2.810

Time Analysis for Manufacturing Systems

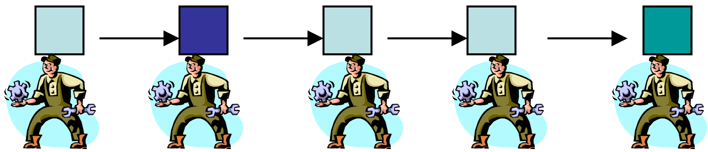
Analytical Tools and Applications

reference:
Manufacturing Systems Engineering
Prentice Hall 1994 By Stanley B. Gershwin

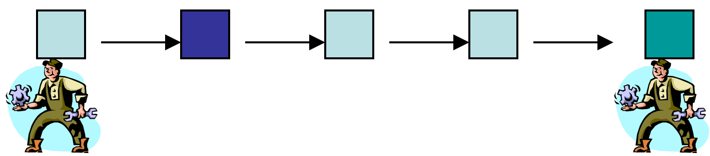
Outline

1. Tools from Operations Research
    - Little’s Law
    - Unreliable Machine(s)
    - Buffers
    - M/M/1 Queue
2. Applications
    - Transfer Lines, FMS, TPS Cells, Push Vs Pull, …

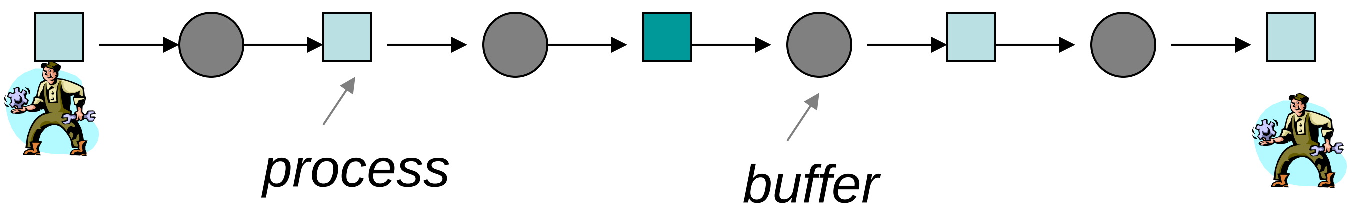
Example Mfg Systems



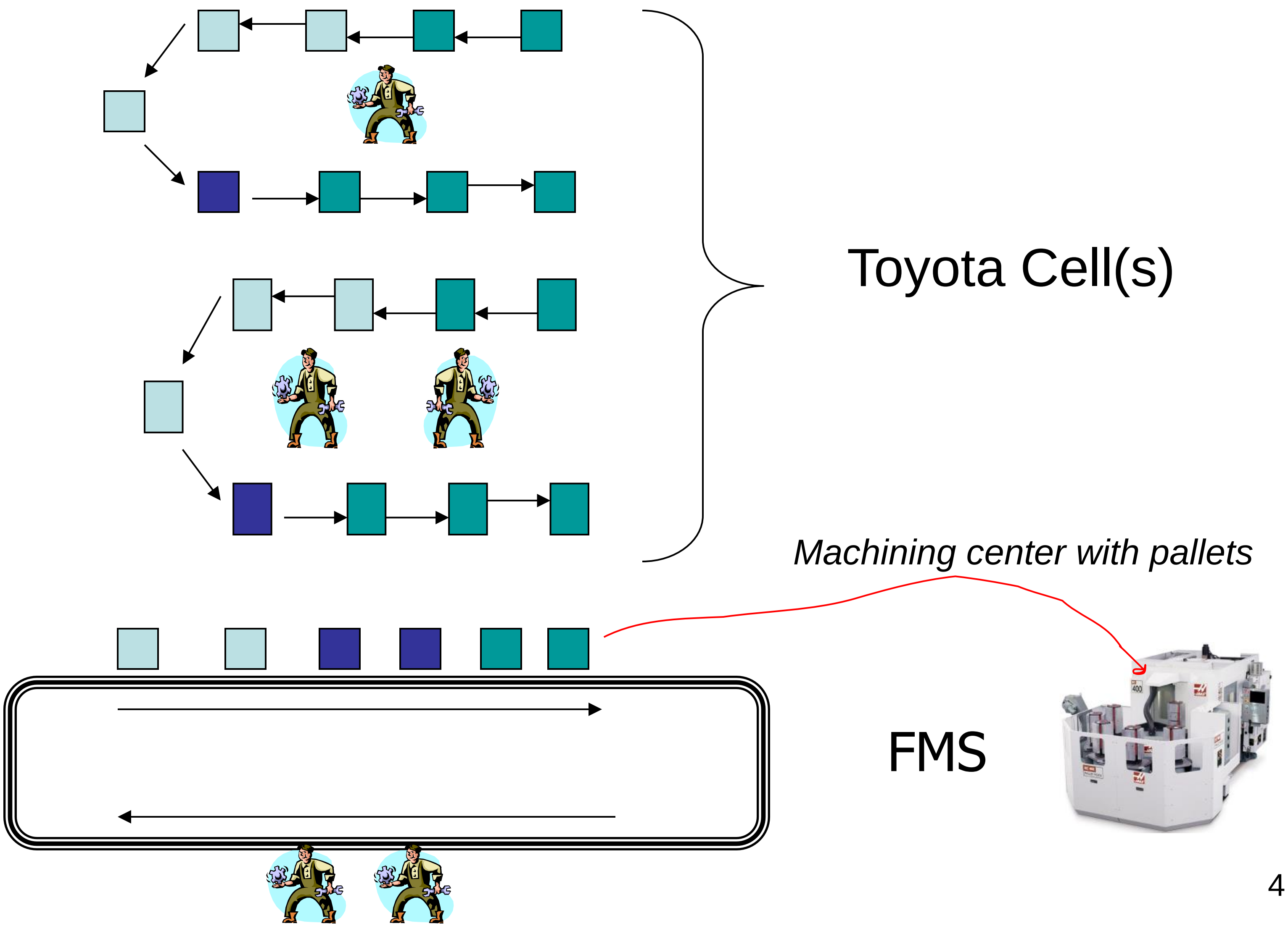
Flow Line(s)



Transfer line



Example Mfg Systems



Little's Law

$$N = \lambda T$$

N = Average parts in the system

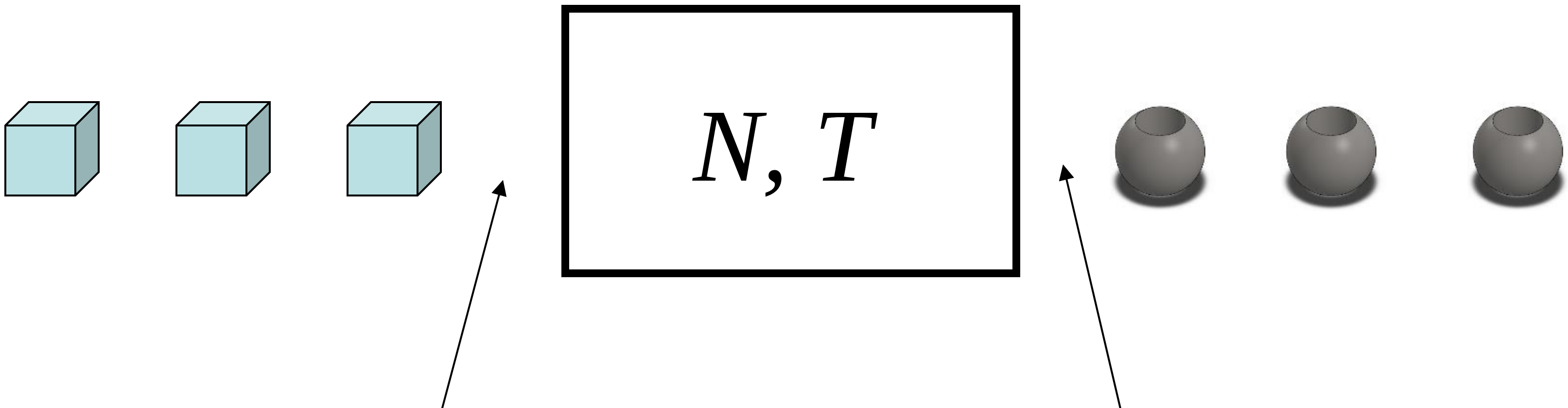
λ = Average arrival rate

T = Average time in the system

Ref. L. Kleinrock, "Queueing System, Vol 1 Theory, Wiley, 1975

Figure 2.1 A general queueing system.

Queueing Systems



Observer here
Observing arrivals;
$\alpha(t)$ = number of arrivals in (0,t)

Observing departures;
$\delta(t)$ = number of departures in (0,t)

$$N(t) = \alpha(t) - \delta(t)$$

Number in the system, parts or customers

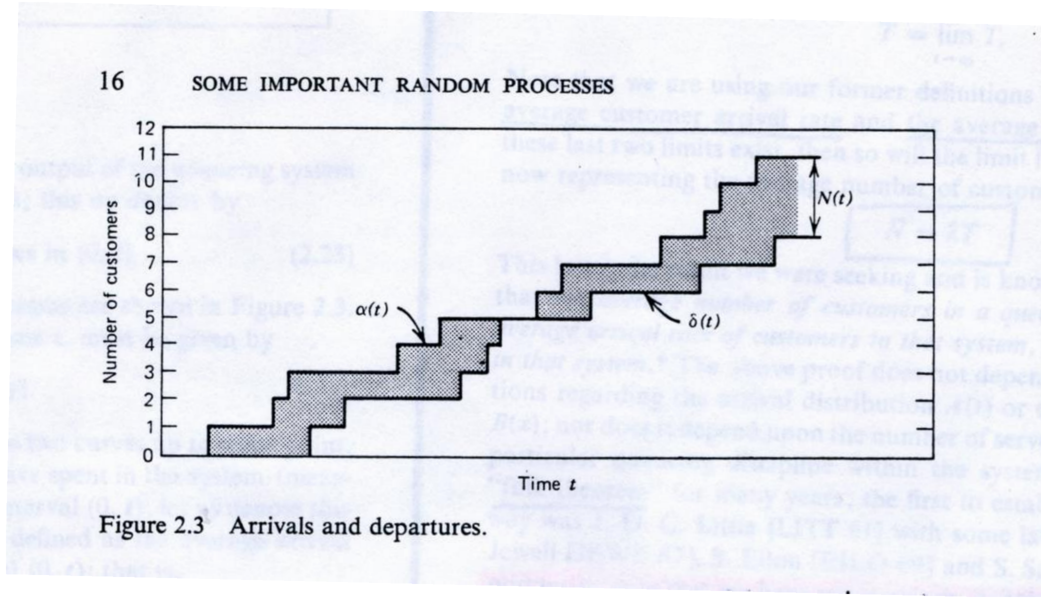


Figure 2.3 Arrivals and departures.

