y \geqslant 0, \\ \Theta_1^* + 180^\circ & \text{for } p_y < 0, \\ \Theta_1^* + 360^\circ & \text{for } p_x > 0 i p_y \geqslant 0, \end{cases}$$

$$\Theta_1^* = \text{arc tg}\left(\frac{-p_x}{p_y}\right). \tag{7a}$$

$$\Theta_3' = \text{arc tg}\frac{S_3}{C_3}, \quad S_3 = \frac{w_1^2 + w_2^2 - (l_2^2 + l_3^2)}{2 l_2 l_3}, \quad C_3 = (1 - S_3^2)^{1/2}. \tag{7b}$$

$$\Theta_2' = \text{arc tg}\frac{S_2}{C_2}, \quad S_2 = \frac{w_2 l_3 C_3 - w_1 (l_3 S_3 + l_2)}{l_3^2 C_3^2 + (l_3 S_3 + l_2)^2}, \quad C_2 = \frac{w_1 l_3 C_3 + w_2 (l_3 S_3 + l_2)}{l_3^2 C_3^2 + (l_3 S_3 + l_2)^2},$$

$$w_1 = -S_1 p_x + C_1 p_y + \lambda_5 S_1 a_x - \lambda_5 C_1 a_y,$$

$$w_2 = p_2 - \lambda_1 - \lambda_5 a_z. \tag{7c}$$

$$\Theta_{34}' = \begin{cases} \Theta_{34}^* & \text{for } \lambda_5 C_{34} \geqslant 0, \\ \Theta_{34}^* + 180^\circ & \text{for } \lambda_5 S_{34} > 0 i \lambda_5 C_{34} < 0, \\ \Theta_{34}^* - 180^\circ & \text{for } \lambda_5 S_{34} < 0 i \lambda_5 C_{34} < 0, \end{cases}$$

$$\lambda_5 S_{34} = S_1 S_2 p_x - C_1 S_2 p_y + C_2 p_z - \lambda_1 C_2 - l_2 - l_3 S_3,$$

$$\lambda_5 C_{34} = -S_1 C_2 p_x + C_1 C_2 p_y + S_2 p_z - \lambda_1 S_2 - l_3 C_3,$$

$$\Theta_{34}^* = \text{arc tg}\frac{\lambda_5 S_{34}}{\lambda_5 C_{34}}. \tag{7d}$$

$$\Theta_4' = \Theta_{34}' - \Theta_3' \tag{7e}$$

$$S_5 = C_1 n_x + S_1 n_y, \quad C_5 = C_1 o_x + S_1 o_y,$$

$$\Theta_5^* = \text{arc tg}\frac{S_5}{C_5}. \tag{7f}$$

Θ_{5min}' and Θ_{5max}' boundary angles depended on Θ_2'–Θ_4' as well as Θ_{5max}'–$\Theta_{5min}' = 360^\circ$ angles [see formulae (2)] and so S_5 and C_5 signs must be examined in order to determine Θ_5' angle. The analysis of the above formulae makes it clear that for explicitly determined elements of the $\mathbb{T}_{5ref}$ matrix, there may be two solutions for Θ_5'–$\Theta_5' = \Theta_{5min}'$ or $\Theta_5' = \Theta_{5max}'$. This is the kinematic singularity of the first kind. There is no singularity of the second kind of the IRb-6 manipulator [4].

4. EQUATIONS OF ACTUATORS KINEMATICS

Driving kinematics has been illustrated in Figs 3–6. The transform matrices illustrated in these figures have been taken from paper [17]. The $x_{hi} y_{hi} z_{hi}$ coordinates are associated with the body of

(a)

(b)

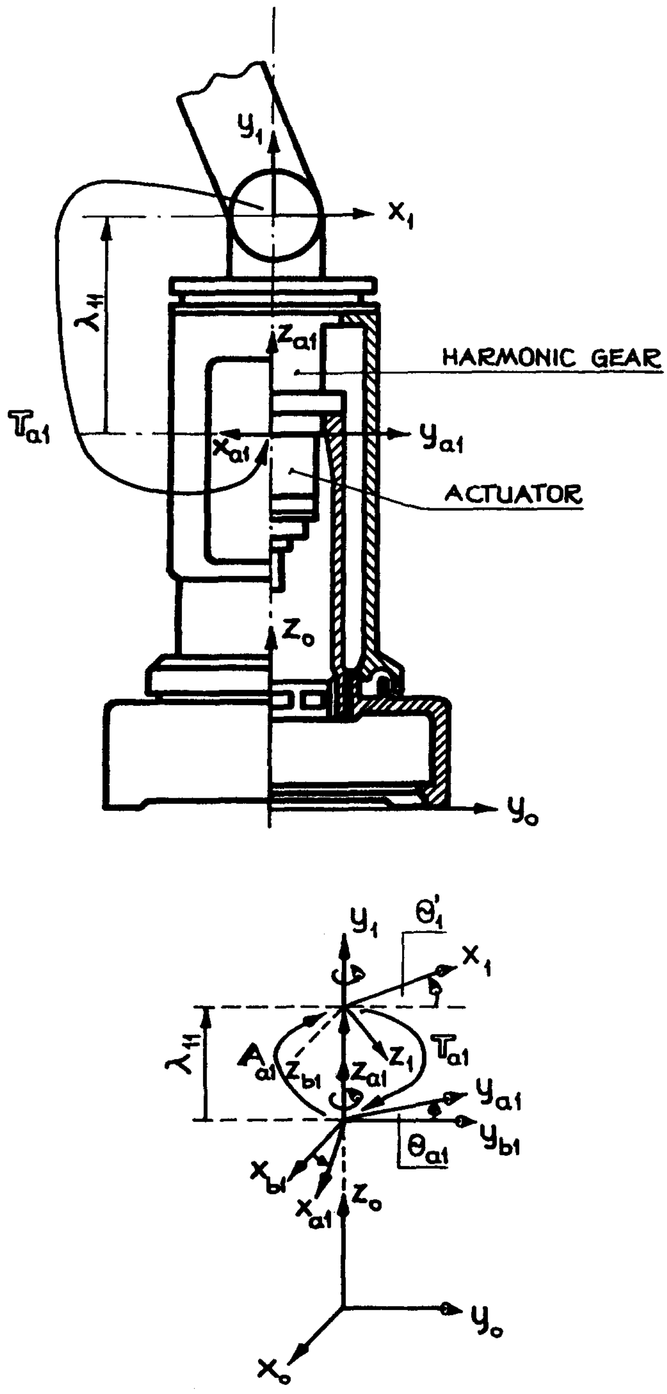

Fig. 3(a). The first degree of freedom drive unit.
Fig. 3(b). The first degree of freedom drive unit: $\mathbb{A}_{a1} = \text{Rot}(z, -\Theta_{a1} + \Theta'_1 + 90°)\text{Trans}(0, 0, \lambda_{11})$ $\text{Rot}(x, 90°)$, $\mathbb{T}_{a1} = \mathbb{A}_{a1}^{-1}$.

the ith actuator. The $x_{ai}y_{ai}z_{ai}$ coordinates are associated with the rotor of the ith actuator. Θ_{ai} angle of rotation of the $x_{ai}y_{ai}z_{ai}$ system coordinates around the z_{bi} axis is a natural coordinate of ith actuator.

