MIT 2.810 Manufacturing Processes and Systems

Fall 2013

Solutions to Homework 8

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Problem 1
Process Capability

We are studying two processes for machining a part. Process A produces parts which have a mean length of 100 and a standard deviation of 3. Process B produces parts which have a mean length of 105 and standard deviation 1. The design specifications for the part are 100 ± 10. Calculate:

1. C_p for each process,
2. C_{pk} for each process,
3. The percentage of parts which are out of specification limits for each process. State the assumptions you need to make to estimate this percentage.

Answer:

We have, USL = 110, LSL = 90.

	Process A ($\mu = 100, \sigma = 3$)	Process B ($\mu = 105, \sigma = 1$)
$C_p = (USL - LSL)/6\sigma$	1.111	3.333
$C_{pk} = \min((USL - \mu)/3\sigma, (\mu - LSL)/3\sigma)$	1.111	1.667

Table 1: Process capability indices for both processes, for individual parts

Assume:

1. Part dimension is distributed normally with the means and standard deviations for each process as stated in the problem.
2. The process is in statistical control.

For Process A:

By symmetry, we will have equal number of parts out-of-spec on both sides of the specification limits.

For the LSL: $z = (x - \mu)/\sigma = (90-100)/3 = -3.333$.

Then, $\Phi(-3.333) = 0.00043$ or 0.043%.

Since we have the same fraction of parts out-of-spec on the upper limit, the total fraction of parts which will fall outside specification limits is: 0.086% or 860 parts per million.

For Process B:

In this case, the process is not centered between the specification limits. We calculate the out-of-spec fraction for each spec limit separately as follows:

For the LSL: $z = (x - \mu)/\sigma = (90-105)/1 = -15$.

Then, $\Phi(-15) \sim= 0$.

For the USL: $z = (x - \mu)/\sigma = (110-105)/1 = -5$.

Then, $\Phi(-5) \sim= 2.86652E-07$ i.e., 2.86652E-05%.

Thus, the total fraction of parts out-of-spec for process B is 0.286 parts per million.

Problem 2
Process Capability and Tolerance Stack-up

Suppose that 20 parts manufactured by the processes in problem 1 were assembled so that their dimensions were additive. That is,

$$L = L_1 + L_2 + \ldots + L_{20}$$

The specifications on the final length are 2000 ± 200. Which process would you prefer to produce the parts? Why? Do the process capability indices provide any guidance in selecting the process?

Answer:

We assume that the product dimensions can be treated as independent random variables. Then, using the formulae from lecture 4 [process performance], slides 63-66, we get,

For process A,

$$E[L] = E[L_1] + E[L_2] + \ldots + E[L_{20}] = 20 * 100 = 2000.$$

$$\sigma_L = \sqrt{N} * \sigma_1 = \sqrt{20} * 3 = 13.41.$$

For process B,

$$E[L] = E[L_1] + E[L_2] + \ldots + E[L_{20}] = 20 * 105 = 2100.$$

$\sigma_L = \sqrt{N} * \sigma_1 = \sqrt{20} * 1 = 4.47.$

We can again estimate the fraction out-of-spec parts based on the final dimension, *L*.

For process A, we get, $2 * \Phi((1800 – 2000)/13.41) = 2 * \Phi(-14.91) \sim= 0$.

Thus, on either side, we expect almost no parts out-of-spec.

For process B, we get,

LSL: $\Phi((1800 – 2100)/4.47) = \Phi(-67.11) \sim= 0$.

USL: $\Phi((2200 – 2100)/4.47) = \Phi(22.37) \sim= 0$.

Thus, on either side, we expect almost no parts out-of-spec.

For individual parts, C_{pk} provides a reasonable estimate of process quality. However, neither the C_p nor C_{pk} of the individual parts predict the performance for the assembled part. Also, if the distribution for the part dimension is not normal, then we cannot attribute a particular percent of out-of-spec parts for a C_p or C_{pk} value. Calculating the process capability indices for the assembled part yields:

	Process A ($\mu = 2000, \sigma = 13.41$)	Process B ($\mu = 2100, \sigma = 4.47$)
$C_p = (USL – LSL)/6\sigma$	4.97	14.91
$C_{pk} = \min((USL - \mu)/3\sigma, (\mu - LSL)/3\sigma)$	4.97	7.457

Table 2: Process capability indices for both processes, for the final product

Again, C_p does not prove to be a good indicator if the process is not centered. We can say that the process is centered about the mean if $C_p = C_{pk}$. A large value of C_{pk} does not necessarily imply that the process is centered between the LSL and USL.