Ref. Kleinrock
Vol 1, 1975
See p.16, 17

$$N(t) = \alpha(t) - \delta(t)$$

$$\gamma(t) = \int_0^t N(t)\,dt$$

$\gamma(t) =$ customer -seconds (shaded area in figure)

average number of customers in the system $= \bar{N} = \dfrac{\gamma(t)}{t}$

average arrival rate $= \lambda_t = \dfrac{\alpha(t)}{t}$

average time per customer $= T_t = \dfrac{\gamma(t)}{\alpha(t)} = \dfrac{\bar{N}}{\lambda_t}$

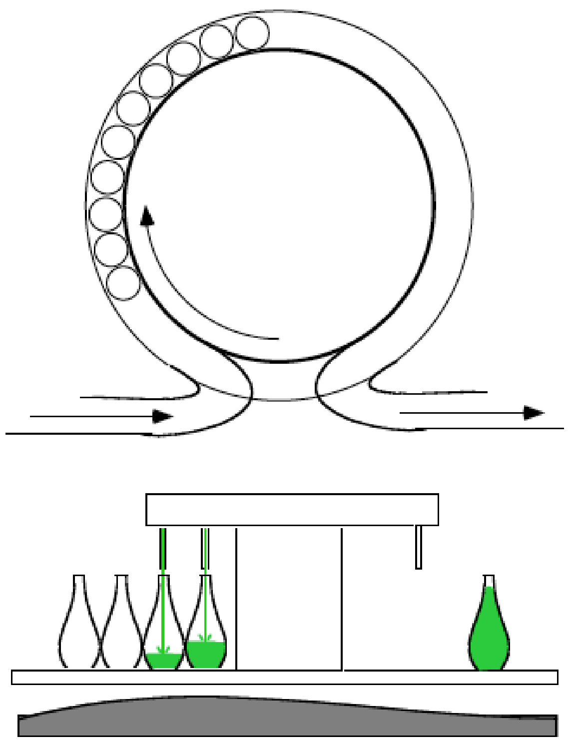
this gives $\bar{N} = \lambda_t \cdot T_t$

$$\left.\begin{array}{l} T = \lim_{t\to\infty} T_t \\ \lambda = \lim_{t\to\infty} \lambda_t \end{array}\right\}$$

assuming the limits exist gives $\bar{N} = \lambda \cdot T$ (or $L = \lambda W$)

$$\bar{N} = \lambda \cdot T$$

Typical Dial Machine



10/23/13 © Daniel E Whitney

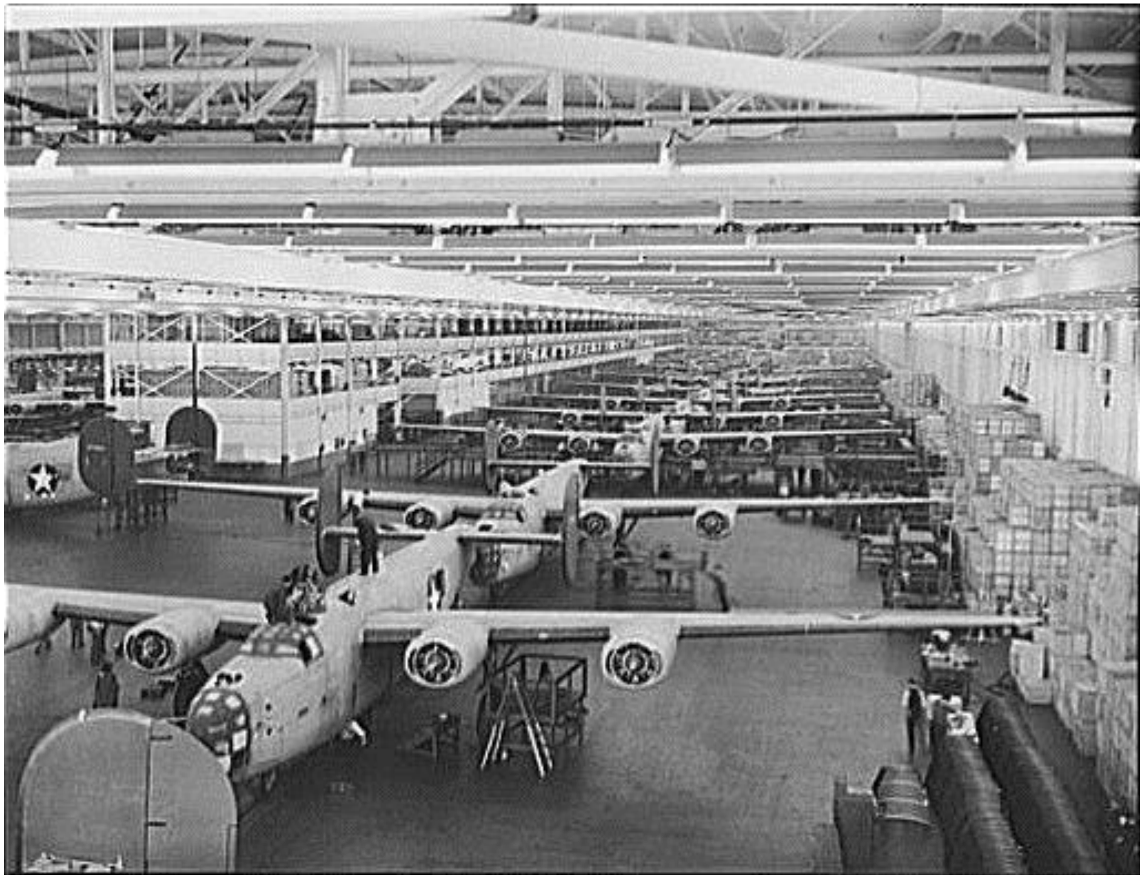
Q. You want a high rate of production λ, but if you fill too fast the liquid comes out. What do you do?

A. Fill while the bottle is moving making T long enough to avoid losing any liquid.

This results in long lines and large factories $\bar{N} = \lambda \cdot T$

Ford's Willow Run Factory

Moving assembly line production of B-24s



Ford's Willow Run plant - 10 mo delay, but in 1944 produced 453 airplanes in 468 hrs

About 1 plane every hour!

How long did they work on assembly?

- Production rate when fully running was about 1 plane very hour

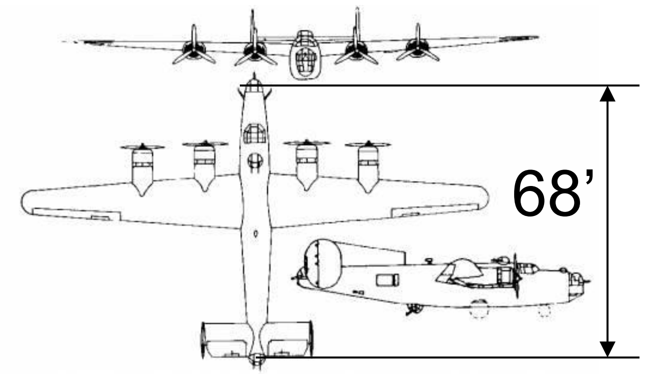
How long did they work on assembly?

- Production rate when fully running was about 1 plane very hour
- Little's Law: $L = \lambda W$
- λ = 1 plane/hr
- L = ? "Assembly line was over one mile"
- W = ?

How long did they work on assembly?

- Production rate when fully running was about 1 plane very hour
- Little’s Law: $L = \lambda W$
- λ = 1 plane/hr
- L = 5280’/68’ = 78 planes,

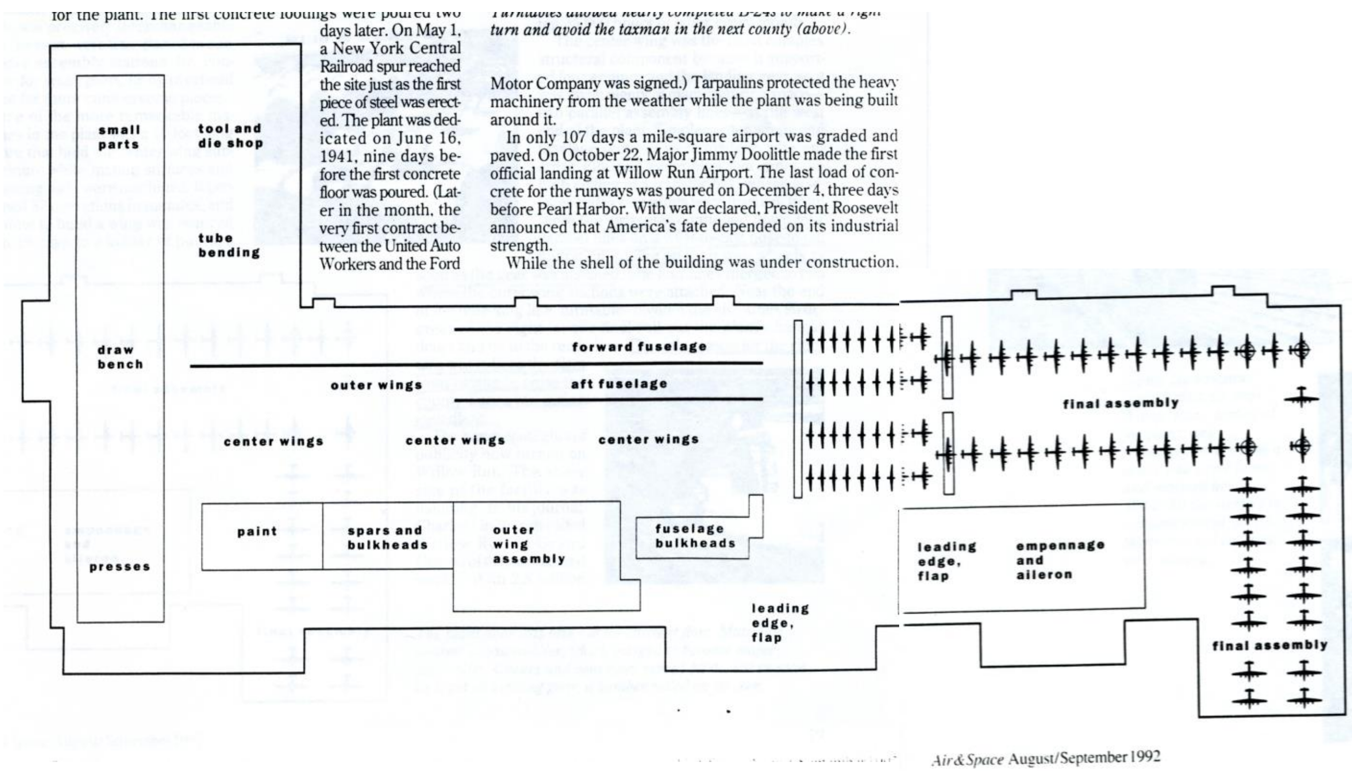
  (if heel to toe for one mile)
- $W = L/\lambda \approx 78$ hours