Formulae binding the Θ'_1–Θ'_5 natural coordinates of links to the Θ_{a1}–Θ_{a5} natural coordinates of actuators are as follows [4, 17]:

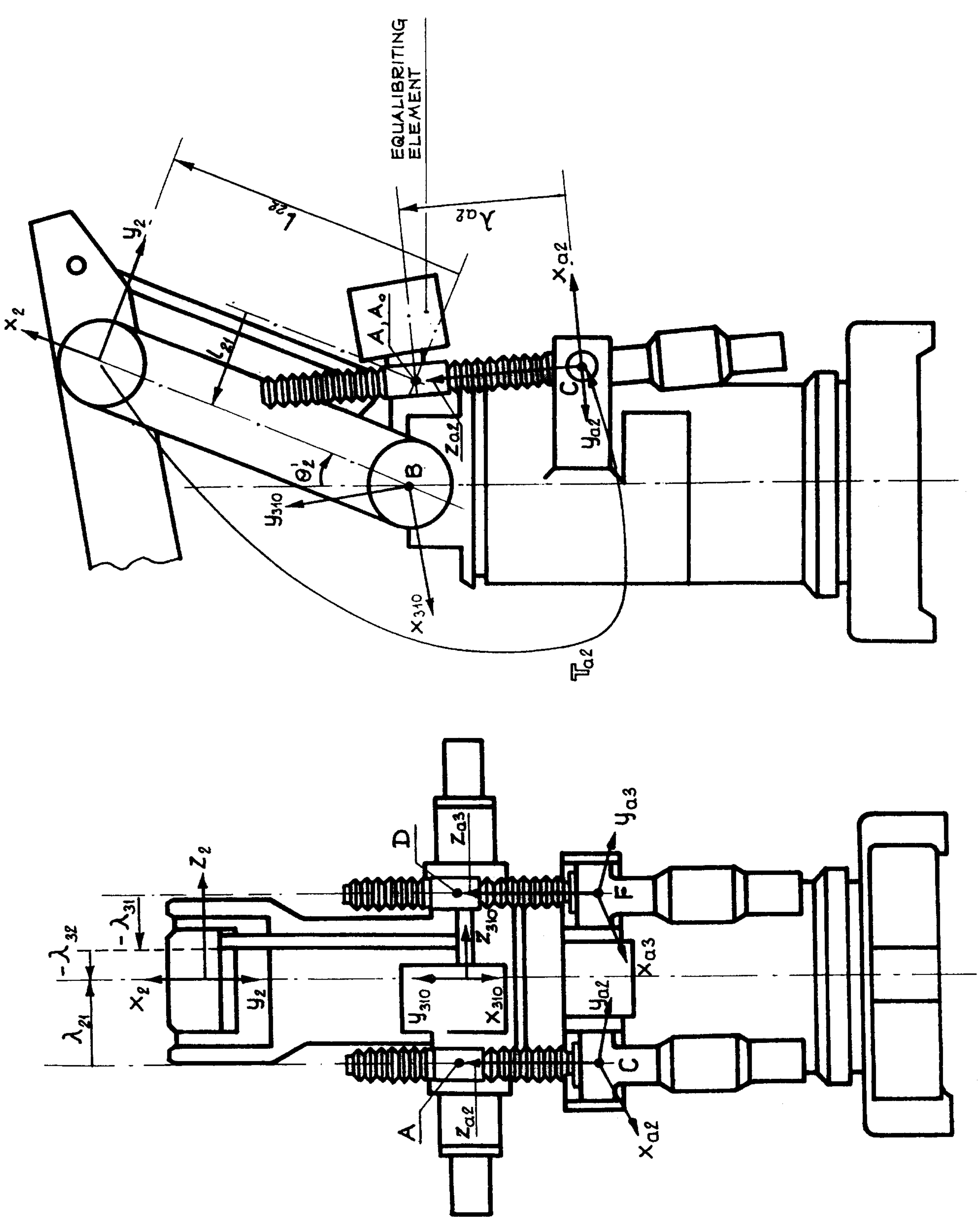

Fig. 4(a). The second degree of freedom drive unit.

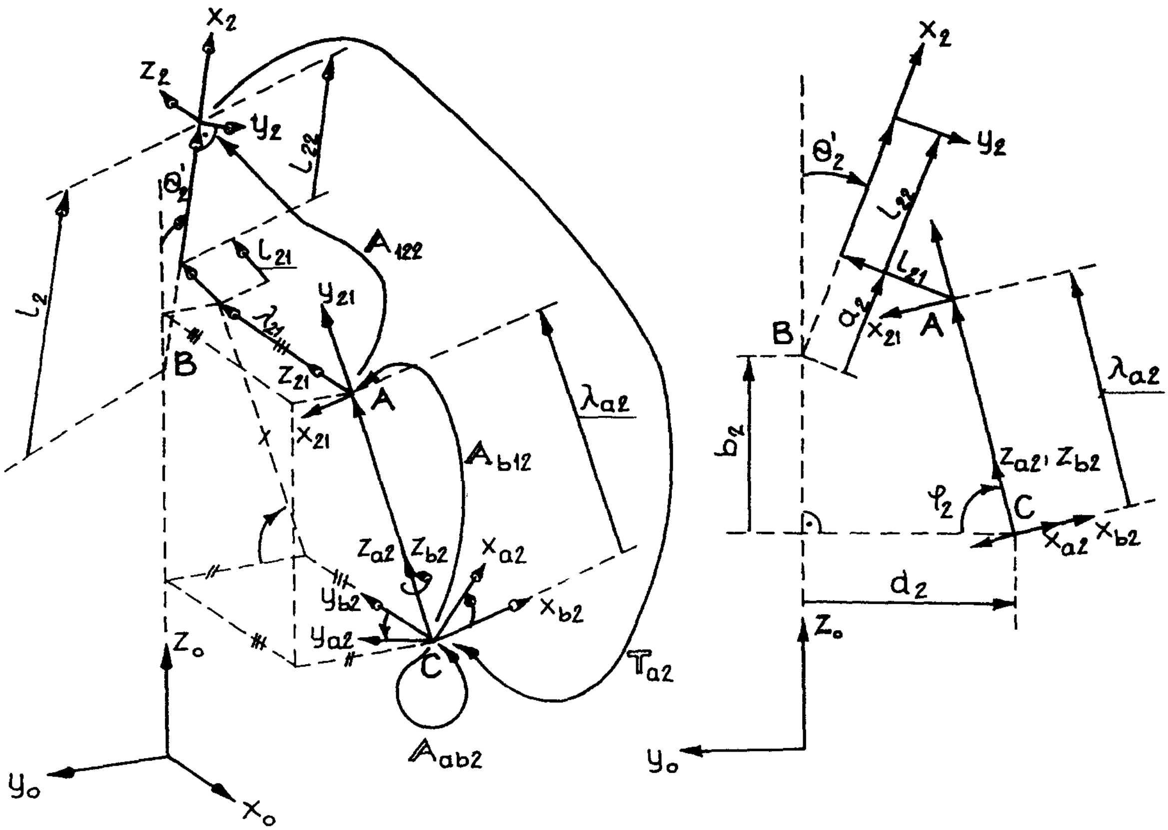

Fig. 4(b). The second degree of freedom drive unit: $\mathbb{A}_{ab2} = \mathrm{Rot}(z, -\Theta_{a2})$, $\mathbb{A}_{ab12} = \mathrm{Rot}(z, 180°)\mathrm{Trans}(0, 0, \lambda_{a2})\mathrm{Rot}(x, 90°)$, $\mathbb{A}_{122} = \mathrm{Rot}(z, 90° - \varphi_2 + \Theta'_2)\mathrm{Trans}(0, 0, \lambda_{21})\mathrm{Trans}(l_{21}, 0, 0)\mathrm{Rot}(z, 90°)\mathrm{Trans}(l_{22}, 0, 0)$, $\mathbb{T}_{a2} = (\mathbb{A}_{ab2}\mathbb{A}_{b12}\mathbb{A}_{122})^{-1}$.