The fraction of parts out-of-spec provides a clearer picture of process quality.

Problem 3
Interchangeable Parts

A shaft and bearing pair that are assembled into a single unit are manufactured as follows. The shaft has diameter that is normally distributed with mean 1.0 in. and standard deviation 0.003 in. The bearing has inside diameter normally distributed with mean 1.01 in. and standard deviation 0.004 in.

1. If the bearing and shaft that are to be assembled are selected at random, what is the probability that they will not fit?
2. If instead we want a fit with at least 0.002 in. clearance, how must the standard deviation of the bearing change such that 99% of the assemblies will succeed?

Answer:

Let μ_s, σ_s be the mean and standard deviation of the shaft diameter, and μ_b, σ_b be the mean and standard deviation of the bearing inner diameter. We assume that the shaft and bearing diameters are independent random variables. Therefore, they are also **uncorrelated**.

1.

We define a new random variable x as the difference in the diameters of the bearing and the shaft.

We want to find the probability that x is less than zero.

$$\mu_x = \mu_b - \mu_s = 0.01 \text{ in.} \quad \text{(By linear sum of expectations)}$$
$$\sigma_x = \sqrt{\sigma_b^2 + \sigma_s^2} = 0.005 \text{ in.}$$

Then,

$$\begin{aligned} \mathrm{P}(x < 0) &= \Phi\left(\frac{0 - 0.01}{0.005}\right), \\ &= 1 - \Phi\left(\frac{0.01 - 0}{0.005}\right), \\ &= 1 - 0.97725, \\ &= 0.02275 \text{ i.e., } 2.275\%. \end{aligned}$$

Thus, we have a 2.275% chance that a randomly selected bearing and shaft will not fit.

2.

Here, we are given a probability of x being less than 0.002 in. as equal to 0.01. Therefore,

$$\begin{aligned} \mathrm{P}(x < 0.002) &= \Phi\left(\frac{0.002 - 0.01}{\sigma}\right), \\ &= 1 - \Phi\left(\frac{0.01 - 0.002}{\sigma}\right), \\ &= 0.01. \quad \text{(Given)} \end{aligned}$$

Therefore,

$$\Phi\left(\frac{0.01 - 0.002}{\sigma}\right) = 0.99.$$

Looking up $\Phi = 0.99$ to find the desired value of z, we get,

$$\left(\frac{0.01 - 0.002}{\sigma}\right) = 2.33\,,$$
$$\sigma = 0.00343\,.$$

That is,

$$\sqrt{\sigma_b^2 + \sigma_s^2} = 0.00343\,,$$
$$\sigma_b = 0.00170 \text{ in.}$$

So, for 99% of the assemblies to succeed, we need σ_b to be 0.00170 in.

Problem 4
Control Charts

We are monitoring a process by plotting x-bar and S charts. Table 1 shows the measurement data from 25 samples, each of size 6. Plot x-bar and S charts for this data. Is the process in control?

Sample Number	Observation					
	1	2	3	4	5	6
1	1.324	1.413	1.674	1.457	1.691	1.515
2	1.431	1.359	1.608	1.467	1.611	1.478
3	1.428	1.487	1.493	1.432	1.567	1.471
4	1.503	1.635	1.384	1.283	1.551	1.434
5	1.560	1.274	1.527	1.436	1.644	1.412
6	1.596	1.545	1.357	1.328	1.420	1.410
7	1.627	1.506	1.837	1.418	1.514	1.587
8	1.419	1.430	1.664	1.607	1.552	1.567
9	1.388	1.728	1.536	1.518	1.369	1.594
10	1.404	1.670	1.509	1.463	1.522	1.547
11	1.416	1.767	1.428	1.593	1.418	1.596
12	1.582	1.336	1.578	1.391	1.756	1.435
13	1.286	1.411	1.445	1.640	1.193	1.498
14	1.495	1.404	1.589	1.646	1.497	1.546
15	1.359	1.286	1.600	1.250	1.547	1.379
16	1.575	1.530	1.517	1.184	1.866	1.410
17	1.368	1.727	1.396	1.501	1.445	1.541
18	1.416	1.386	1.306	1.621	1.557	1.438