Willow Run

Two lines converge into one

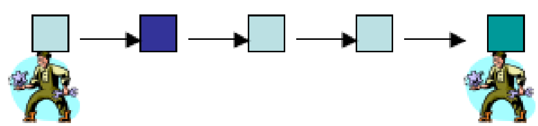
Ford's Willow Run Factory



Assembly Line, L ~ 81 planes, implies around 81 hrs/plane

Applying Little's Law

- Boundaries are arbitrary, but you must specify eg. waiting time + service time
- Internal details are not considered eg. first in first out, flow patterns etc..

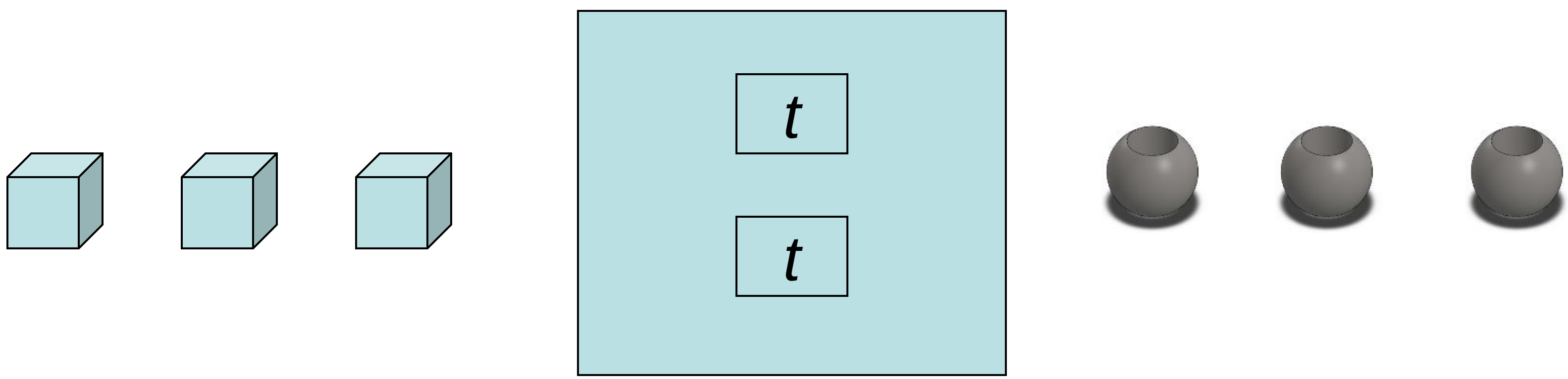


Transfer line

FMS *(Non-synchronous)*

Confusion when talking about time

Time to process one part = t
What is the cycle time?
a) t, or b) t/2?

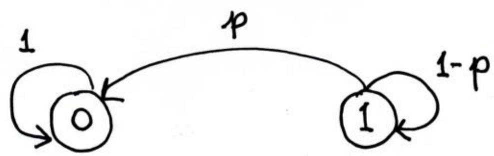


Unreliable Machine

- Ref S. B. Gershwin (handout - see web)
- Preliminaries: conditional probability and Markov chains - transition probabilities
- Probability machine is down - exponential distribution

Exponential distribution

State	Meaning
0	Machine down
1	Machine up



$$\begin{bmatrix} P(0,t+1) \\ P(1,t+1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & p \\ 0 & 1-p \end{bmatrix}}_{\text{transition matrix}} \begin{bmatrix} P(0,t) \\ P(1,t) \end{bmatrix}$$

Exponential distribution for failure rate

$$\text{p.d.f.} = pe^{-pt}$$

$$\text{Expectation} = \frac{1}{p} = \text{MTTF} \; ; \quad \text{Var} = \frac{1}{p^2}$$

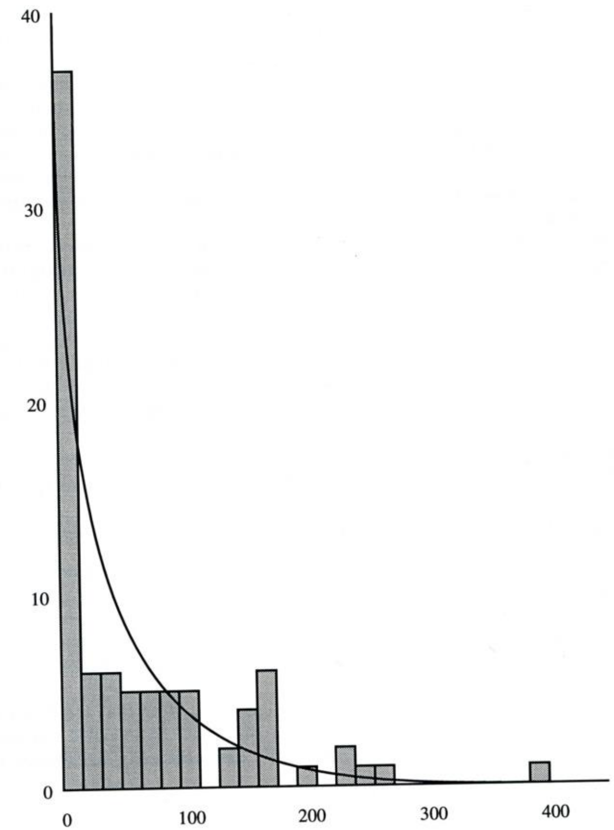
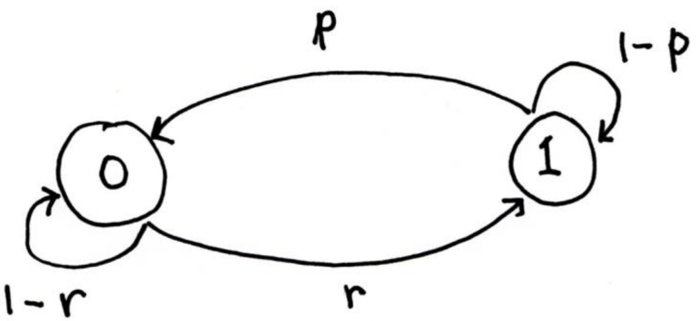


Figure 2.7: Exponential Density Function and Samples

Note: MTTF = mean time to failure

Unreliable Machine with Repair



$$\begin{Bmatrix} p(0,t+1) \\ p(1,t+1) \end{Bmatrix} = \begin{bmatrix} 1-r & p \\ r & 1-p \end{bmatrix} \begin{Bmatrix} p(0,t) \\ p(1,t) \end{Bmatrix}$$

$$\frac{1}{p} = \text{MTTF} \quad ; \quad \frac{1}{r} = \text{MTTR}$$

Note: MTTR = mean time to repair

Operation dependent

e.i. machine can only fail when it is operating

Consider a long time interval T,

say there are m_i failures for machine i

$$\therefore \text{ Total downtime } = D = \sum_{i=1}^{k} m_i \, MTTR_i = \sum_{i=1}^{k} \frac{m_i}{r_i}$$

$$\text{Total up time } \quad U = T - D$$

$$\#\text{ failures} = m_i = \frac{U}{MTTF_i} = p_i U$$

this gives

$$U = T - U \sum_{i=1}^{k} \frac{p_i}{r_i}$$

or

$$\frac{U}{T} = \frac{1}{1 + \sum_{1}^{k} \frac{p_i}{r_i}}$$

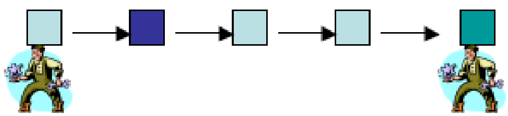
Note: MTBF = mean time between failures = MTTF + MTTR

for one machine

$$\frac{U}{T} = \frac{r_i}{r_i + p_i} = \frac{MTTF_i}{MTTF_i + MTTR_i}$$

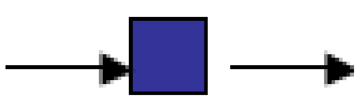
Operation Dependent

- Multiple machines (Transfer line)



Buzacott's formula, $$\mu = \frac{1}{\tau} \times \frac{1}{1 + \sum_{1}^{k} \frac{MTTR}{MTTF}}$$

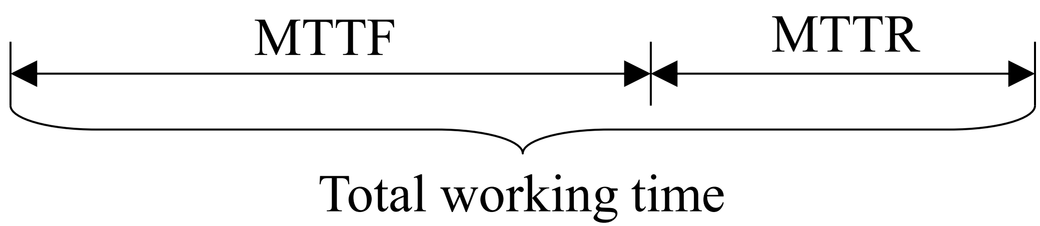
- Single Machine



$$\mu = \frac{1}{\tau} \times \frac{MTTF}{MTTF + MTTR}$$

τ = service time without failures

Time dependent approach



$$\text{Machine up} = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

$$\text{Machine down} = \frac{\text{MTTR}}{\text{MTTF} + \text{MTTR}}$$

$$\text{Average Production rate} = \frac{1}{\tau} \times \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