$$\Theta'_1 = k_1^{-1}\Theta_{a1},$$

$$\Theta'_2 = -\arccos\frac{AB^2 + BC^2 - [A_0C - (h_2/2\pi)\Theta_{a2}]^2}{2 \cdot AB \cdot BC} + \alpha,$$

$$\Theta'_3 = -\arccos\frac{DE^2 + EF^2 - [D_0F - (h_3/2\pi)\Theta_{a3}]^2}{2 \cdot DE \cdot EF} + \beta - \Theta'_2,$$

$$\Theta'_4 = k_4^{-1}\Theta_{a4} - (\Theta'_2 + \Theta'_3),$$

$$\Theta'_5 = k_4^{-1}k_5^{-1}(\Theta_{a4} - \Theta_{a5}). \tag{8}$$

These formulae are corresponded by the following equations:

$$\Theta_{a1} = k_1\Theta'_1,$$

$$\Theta_{a2} = (2\pi/h_2)\{-[AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos(\alpha - \Theta'_2)]^{1/2} + A_0C\},$$

$$\Theta_{a3} = (2\pi/h_3)\{-[DE^2 + EF^2 - 2 \cdot DE \cdot EF \cdot \cos(\beta - \Theta'_2 - \Theta'_3)]^{1/2} + D_0F\},$$

$$\Theta_{a4} = [k_4(\Theta'_2 + \Theta'_3 + \Theta'_4),$$

$$\Theta_{a5} = k_4(\Theta'_2 + \Theta'_3 + \Theta'_4) - k_4k_5\Theta'_5. \tag{9}$$

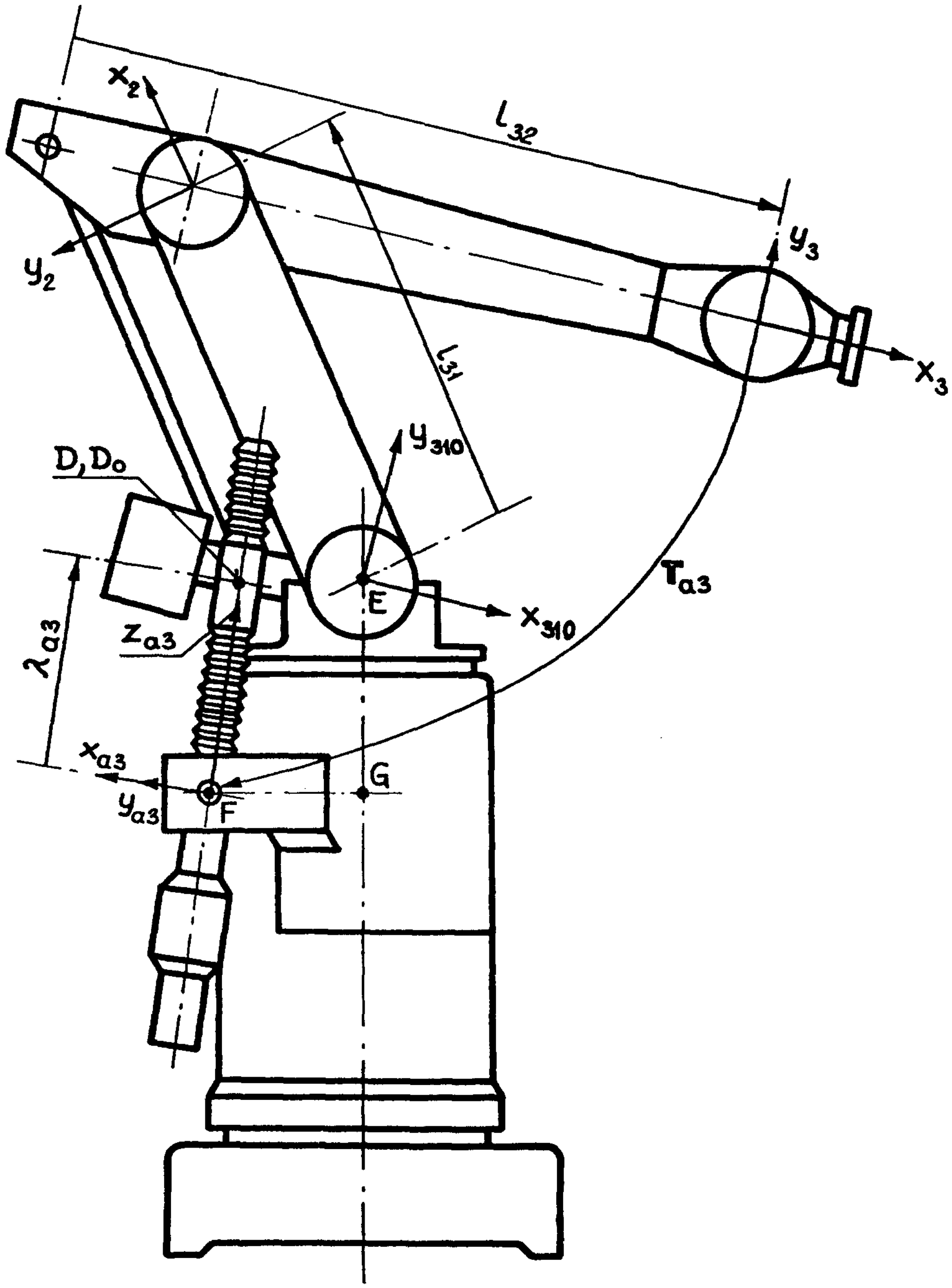

Fig. 5(a). The third degree of freedom drive unit.

$$\alpha = \arccos \frac{AB^2 + BC^2 - A_0C^2}{2 \cdot AB \cdot BC},$$

$$\beta = \arccos \frac{DE^2 + EF^2 - D_0F^2}{2 \cdot DE \cdot EF}.$$

The $\mathbb{T}_{a1}$–$\mathbb{T}_{a5}$ matrices describing the manipulator driving kinematics are in the form of [17]:

$$\mathbb{T}_{a1} = \begin{bmatrix} -S_\varphi & C_\varphi & 0 & 0 \\ 0 & 0 & 1 & -\lambda_{11} \\ C_\varphi & S_\varphi & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{10a}$$