19	1.580	1.419	1.654	1.512	1.725	1.528
20	1.711	1.441	1.236	1.382	1.760	1.353
21	1.437	1.505	1.349	1.567	1.488	1.474
22	1.474	1.594	1.658	1.497	1.472	1.583
23	1.592	1.433	1.555	1.530	1.687	1.506
24	1.640	1.524	1.571	1.556	1.553	1.550
25	1.580	1.366	1.624	1.373	1.689	1.455

Table 3: Sample data for problem 4

Answer:

We calculate the sample mean, sample standard deviation and the grand average and the average standard deviation as shown in Table 4.

Sample Number	Statistics	
	Sample mean	Sample standard deviation
1	1.5124	0.1462
2	1.4922	0.0996
3	1.4799	0.0507
4	1.4650	0.1252
5	1.4755	0.1301
6	1.4427	0.1057
7	1.5816	0.1444
8	1.5398	0.0973
9	1.5219	0.1334
10	1.5191	0.0895
11	1.5362	0.1415
12	1.5128	0.1553
13	1.4120	0.1578
14	1.5295	0.0842
15	1.4034	0.1408
16	1.5137	0.2226
17	1.4964	0.1300
18	1.4541	0.1156
19	1.5694	0.1088
20	1.4805	0.2090
21	1.4699	0.0732
22	1.5463	0.0766

23	1.5504	0.0853
24	1.5657	0.0393
25	1.5144	0.1357

Table 4: Sample mean and sample average

The grand average and the average standard deviation are calculated as follows:

$$\bar{\bar{x}} = \frac{1}{25}\sum_{i=1}^{25} \bar{x}_i = 1.5034,$$
$$\bar{s} = \frac{1}{25}\sum_{i=1}^{25} s_i = 0.1199.$$

The factor C_4 can be estimated from the standard tables[1] for sample size, n = 6, as 0.9515. Alternatively, we may also use the approximation of C_4 = 4(n-1)/(4n-3) = 0.9523. We will use the exact value 0.9515 as obtained from the table.

Then, the parameters for the x-bar chart are:

$$\text{UCL} = \bar{\bar{x}} + 3\frac{\bar{s}}{C_4\sqrt{n}} = 1.5034 + 3\frac{0.1199}{0.9515 \times \sqrt{6}} = 1.6577,$$
$$\text{Center Line} = \bar{\bar{x}} = 1.5034,$$
$$\text{LCL} = \bar{\bar{x}} - 3\frac{\bar{s}}{C_4\sqrt{n}} = 1.5034 - 3\frac{0.1199}{0.9515 \times \sqrt{6}} = 1.3490.$$

And, the parameters for the S-chart are:

$$\text{UCL} = \bar{s} + 3\frac{\bar{s}}{C_4}\sqrt{1 - C_4^2} = 0.1199 + 3\frac{0.1199}{0.9515}\sqrt{1 - 0.9515^2} = 0.2362,$$
$$\text{Center Line} = \bar{s} = 0.1199,$$
$$\text{LCL} = \bar{s} - 3\frac{\bar{s}}{C_4}\sqrt{1 - C_4^2} = 0.1199 - 3\frac{0.1199}{0.9515}\sqrt{1 - 0.9515^2} = 0.0035.$$

Using these parameters, we plot the x-bar and S-charts as shown in figures 1 and 2 below. Based on these charts, we conclude that the process is in statistical control.