Where, τ = operation time

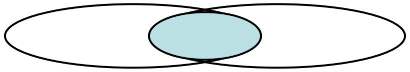
Estimation of μ

A B

"up" P_A P_B

Assumption: time dependent failure
(not quite right, but pretty close)

Probability that both A and B are up is $A \cap B$



$$A \cap B = P_A P_B$$

$$\text{Production rate} = \boxed{\frac{1}{\tau} P_A P_B} = \frac{1}{\tau} \frac{MTTF_A}{MTTF_A + MTTR_A} \times \frac{MTTF_B}{MTTF_B + MTTR_B}$$

$$= \frac{1}{\tau} \frac{1}{1 + \alpha_A} \times \frac{1}{1 + \alpha_B}$$

Where, $\alpha_i = \dfrac{MTTR_i}{MTTF_i}$

$$= \frac{1}{\tau} \frac{1}{1 + \alpha_A + \alpha_B + \alpha_A \alpha_B}$$

Estimation of μ (continued)

$$= \frac{1}{\tau} \frac{1}{1 + \alpha_A + \alpha_B + \alpha_A \alpha_B}$$

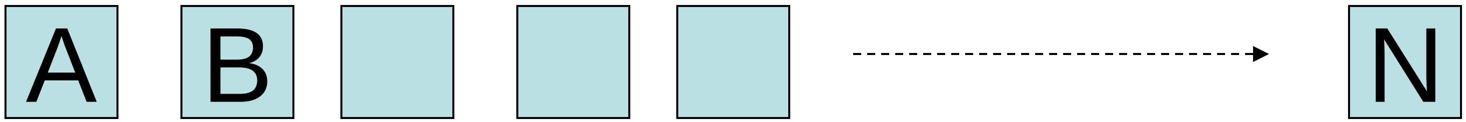
Note: $\alpha_A \alpha_B << 1$

Ignoring higher order terms,
Same as Buzacott's result

$$\mu \approx \frac{1}{\tau} \frac{1}{1 + \sum_{1}^{2} \alpha_i}$$

Note: Buzacott's formula is also from an approximation:
Doesn't take into account two simultaneous failures,
nor buffer capacity in machine

Example:Transfer Line



infinite buffer $\mu_0 = (1/\tau \text{ x } p)_{bottleneck}$

zero buffer $\mu_\infty = 1/\tau \text{ x } p_A p_B \ldots p_N$

example; transfer line, all p = 0.9

$\mu = (0.9)^N \text{ x } 1/\tau$

N=1 $\mu = .9 \text{ x } 1/\tau$

N=10 $\mu = .35 \text{ x } 1/\tau$

N=100 $\mu = .00003 \text{ x } 1/\tau$

Time dependent

$\mu = (1/(1+0.111N)) \text{ x } 1/\tau$

N=1 $\mu = .9 \text{ x } 1/\tau$

N=10 $\mu = .47 \text{ x } 1/\tau$

N=100 $\mu = .0825 \text{ x } 1/\tau$

Operation dependent

Case Study Example

Hewlett-Packard Uses Operations Research to Improve the Design of a Printer Production Line

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INTERFACES **28:** 1 January–February 1998 (pp. 24–36)

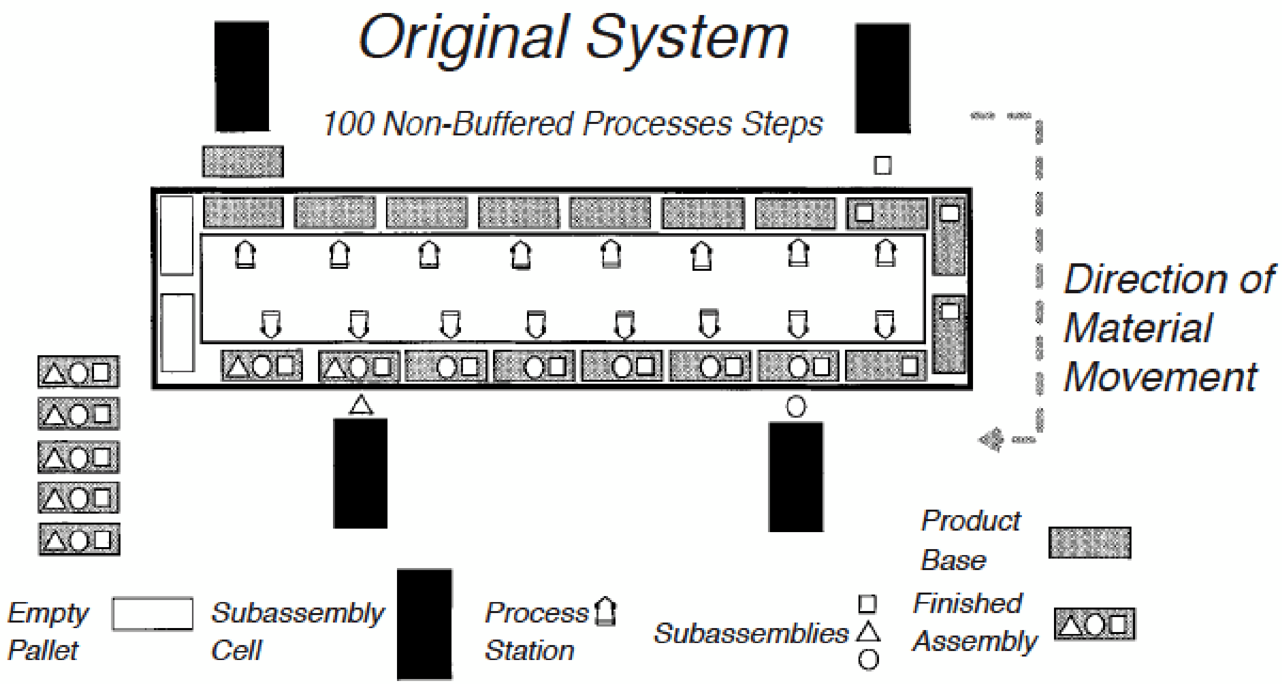


Figure 2: In the original system design, the base is assembled in the upper left subassembly cell (black rectangle). It is attached to a pallet and it moves clockwise on the main loop as various operations are performed. The first subassembly is added at the upper right. After additional operations take place and further subassemblies are added, the completed print mechanism is separated from the pallet. The pallet stays in the system and the completed assemblies are moved downstream. The main loop contains 30 automated work stations. Each subassembly cell does approximately as much work as four main loop stations. Essentially no in-process inventory space was designed in the system.

Issues

- Huge demand for printers
- 200,000 (manual) $\rightarrow$ 300,000/mo
- Add automation
- Build one station: reliability 0.99 not 0.995
- $(0.995)^{100} = 0.6057$; $(0.99)^{100} = 0.366$
- Simulation Vs Analytical Models

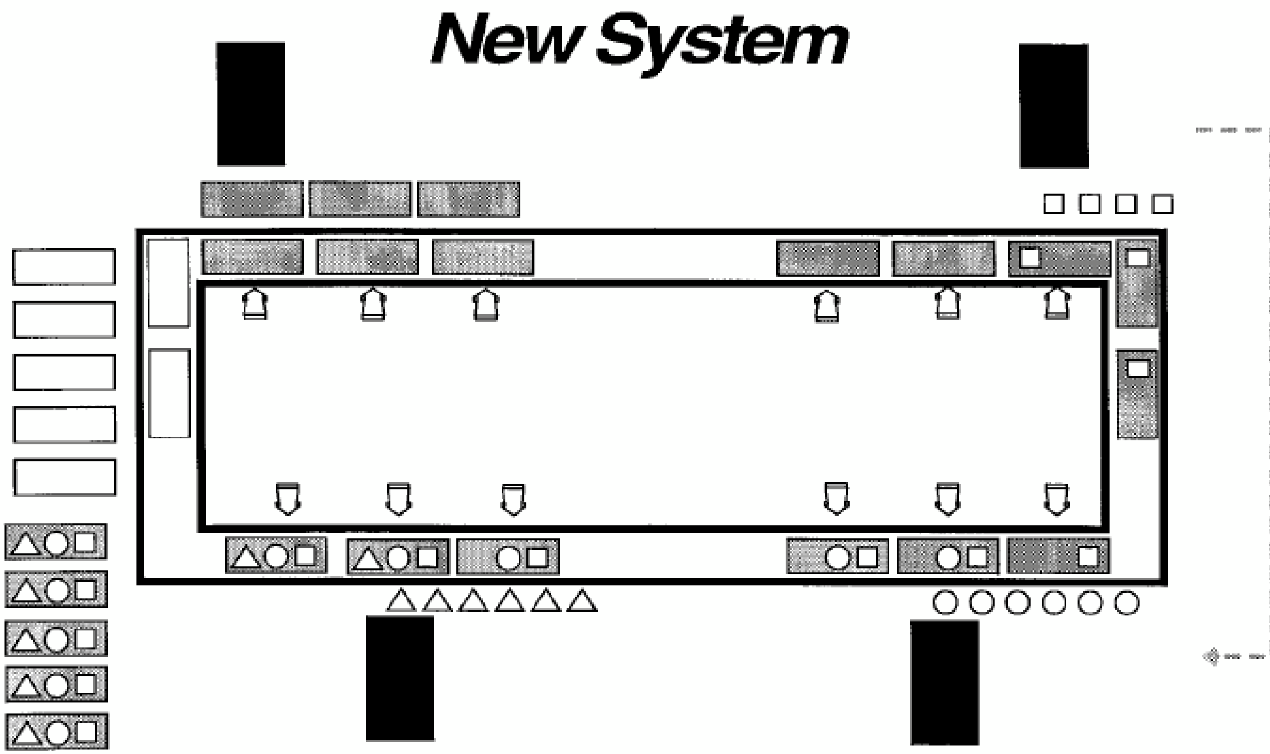
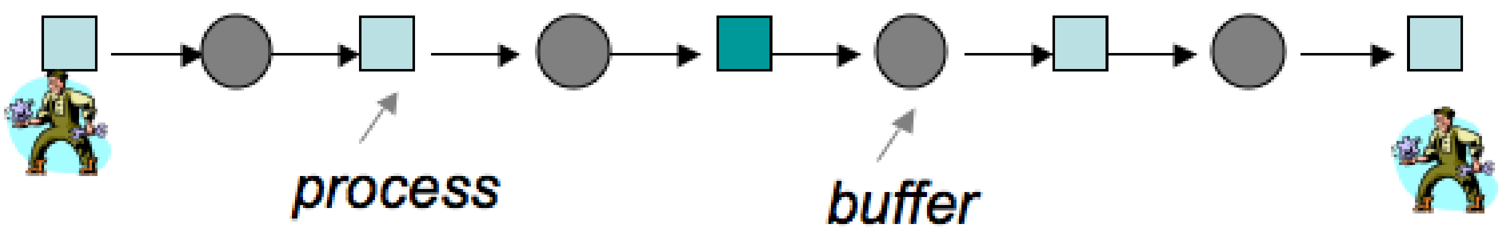