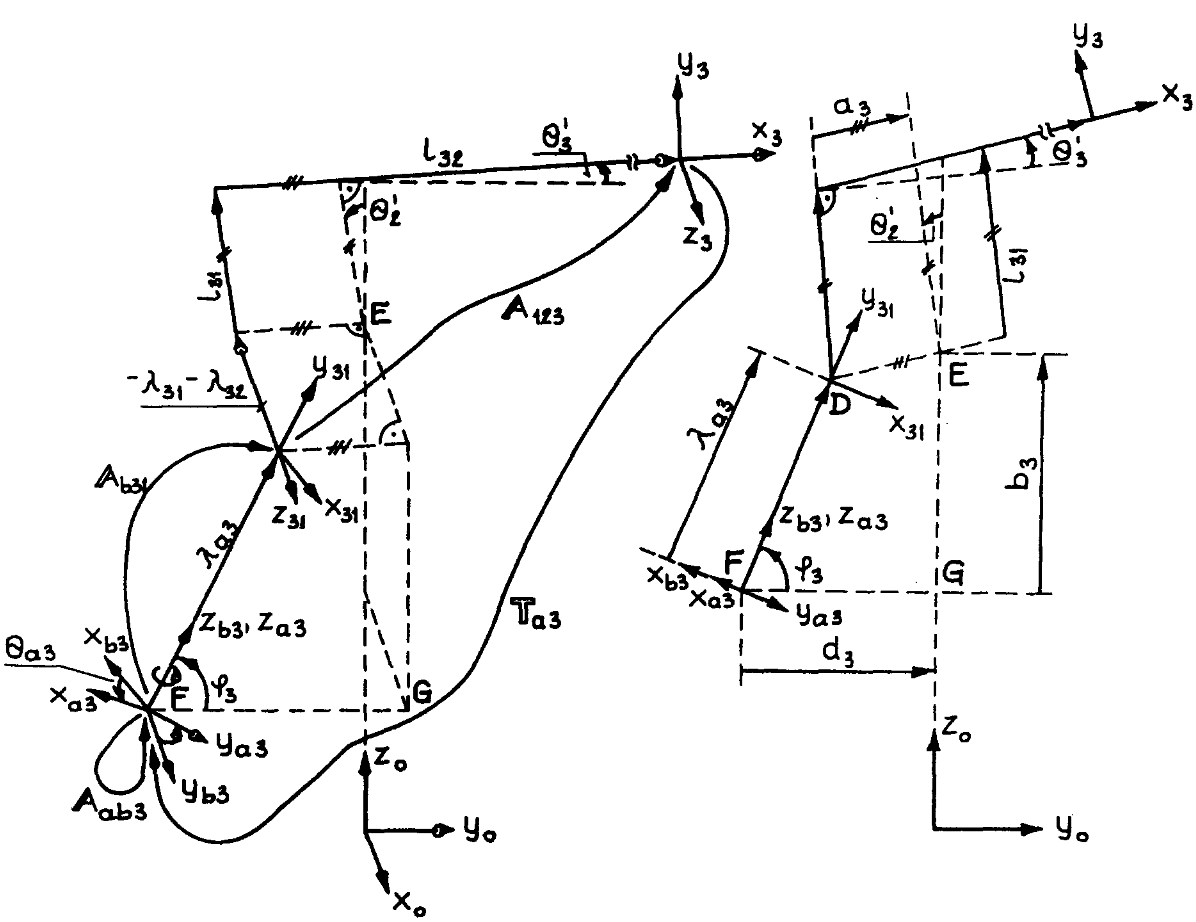

Fig. 5(b). The third degree of freedom drive unit: $\mathbb{A}_{ab3} = \mathrm{Rot}(z, -\Theta_{a3})$, $\mathbb{A}_{b31} = \mathrm{Rot}(z, 180)\mathrm{Trans}(0, 0, \lambda_{a3})\mathrm{Rot}(x, 90)$, $\mathbb{A}_{123} = \mathrm{Rot}(z, 180° - \varphi_3 + \Theta'_2)\mathrm{Trans}(0, 0, \lambda_{32} + \lambda_{31})\mathrm{Trans}(l_{31}, 0, 0)$ $\mathrm{Rot}(z, -90° + \Theta'_3)\mathrm{Trans}(l_{32}, 0, 0)$, $\mathbb{T}_{a3} = (\mathbb{A}_{ab3}\mathbb{A}_{ab31}\mathbb{A}_{123})^{-1}$.

where S_φ, $C_\varphi = \sin\varphi$, $\cos\varphi \cdot \varphi = \Theta'_1 - \Theta_{a1}$.

$$\mathbb{T}_{a2} = \begin{bmatrix} C_a C_\varphi & -S_a C_\varphi & S_\varphi & -l_{22} - \lambda_{a2} S_\varphi \\ C_a S_\varphi & -S_a S_\varphi & -C_\varphi & l_{21} + \lambda_{a2} C_\varphi \\ S_a & C_a & 0 & -\lambda_{21} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{10b}$$

where S_a, $C_a = \sin\Theta_{a2}$, $\cos\Theta_{a2} \cdot S_\varphi$, $C_\varphi = \sin\varphi$, $\cos\varphi$, $\varphi = \varphi_2 - \Theta'_2$,

$$\varphi_2 = \operatorname{arc\,tg} \frac{b_2 + a_2 C_2 - l_{21} S_2}{d_2 - l_{21} C_2 - a_2 S_2}.$$

$$\lambda_{a2} = [(b_2 + a_2 C_2 - l_{21} S_2)^2 + (d_2 - l_{21} C_2 - a_2 S_2)^2]^{1/2}.$$

$$\mathbb{T}_{a3} = \begin{bmatrix} C_a S_\varphi & -S_a S_\varphi & C_\varphi & -l_{32} - l_{31} S_3 - \lambda_{a3} C_\varphi \\ C_a C_\varphi & -S_a C_\varphi & -S_\varphi & -l_{31} C_3 + \lambda_{a3} S_\varphi \\ S_a & C_a & 0 & -(\lambda_{31} + \lambda_{32}) \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{10c}$$

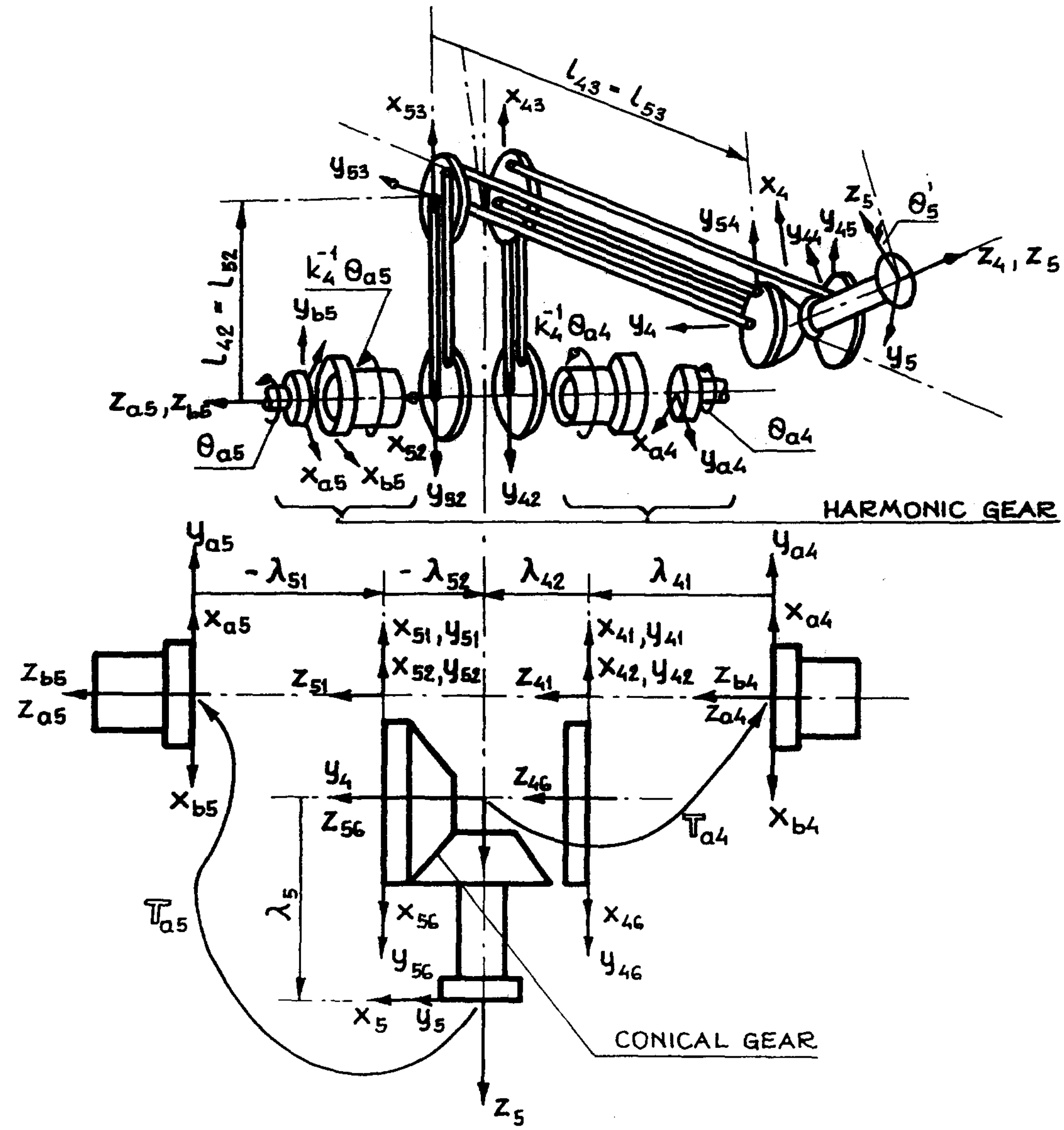

Fig. 6(a). The fourth and fifth degree of freedom drive unit.