[1] http://onlinelibrary.wiley.com/doi/10.1002/0471790281.app6/pdf

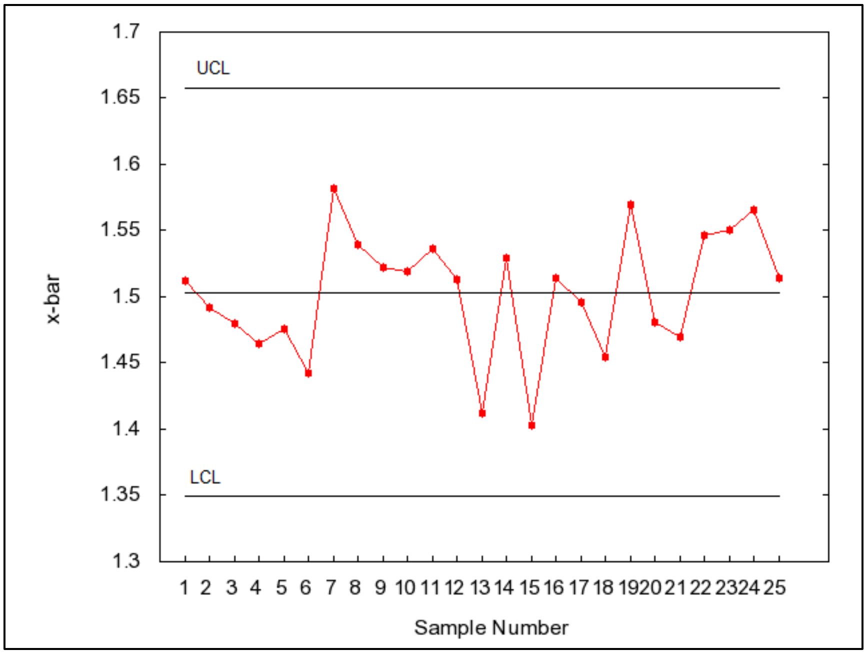

Figure 1: x-bar chart

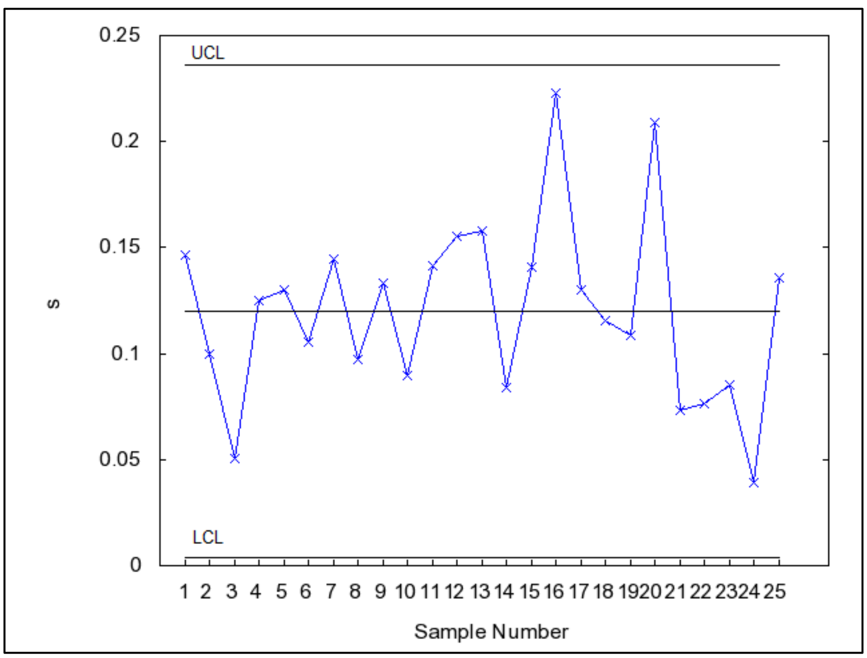

Figure 2: S chart

Note:

The S-chart makes use of all the observations in a sample to calculate the sample standard deviation. Contrast this with the R-chart, where we only use the maximum and minimum values in a sample. This was done mostly in the past for ease of calculation. S-charts are now generally preferred because of the above-mentioned point, and because the proliferation of scientific calculators has made standard deviation calculations easy. For sample sizes smaller than 10, the

R-chart can be used without significantly compromising accuracy or speed. For samples of size 10 or greater, S-charts are recommended.

Problem 5
Tolerance Stack-up

A certain product requires assembling 5 blocks in series. Each block is 100 mm in length. We are considering two processes – milling and sand casting – for manufacturing each 100 mm block. Assume that for each process, the variation is mean centered with $C_p = 1$.
Estimate:

1. Mean length and standard deviation of the length of the final part assuming the lengths are uncorrelated,
2. Mean length and standard deviation of the length of the final part assuming the lengths are correlated.

Hint: Estimate the dimensional tolerances for a part of 100 mm size produced by each process.