Figure 4: We recommended the addition of (1) an empty pallet buffer, (2) space for in-process inventory between the subassembly systems and the main line, and (3) space on the main line.

Summary: Production Rates

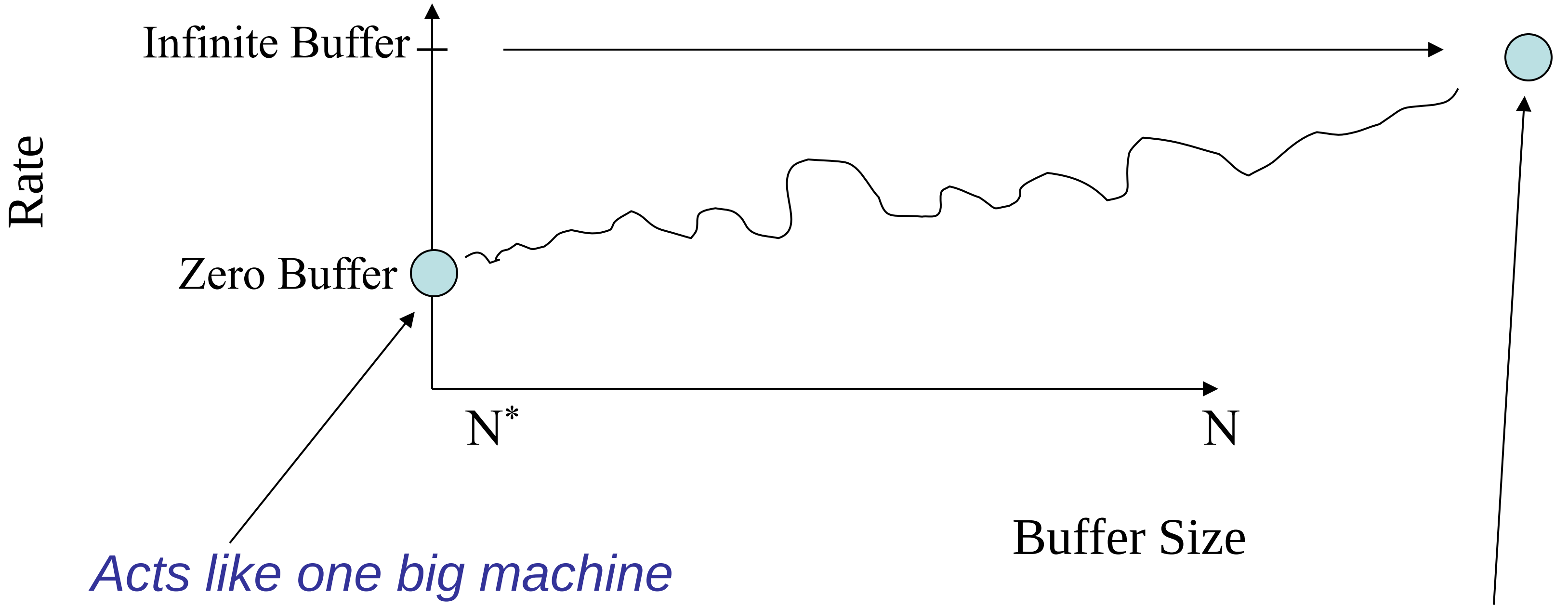
Zero Buffer: $$\frac{1}{\tau}\cdot\frac{1}{1+\sum_{1}^{n}\frac{MTTR_i}{MTTF_i}}$$ *Transfer line*

Infinite Buffer: $$\min\left(\frac{1}{\tau_i}\cdot\frac{MTTF_i}{MTTF_i+MTTR_i}\right)$$ *Bottleneck*



Finite Buffer Size

How do the two cases connect for finite buffers?



Acts like one big machine

Machines are independent rate is controlled by the slowest machine "bottleneck"

A small amount of buffer space helps a lot, but too much is costly

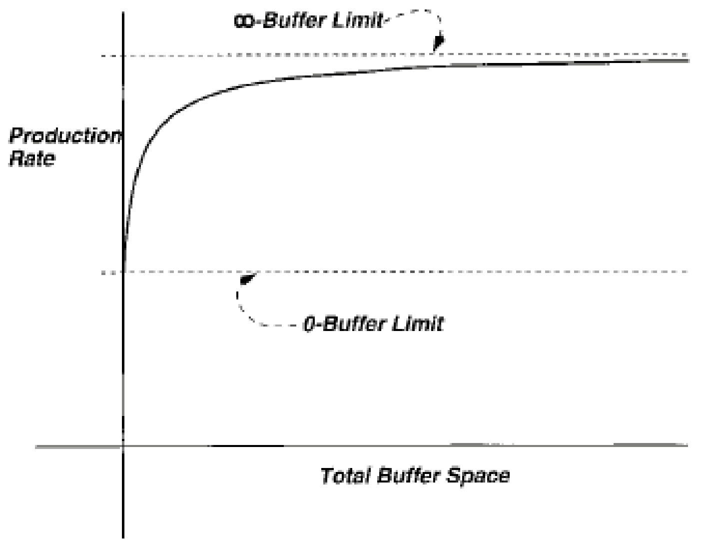
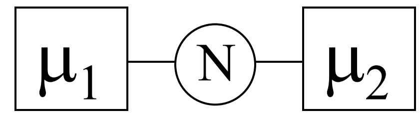


Figure 3: The production rate increases as in-process inventory space increases. This increase is rapid at first and then small. The upper and lower limits are easy to calculate, but the rest of the curve requires the decomposition method.

Finite buffer approximation

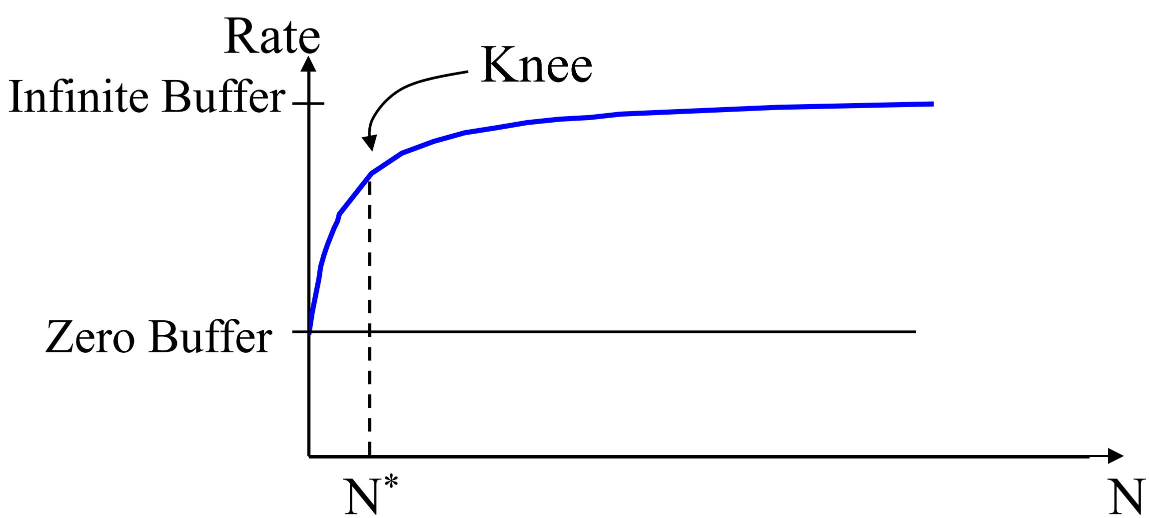
For a two machine system :



Average Downtime is $\dfrac{MTTR_1 + MTTR_2}{2}$

and, $\mu 1 \approx \mu 2$, call the rate μ.