where S_φ, $C_\varphi = \sin\varphi$, $\cos\varphi \cdot \varphi = \Theta_2' + \Theta_3' - \varphi_3$,

$$\varphi_3 = \operatorname{arc\,tg}\frac{b_3 - a_3 S_{23}}{d_3 - a_3 C_{23}},$$

$$\lambda_{a3} = [(d_3 - a_3 C_{23})^2 + (b_3 - a_3 S_{23})^2]^{1/2}.$$

$$\mathbb{T}_{a4} = \begin{bmatrix} -S_\varphi & C_\varphi & 0 & l_{43} S_4 - l_{42} C_{34} \\ 0 & 0 & 1 & -(\lambda_{41} + \lambda_{42}) \\ C_\varphi & S_\varphi & 0 & -l_{43} C_4 - l_{42} S_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{10d}$$

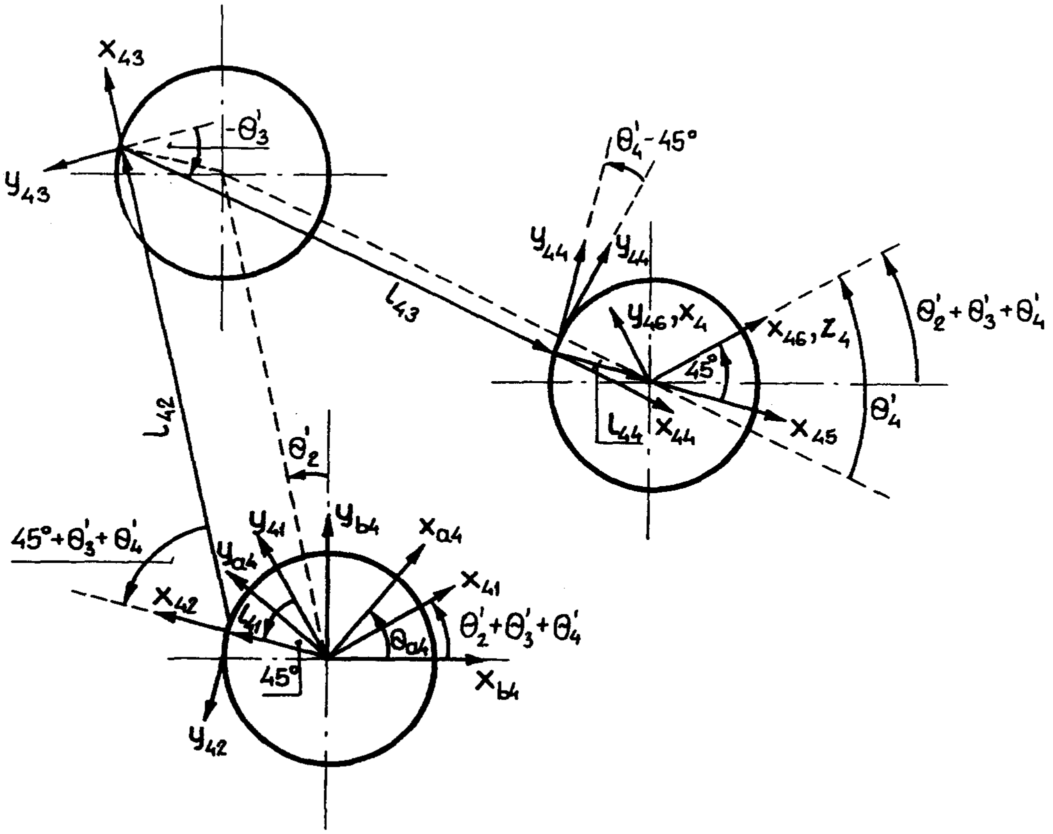

Fig. 6(b). The fourth degree of freedom drive unit.

where S_φ, $C_\varphi = \sin\varphi$, $\cos\varphi \cdot \varphi = \Theta_2' + \Theta_3' + \Theta_4' - \Theta_{a4}$, $l_{41} = l_{44}$.

$$\mathbb{T}_{a5} = \begin{bmatrix} -C_5S_\varphi & C_5C_\varphi & S_5 & l_{53}S_4C_5 - l_{52}C_{34}C_5 - (\lambda_{51}+\lambda_{52})S_5 \\ S_5S_\varphi & -S_5C_\varphi & C_5 & -l_{53}S_4S_5 + l_{52}C_{34}S_5 - (\lambda_{51}+\lambda_{52})C_5 \\ C_\varphi & S_\varphi & 0 & -l_{53}C_4 - l_{52}S_{34} - \lambda_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{10e}$$

where S_φ, $C_\varphi = \sin\varphi$, $\cos\varphi \cdot \varphi = \Theta_2' + \Theta_3' + \Theta_4' - \Theta_{a5}$, $l_{51} - l_{54}$.

Figures 4 and 5 illustrate the $x_{310}y_{310}z_{310}$ coordinates describing an element equilibrating, the third link. Kinematics of this element are illustrated in Fig. 7. The $\mathbb{T}_{031}$ matrix describing these coordinates in relation to the base coordinate system has the form of:

$$\mathbb{T}_{031} = \begin{bmatrix} -S_1C_{23} & S_1S_{23} & C_1 & 0 \\ C_1C_{23} & -C_1S_{23} & S_1 & 0 \\ S_{23} & C_{23} & 0 & \lambda_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{11}$$

5. EXAMPLE

The formulae presented in Section 3 are the solution of inverse problem of IRb-6 manipulator kinematics. On the basis of these formulae algorithms generating natural coordinates of actuators are designed. The coordinates correspond to the reference trajectory of a manipulation object as