Answer:

For each process, we can determine what the dimensional tolerances are by using the graph in figure 35.20 of Kalpakjian-Schmidt (7th edition) or lecture 4, slide 32 [figure 35.19 in the 5th edition]. For a part dimension of 100 mm, we get the following intercepts on the Y-axis of the graph:

Turning and milling: 0.02848 mm.

Sand casting: 1.5738 mm.

We interpret these tolerances as the half-range between our specification limits i.e., (USL – LSL)/2. Then, for each process, using the equation for determining C_p, we calculate the standard deviation of the length of part as follows:

For milling:

$C_p = (USL - LSL)/6\sigma = 0.02848*2/6\sigma = 1.$

Therefore, $\sigma = 0.009493$.

And for sand casting,

$C_p = (USL - LSL)/6\sigma = 1.5738*2/6\sigma = 1.$

Therefore, $\sigma = 0.5246$.

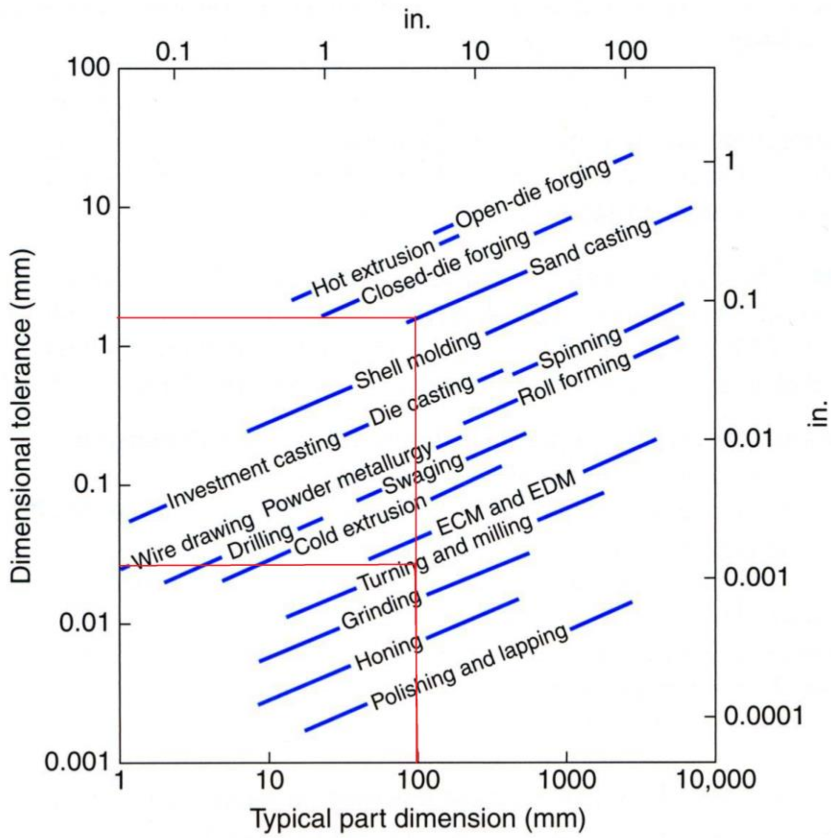

FIGURE 35.20 Dimensional tolerances as a function of part size for various manufacturing processes; note that because many factors are involved, there is a broad range for tolerances.

Figure 3: Dimensional tolerance intercepts for milling and sand-casting for a 100mm block

The final part is produced by assembling five such blocks. We again refer to slides 63-66 from lecture 4 to determine the mean and standard deviation of the final dimension. For n blocks, each having length L_i (i = 1,2,…n) with the same standard deviation, we have:

$$\mathrm{E}[L] = \mathrm{E}[L_1] + \mathrm{E}[L_2] + \cdots + \mathrm{E}[L_n],$$

and,

$$\sigma = \begin{cases} n\sigma_i & \text{if the i} = 1,2,...,n \text{ lengths are correlated,} \\ \sqrt{n}\sigma_i & \text{if the i} = 1,2,...,n \text{ lengths are uncorrelated.} \end{cases}$$

Therefore, for n = 5, we get the following results for each process:

Process	Mean length (mm)	Standard Deviation of the length (mm)	
		L_i correlated	L_i uncorrelated
Milling	500	0.0474	0.0212
Sand Casting	500	2.623	1.173

Table 5: Mean and standard deviation of the assembled part