Gershwin's Approximation: $N^* \approx 2 \text{ to } 6 \times \overline{MTTR} \times \mu$



Simulation of a 20 machine, 19 buffer (cap = 10 parts) Transfer line. Each machine with one minute cycle time could produce 4800 parts per week. MTTF 3880 minutes, MTTR 120 minutes. *See Gershwin p63-64*

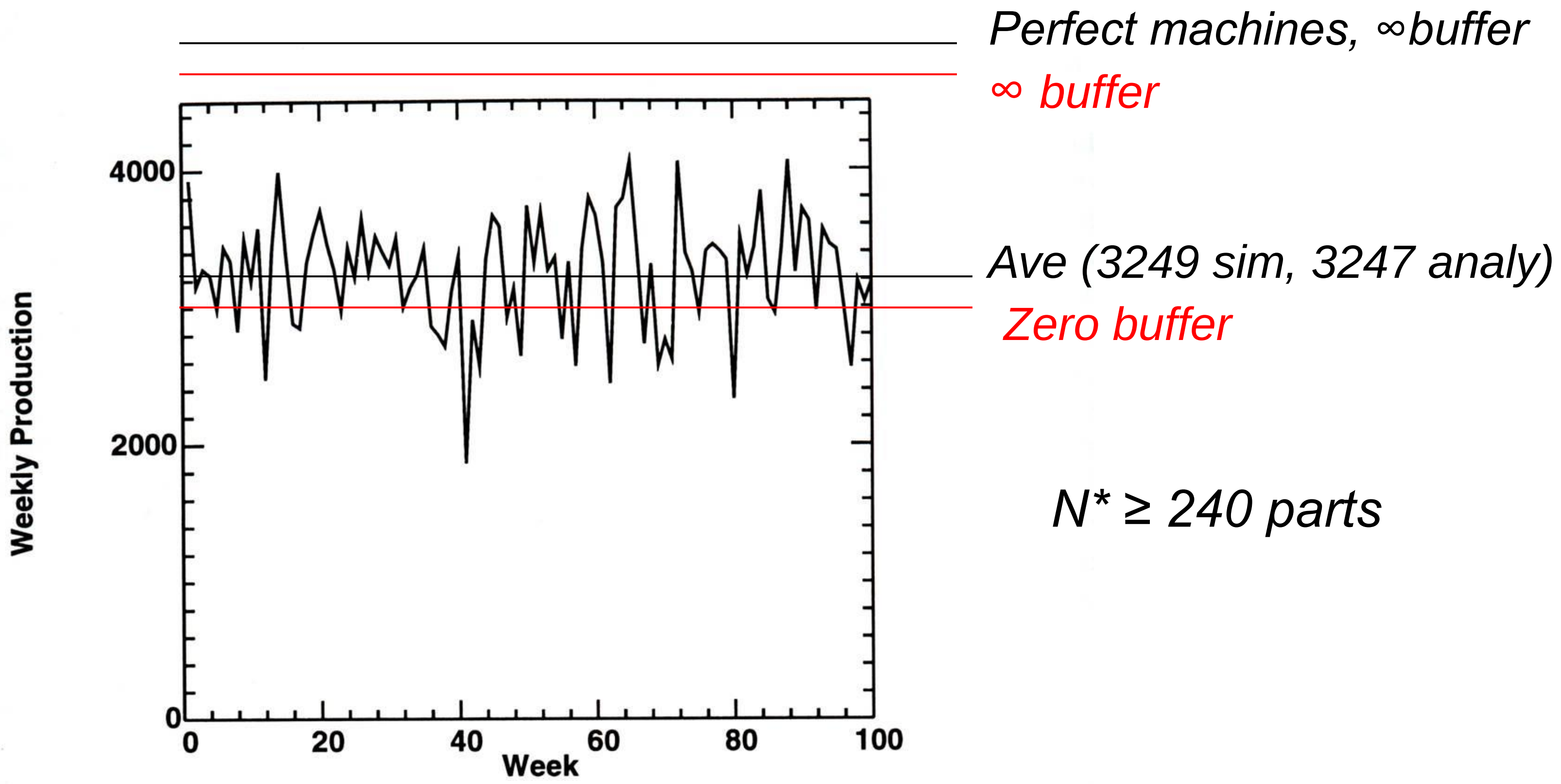
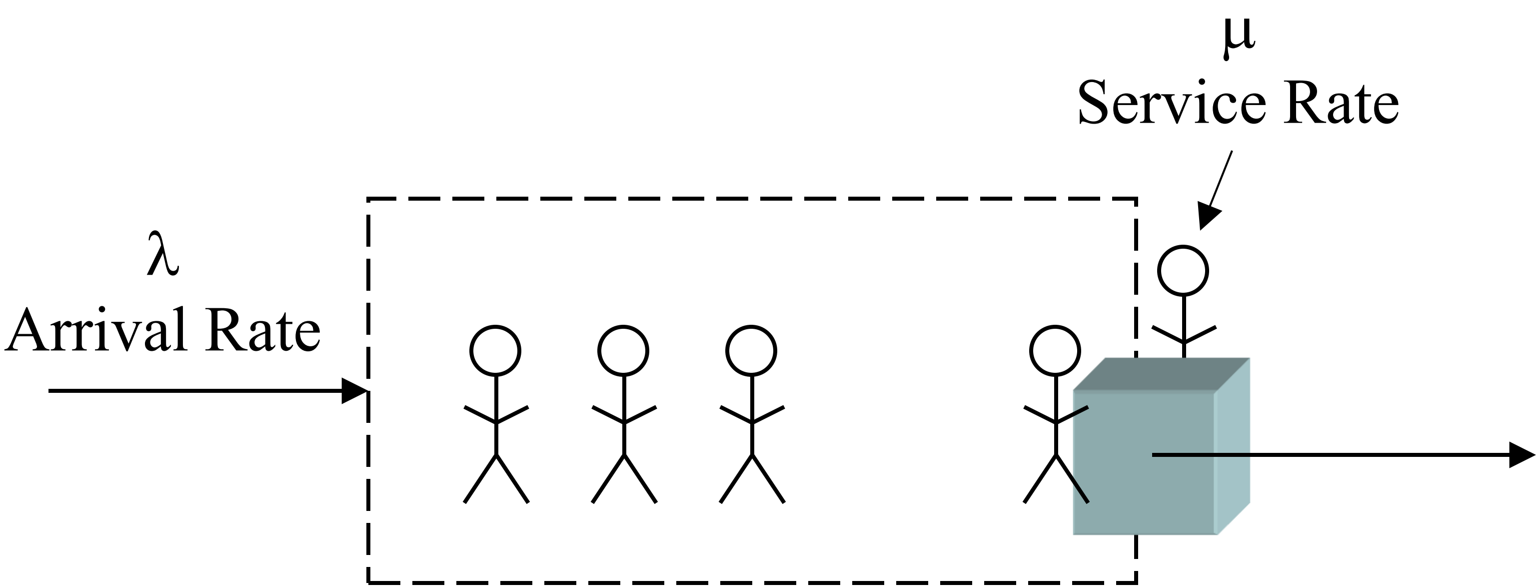


Figure 3.2: Production Variability

N ≥ 240 parts*

M/M/1 Queue



..how the inventory in the system grows as you approach capacity

(λ & μ vary according to exponential distribution)

Steady State $(\lambda < \mu)$

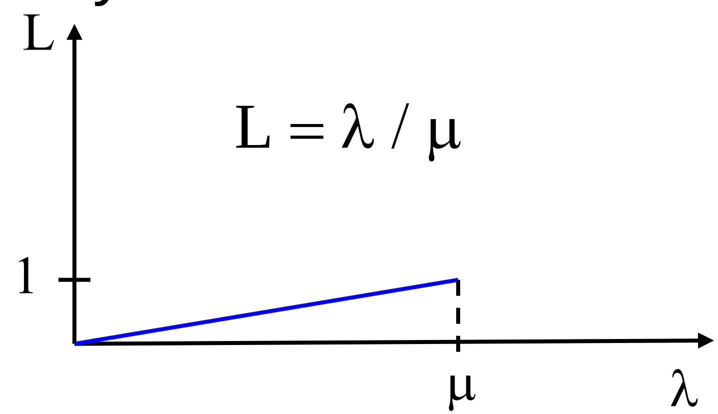
$\lambda < \mu$

Consider the <u>deterministic</u> case:

- How many people are in the system?

A.

$\lambda = 0$	L = 0
$0 < \lambda < \mu$	0 < L < 1
$\lambda = \mu$	L = 1



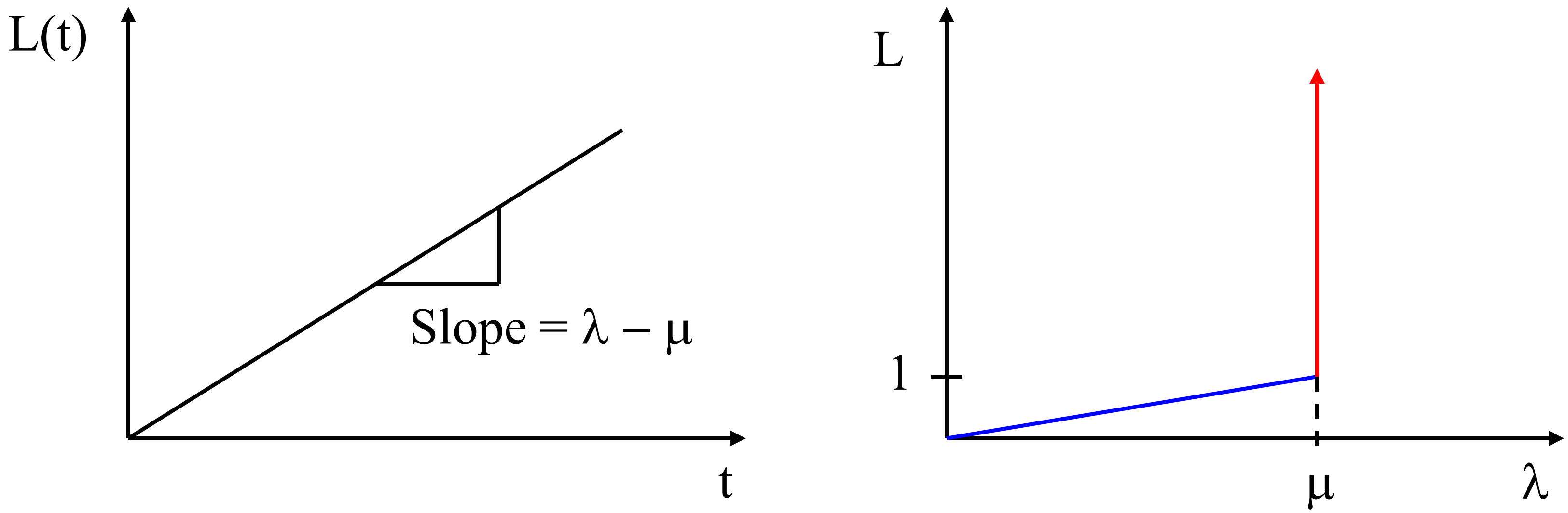
Note: From Little's Law : Time in system, W = L / λ

Since L = λ / μ for $\lambda < \mu$ ➔ W = 1 / μ

When $\lambda > \mu$

- What happens at $\lambda > \mu$?
- There is no steady state, parts in the system grow without limit.