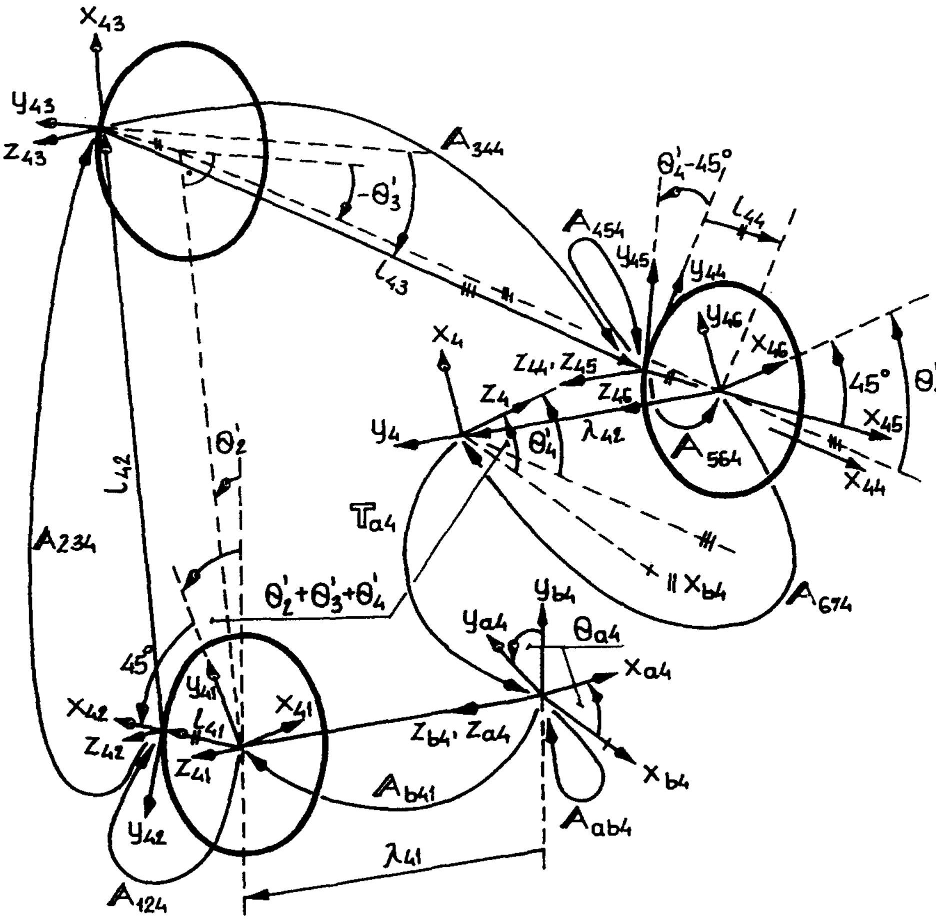

Fig. 6(c). The fourth degree of freedom drive unit: $\mathbb{A}_{ab4} = \mathrm{Rot}(z, -\Theta_{a4})$, $\mathbb{A}_{b14} = \mathrm{Rot}(z, \Theta_2' + \Theta_3' + \Theta_4')\mathrm{Trans}(0, 0, \lambda_{41})$, $\mathbb{A}_{124} = \mathrm{Rot}(z, 90° + 45°)\mathrm{Trans}(l_{41}, 0, 0)$, $\mathbb{A}_{234} = \mathrm{Rot}(z, -45° - \Theta_3' - \Theta_4')\mathrm{Trans}(l_{42}, 0, 0)$, $\mathbb{A}_{344} = \mathrm{Rot}(z, \Theta_3' - 90°)\mathrm{Trans}(l_{43}, 0, 0)$, $\mathbb{A}_{454} = \mathrm{Rot}(z, -45° + \Theta_4')$, $\mathbb{A}_{564} = \mathrm{Trans}(l_{44}, 0, 0)\mathrm{Rot}(z, 45°)$, $\mathbb{A}_{674} = \mathrm{Rot}(z, 90°)\mathrm{Trans}(0, 0, \lambda_{42})\mathrm{Rot}(x, 90°)$, $\mathbb{T}_{a4} = (\mathbb{A}_{ab4}\mathbb{A}_{b14}\mathbb{A}_{124}\mathbb{A}_{234}\mathbb{A}_{344}\mathbb{A}_{454}\mathbb{A}_{564}\mathbb{A}_{674})^{-1}$.

described in the manipulator external space. The algorithms computing the actuators natural coordinates form a reference trajectory generating tier which is a functional structure element of the adaption robots control system. These algorithms are indispensable program means interconnecting vision tier and control drives tier [5].

The author of the paper has worked out a computer algorithm PLAN2 generating task trajectories of the IRb-6 manipulator. Reference external coordinates of the points of a generated trajectory will be called main fulcrums. No such generation is possible without an introductory description of a trajectory in the form of external coordinates values of, at least, two main fulcrums optionally distant from each other. PLAN2 algorithm generates additional fulcrums between consecutive main fulcrums.

For the defined reference external coordinates x_{ref}, y_{ref}, z_{ref}, Φ_{ref}, Θ_{ref}, Ψ_{ref} and the reference times T describing consecutive main fulcrums the algorithm determines $\mathbb{T}_{5\mathrm{ref}}$ matrix, checks if constrains equation is satisfied (6) and computes natural coordinates Θ_1'–Θ_5' from the formulae (7). Then the algorithm asks about a coordinate system describing the shape of trajectory segment between the consecutive main fulcrums. For a straight segment, cartesian system should be set; for a curvilinear segment either cylindrical or spherical coordinate system should be set. Once external coordinates of all the main fulcrums have been set, the algorithm asks about parameters defining the accuracy of generating the reference trajectory [18] and parameters l_6 and λ_6 describing the task. After input parameters have read in, the algorithm determines additional fulcrums accepting a linear change

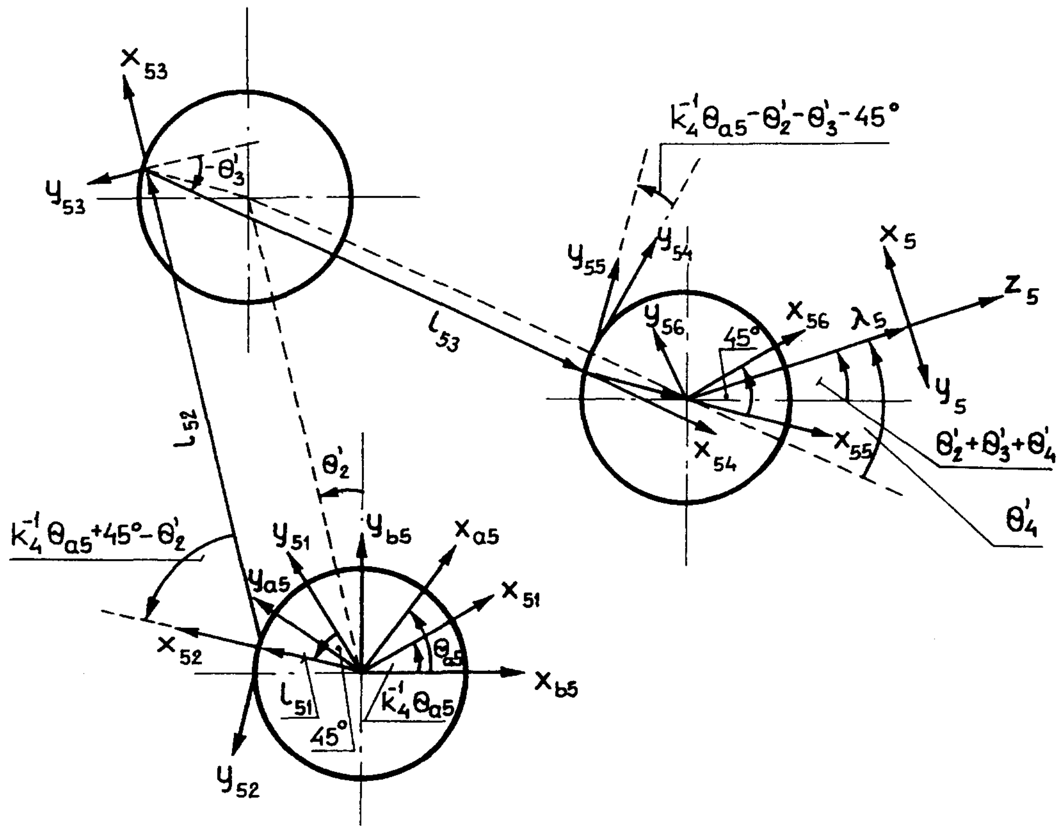

Fig. 6(d). The fifth degree of freedom drive unit.

of external coordinates $x, y, z, \Phi, \Theta, \Psi$ along a length of segment joining consecutive fulcrums. Φ angle is a result of the altered external coordinates and the constrains equation (6).

Figure 8 shows a task rectilinear reference trajectory which is limited by the initial fulcrums P and the final point K. The coordinates of these fulcrums are: $x_{P\mathrm{ref}} = -0.60$ m, $y_{P\mathrm{ref}} = 0.60$ m, $z_{P\mathrm{ref}} = 1.0$ m, $\Phi_{P\mathrm{ref}} = 135°$, $\Theta_{P\mathrm{ref}} = 179°$, $\Psi_{P\mathrm{ref}} = 359°$, $x_{K\mathrm{ref}} = -0.65$ m, $y_{K\mathrm{ref}} = 0.60$ m, $z_{K\mathrm{ref}} = 1.0$ m, $\Phi_{K\mathrm{ref}} = 137.29°$, $\Theta_{K\mathrm{ref}} = 1°$, $\Psi_{K\mathrm{ref}} = 180°$. The following input parameters of the PLAN2 algorithm, defining kinematics of the trajectory as in Fig. 8, will be taken on: the task parameters $l_6 = 0$ and $\lambda_6 = 0.16$ m; time assigned to point P $T_p = 0$; time assigned to point K $T_K = 1.0$ s; trajectory shape—straight line.