As t $\rightarrow \infty$, L $\rightarrow \infty$

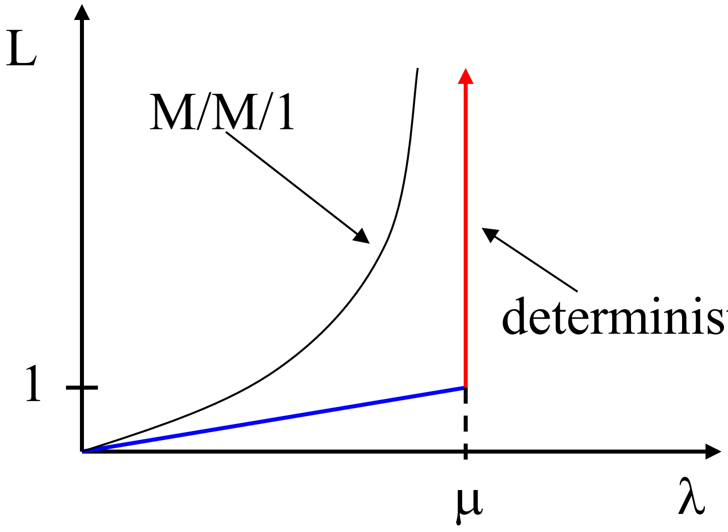


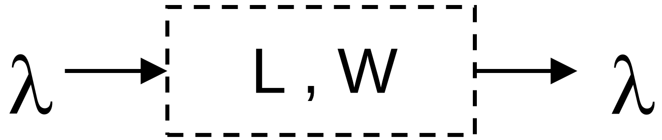
M/M/1 Queue Result

Arrival rate = λ , Service rate = μ, where $\lambda \leq \mu$

L = "Inventory" = $\lambda / (\mu - \lambda)$

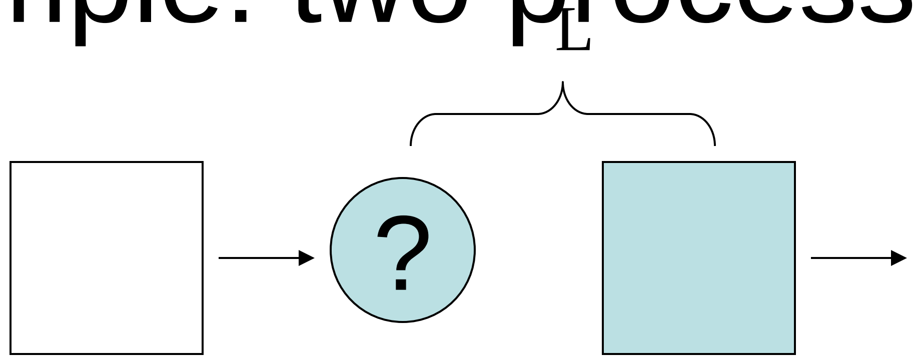
W = Time in system = $1 / (\mu - \lambda)$





See Notes: Principles needed to derive M/M/1 queue result - on website

example: two processes



Process A:
never starved
outputs parts
at average rate λ
with an exponential
distribution

Process B:
with average process
rate $\mu = (5/4)\lambda$ also
with an exponential
distribution

Parts in the system: deterministic: $L = 4/5$; M/M/1: $L = 4$

M/M/1 Queue interpretation

- Overly simplistic but tractable
- Arrivals (always “on”) vs departures (stop when the queue is empty)
- Behavior as you approach capacity

G/G/1 Queue result

Note: this result shows the same nonlinear rise in W and L as the system approaches capacity as the M/M/1 queue did

A more useful queueing result is for the G/G/1 queue

G ⇒ general distributions for arrival and service times with

$$\text{Expectation (arrival)} = \frac{1}{\lambda} \;;\; \text{Exp(service)} = \frac{1}{\mu}$$

$$(\text{Coef of variation})^2 = \frac{\text{Variance}}{\text{mean}^2} \Rightarrow c_\lambda^2 \text{ and } c_\mu^2$$

$$W_q = \text{Time in queue (approx)} = \left(\frac{c_\lambda^2 + c_\mu^2}{2}\right)\left(\frac{u}{1-u}\right)\left(\frac{1}{\mu}\right)$$

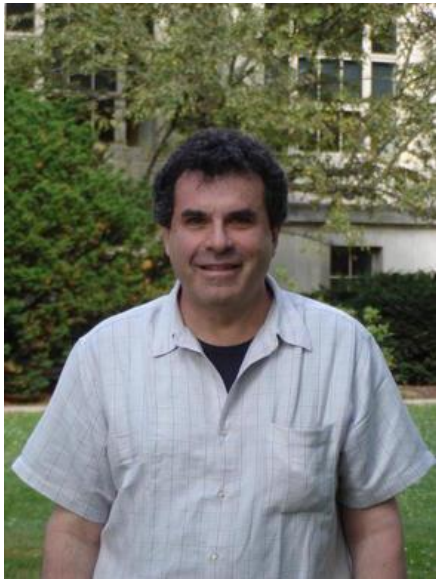
$$u = \text{utilization} = \frac{\lambda}{\mu}$$

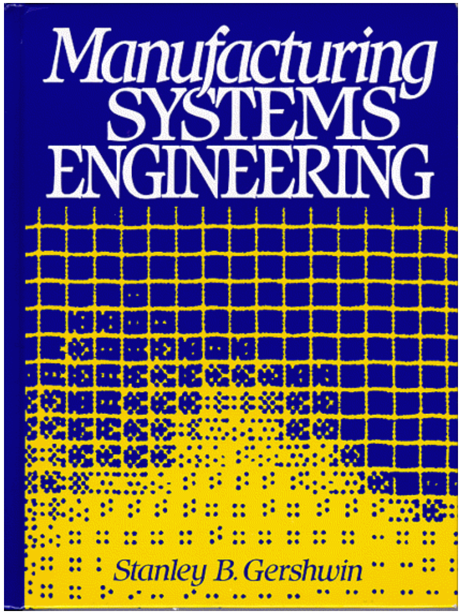
Limitations: $c_\lambda^2,\ c_\mu^2 \le 1$; $\frac{\lambda}{\mu} < 0.95$

Ref. Hopp & Spearman, Factory Physics p. 277
(this approximation used in several commercially available mfg queueing analysis packages)