1283 additional fulcrums shown in Figs 9 and 10, have resulted from the generation. It follows from Figs 9 and 10 that natural coordinates graphs of the links and actuators are similar. Θ'_5 coordinate undergoes a sudden alteration from the minimum to the maximum boundary value, determined by the inequalities (2). It follows from the inequations that Θ'_5 angle jump at $t \cong 0.5$ s time equals 360°. There is a similar change in the Θ_{a5} coordinate. The other coordinates graphs are smooth.

The main fulcrum P is corresponded by the following natural coordinates: $\Theta'_1 = 45°$, $\Theta'_2 = -25°$, $\Theta'_3 = 37.7°$, $\Theta'_4 = -102°$, $\Theta'_5 = -181°$, $\Theta_{a1} = -7110°$, $\Theta_{a2} = -2567.5°$, $\Theta_{a3} = 1360.5°$, $\Theta_{a4} = 11392°$, $\Theta_{a5} = -2364°$.

The main fulcrum K is corresponded by the following natural coordinates: $\Theta'_1 = 47.3°$, $\Theta'_2 = -39°$, $\Theta'_3 = 12°$, $\Theta'_4 = 116°$, $\Theta'_5 = 0°$, $\Theta_{a1} = -7472°$, $\Theta_{a2} = -3818.8°$, $\Theta_{a3} = -2921.3°$, $\Theta_{a4} = -11392°$, $\Theta_{a5} = -11392°$.

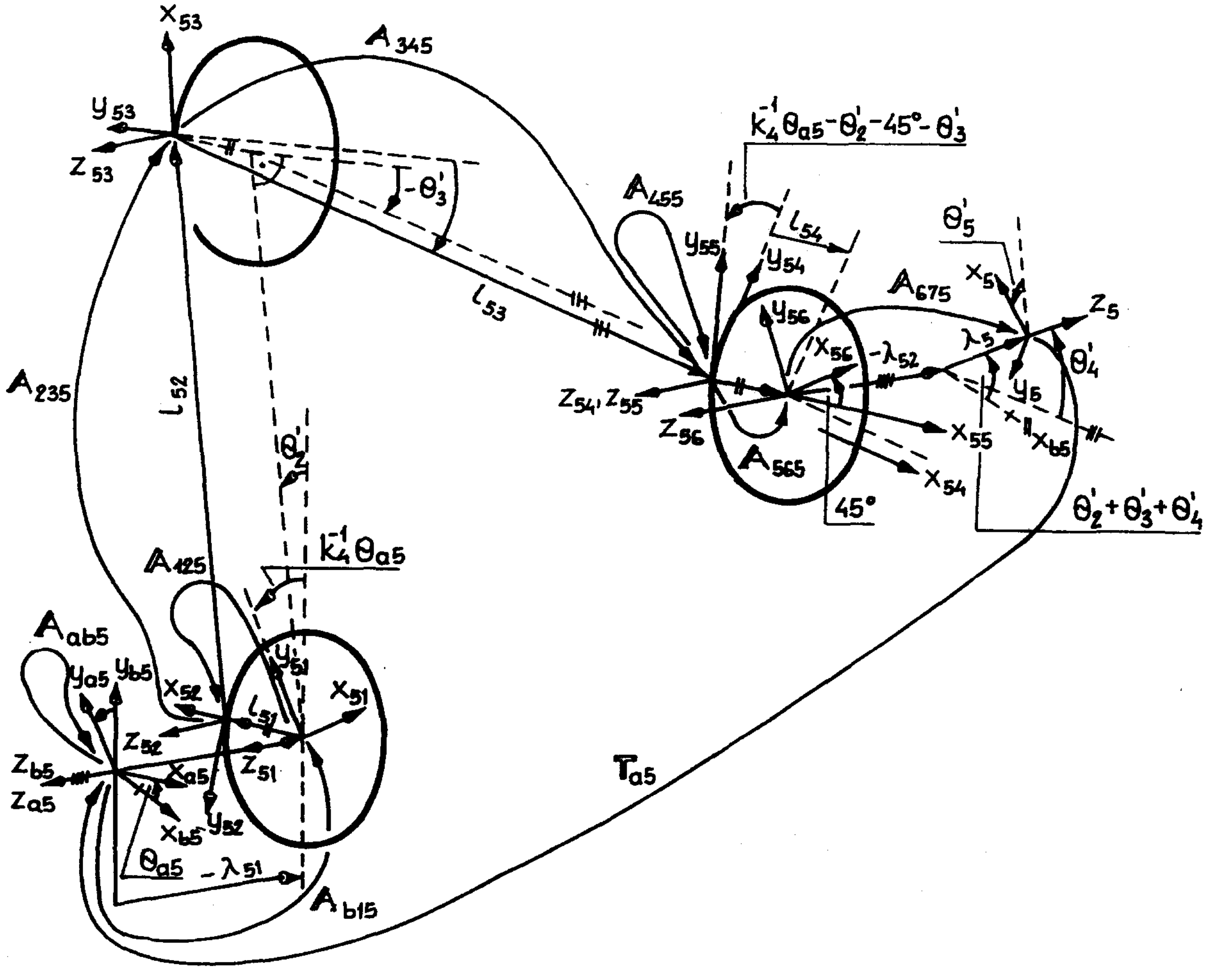

Fig. 6(e). The fifth degree of freedom drive unit: $\mathbb{A}_{ab5} = \mathrm{Rot}(z, -\Theta_{a5})$, $\mathbb{A}_{b15} = \mathrm{Rot}(z, k_4^{-1}\Theta_{a5})\mathrm{Trans}(0, 0, \lambda_{51})$, $\mathbb{A}_{125} = \mathrm{Rot}(z, 90° + 45°)\mathrm{Trans}(l_{51}, 0, 0)$, $\mathbb{A}_{235} = \mathrm{Rot}(z, \Theta_2' - k_4^{-1}\Theta_{a5} - 45°)\mathrm{Trans}(l_{52}, 0, 0)$, $\mathbb{A}_{345} = \mathrm{Rot}(z, \Theta_3' - 90°)\mathrm{Trans}(l_{53}, 0, 0)$, $\mathbb{A}_{455} = \mathrm{Rot}(z, k_4^{-1}\Theta_{a5} - \Theta_2' - \Theta_3' - 45°)$, $\mathbb{A}_{565} = \mathrm{Trans}(l_{54}, 0, 0)\mathrm{Rot}(z, 45°)$, $\mathbb{A}_{675} = \mathrm{Rot}(z, 90° - k_4^{-1}\Theta_{a5} + \Theta_2' + \Theta_3' + \Theta_4')\mathrm{Trans}(0, 0, \lambda_{52})\mathrm{Rot}(x, 90°)\mathrm{Trans}(0, 0, \lambda_5)\mathrm{Rot}(z, \Theta_5')$, $\mathbb{T}_{a5} = (\mathbb{A}_{ab5}\mathbb{A}_{b15}\mathbb{A}_{125}\mathbb{A}_{235}\mathbb{A}_{345}\mathbb{A}_{455}\mathbb{A}_{565}\mathbb{A}_{675})^{-1}$.

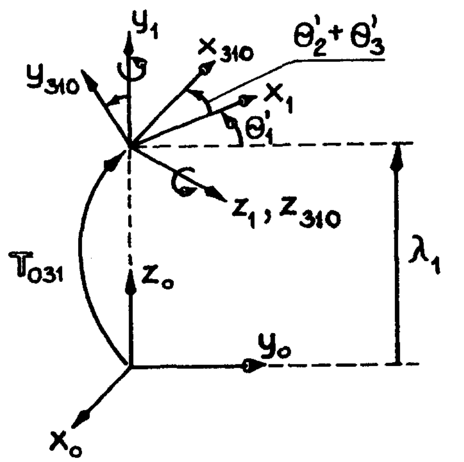

Fig. 7. Description of the third link equilibrator.