Note: $W = W_q + 1/\mu$

For more details take 2.852

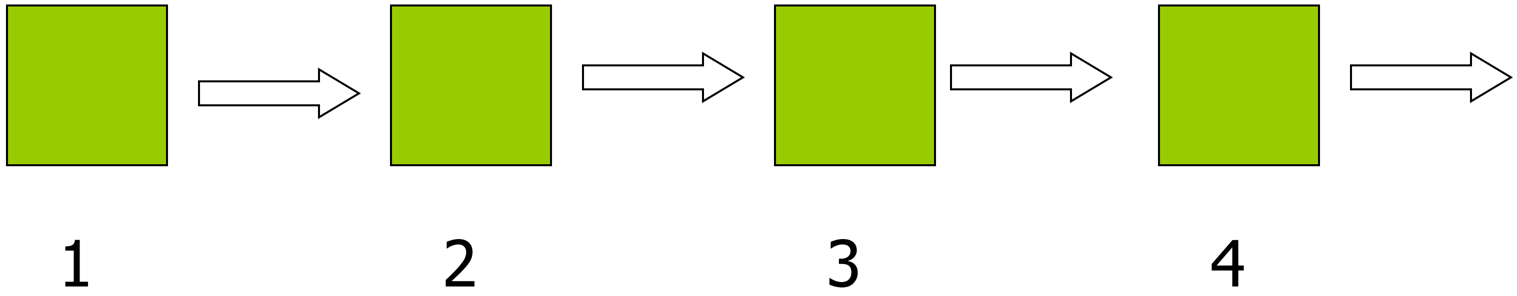


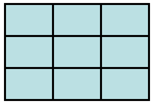


Push Vs Pull

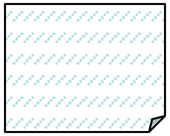
Push and Pull Systems

Machines





Parts

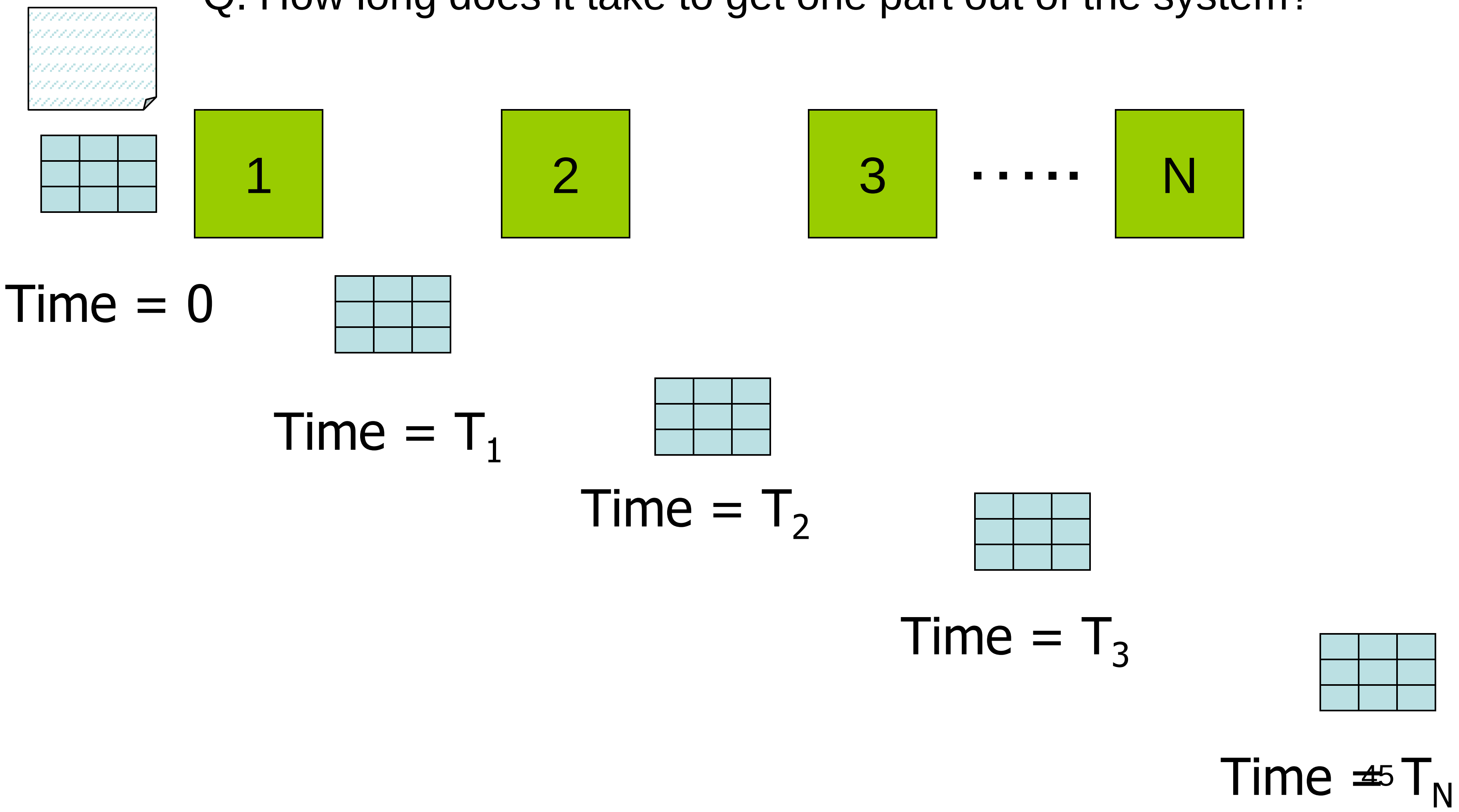


Orders

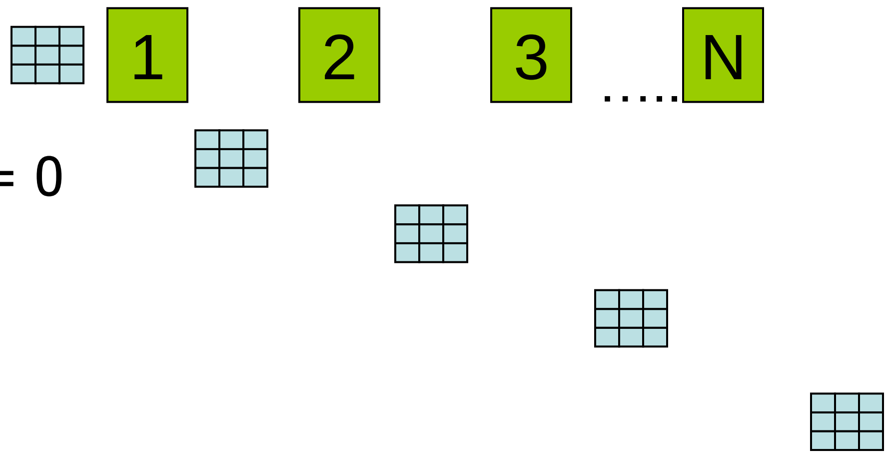
Push Systems –

Order (from centralized decision process) arrives at the front of the system and is produced in batches of size “B”.

Q. How long does it take to get one part out of the system?



Push Systems –



Time = 0

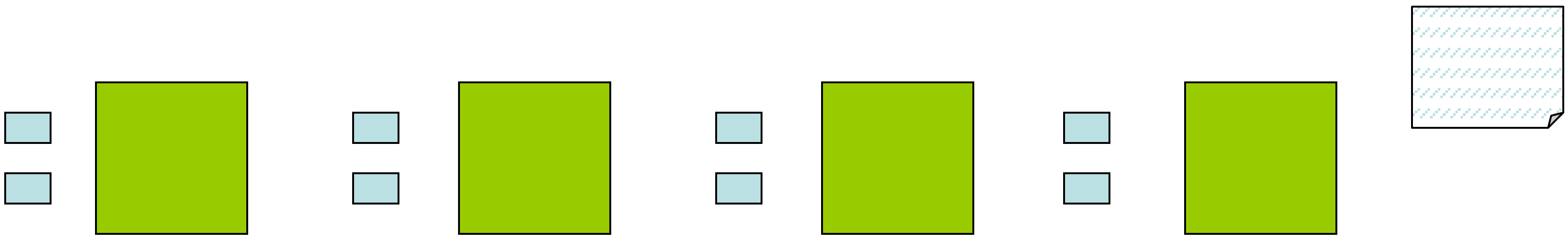
Comment; Of course, this part can come from inventory in a much shorter time, but the point is that the push system is not very responsive.

Time = T_N

If the process time per part is "t" at each of "N" processes, and the batch size is "B", it takes time T_N = "NBt" to get one part through the system.

Pull Systems-

The order arrives at the end of the line and is "pulled" out of the system. WIP between the machines allows quick completion.



Q.How long does it take to pull out one part?

A.The time to finish the last operation "t".

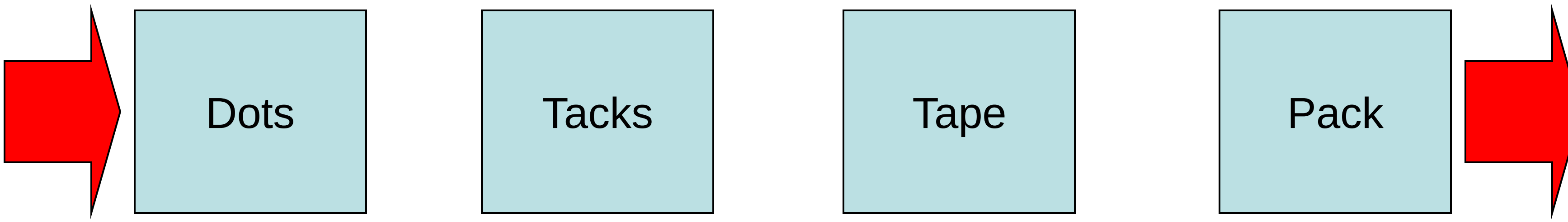
Comparison between Push and Pull Systems

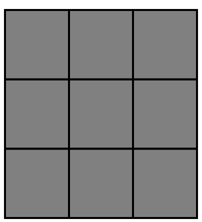
Push system characteristics: Central decision making, local optimization of equipment utilization leads to large batches, large inventories and a sluggish system.

Pull system characteristics: Local decision making, emphasis on smooth flow, cooperative problem solving.

See HP Video

HP Video



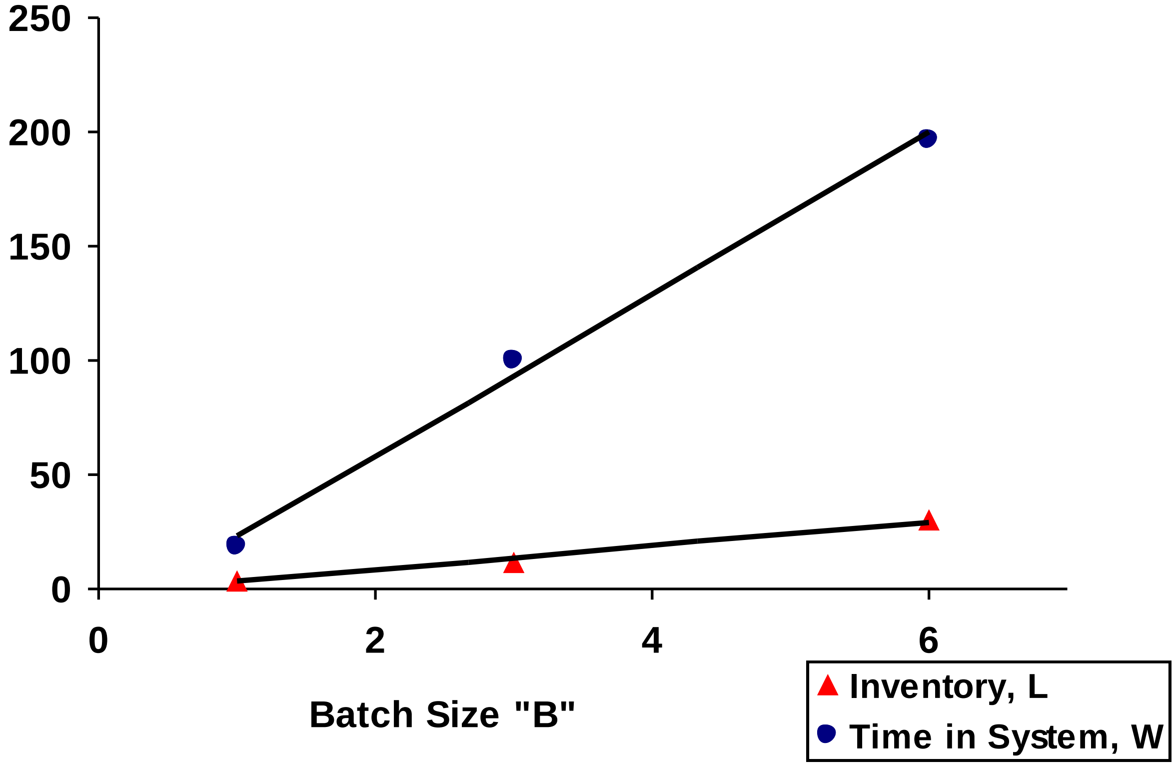


Inventory in the system = L

Time in the system = W

Little's Law $L = \lambda W$

Graphical Interpretation



$L = \lambda W$

$L \approx k_1 B$

$W \approx k_2 B$

$\lambda = L / W = k_1 / k_2$

HP Video Results

	Push system (6)	**Pull (3)**	**Pull (1)**
Space	2 Tables	2 Tables	1 Table
WIP	30	12	4
"Cycle time" = **W**	3:17	1:40	0:19
Rework Units ≈ **WIP***	26	10	3
Quality Problem	Hidden	Visible	Visible
Production Rate λ **= L* / W**	7.9 parts/min	6 parts/min	9.4 parts/min

References

- Kleinrock (Little’s Law)- handout
- Gershwin (exp dist, unreliable machine, M/M/1 queue) - webpage handout