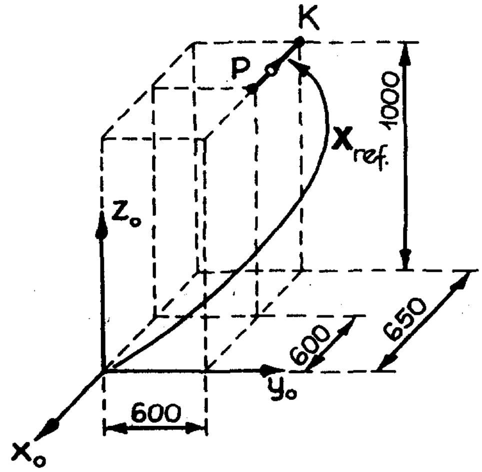

Fig. 8. $\mathbb{X}_{ref}$ reference trajectory.

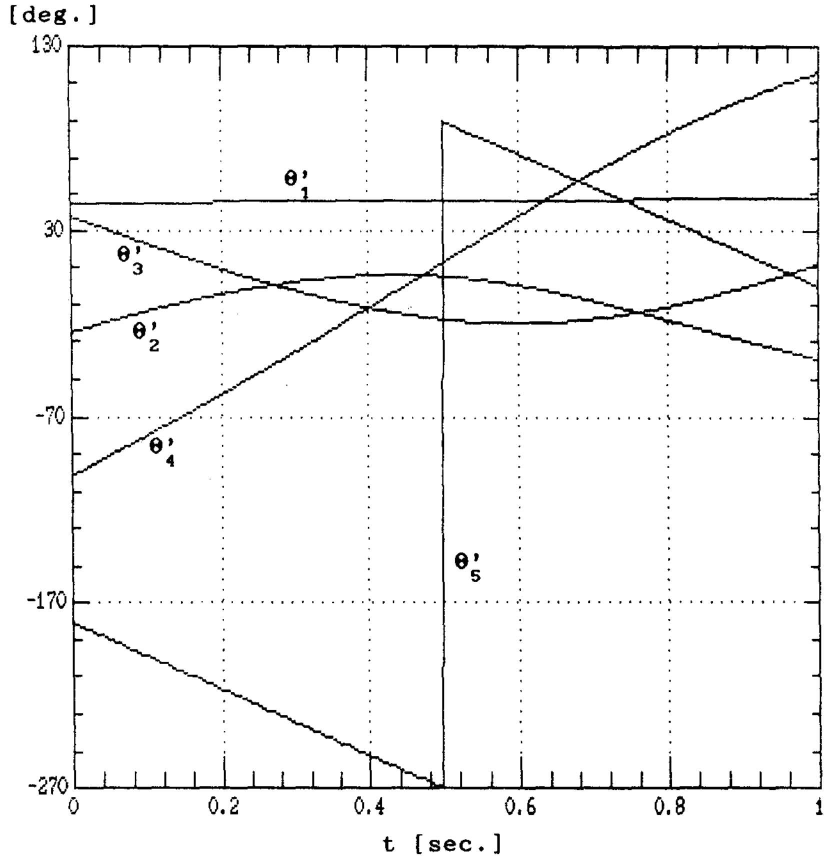

Fig. 9. $\Theta'_1(t)$–$\Theta'_1(t)$ link natural coordinates.

6. CONCLUSIONS

The kinematics models as presented above allow one to:

(a) analyse reference trajectories graphs in the internal space of IRb-6 manipulator;
(b) determine analytic description of the IRb-6 manipulator work space [17];
(c) design reference trajectory generating algorithms for the IRb-6 manipulator (e.g. PLAN2 algorithm [18]);
(d) determine instantaneous advance and angular velocities of links and other elements of IRb-6 manipulator which is essential for their dynamics, stresses and strains analysis.

As the IRb-6, IRb-60, IRp-6, IRp-60 manipulators have similar kinematic structure, the equations of kinematics in Section 2 may be well used to describe each of them.

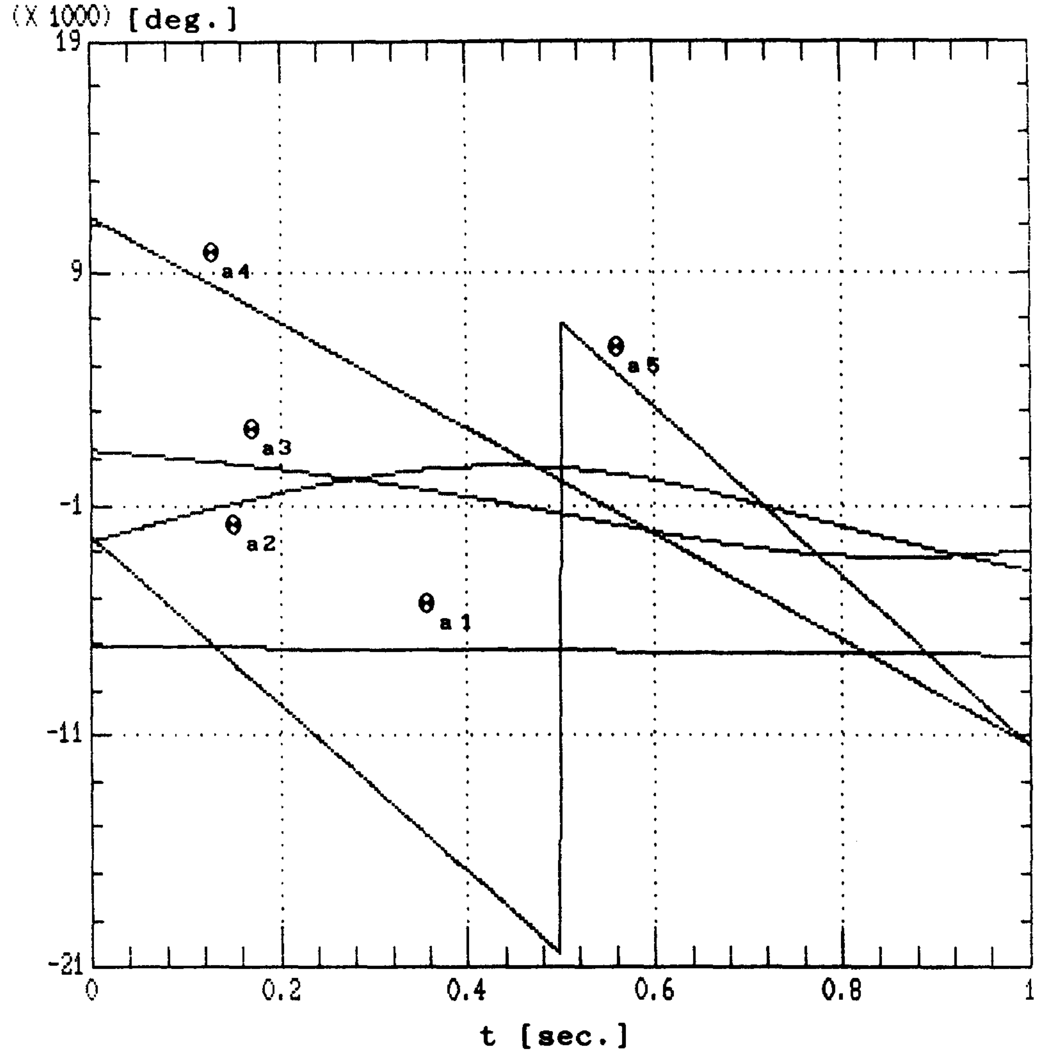

Fig. 10. $\Theta_{a1}(t)$–$\Theta_{a5}(t)$ actuator natural coordinates.

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