MIT 2.810 Manufacturing Processes and Systems

Fall 2013

Solution to Homework 10

November 13, 2013

Problem 1

Consider a transfer line as shown in figure 1 below.

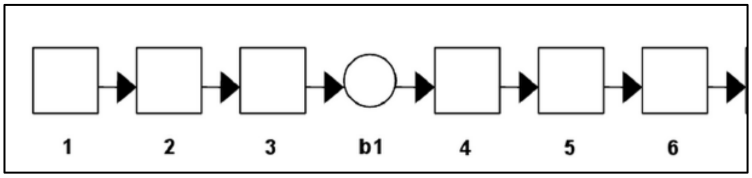

Figure 1: Transfer line layout

The machine parameters are as given in the table below.

	M1	**M2**	**M3**	**M4**	**M5**	**M6**
MTTF (hours)	30	40	20	20	10	50
MTTR (hours)	1	1	1	1	1	1
Processing time (hours)	1	1	1	1	1	1

The buffer has **infinite** storage capacity. Assume operation-dependent failures.

a. Line evaluation

Estimate the production capacity of this system. Assume that the profit per part is $10 and that you sell every part you make. Maintenance costs (not included in the profit calculation) are $1 per hour. Calculate the net profit for this system.

Answer:
The first three machines and the last three machines behave like zero-buffer three-machine lines. We apply Buzzacott's formula to calculate the production rate for each three-machine line.
We get,
$P_1 = 0.9022$, and
$P_2 = 0.8547$.
We can now treat these lines as two machines with an infinite buffer between them.
Then, the production rate is the production rate of the bottleneck = $P_2 = 0.8547$.
Profit = 0.8547 (parts/hr)*10($/part) - 1($/hour) = 7.55 ($/hour).

b. Line improvement: Part I

Identify the bottleneck machine for this line. Suppose that you have a budget of $40,000 to improve the line. Each $10,000 increases the MTTF of a machine by 10 hours. How will you allocate this budget so that you can achieve a production rate of 0.91?

Answer:

We improve the system by identifying the bottleneck and improving it by 10 units until it no longer is the bottleneck. One iteration can be as follows:

1. Increase MTTF of machine M5 from 10 to 20. It is now tied with M4 as the bottleneck. The production rate of the second three-machine line (M4-M5-M6) is 0.8928.
2. Increase MTTF of machine M4 from 20 to 30. Now M5 is the bottleneck. The production rate of the second three-machine line (M4-M5-M6) is 0.9063.
3. Increase MTTF of machine M5 from 20 to 30. The production rate of the second three-machine line (M4-M5-M6) is 0.9202.

Now, this three-machine line is capable of meeting the required demand of 0.91 parts per hour. However, the M1-M2-M3 system has a production rate of 0.9022, which is less than the demand. We continue iterating as follows:

4. Improve the MTTF of machine M3 from 20 to 30. It is now tied with M1 as the bottleneck. The production rate of the first three-machine line (M1-M2-M3) is 0.916.

Thus, this system now meets the required demand rate. The cost of these improvements is: $10,000 * 4 = $40,000. Thus, we are within our budget.

c. Line improvement: Part II

Each 0.005 units decrement in the failure rate of a machine increases the maintenance costs by 3%. Recall that the failure rate, p = 1/MTTF. For example, if you reduce the p-value for machine M2 from (1/40 = 0.025) by 0.005 to 0.02, the maintenance costs would become 1.03 ($/hour). If you could improve only **one** machine, which machine would you apply this improvement to? Set up the equation for the profit of such a system, in terms of the number of decrements, n. What are the upper and lower limits on the value of n? Set up an optimization problem to find the value of n which maximizes profit.

Extra credit: Plot the profit function, and find (manually or using software) the optimum value of n, the resulting failure probability rate for the chosen machine, and the maximum profit.

Answer:
M4-M5-M6 is the bottleneck three-machine line. We could pick any machine to apply the improvement to since the denominator simply consists of a sum of p-values. We focus our attention on machine M5. It has a failure rate, p5 of 0.1. Let n be the number of 0.005 decrements we make to failure rate p5. We can write the production rate and profit of the system as a function of n as follows:

$$\mathrm{P}(n) = \frac{1}{\tau} \times \frac{1}{1 + \frac{0.05}{1} + \frac{0.1 - n \times 0.005}{1} + \frac{0.02}{1}}.$$

Therefore,

$$\mathrm{P}(n) = \frac{1}{1.17 - 0.005n}.$$

Then, the profit per hour (not including maintenance costs) is given by

$$\mathrm{Pr}(n) = \frac{10}{1.17 - 0.005n}.$$

Note that p5 is 0.1. Thus, n can be at most 20 so that p5 is non-negative, and n cannot be negative. Thus, n lies between 0 and 20.

The maintenance cost increases by 3% for every n decrement in p5. That is, the maintenance cost can be written as $(1.03)^n$. Then, the net profit function becomes:

$$\boxed{\text{Net Pr}(n) = \frac{10}{1.17 - 0.005n} - (1.03)^n}$$

Extra Credit:
The profit function is plotted in figure 2 below.
Solving this problem gives n = 10.16 for which, p5 becomes 0.04917. The production rate becomes 0.8935 and the net profit is $7.58/hour.

The function is concave in the region of interest for n in [0, 20]. Thus, we can have two values of n which give the same net profit. A higher value of n means that the failure rate is less. In that case, we would expect less inventory to accumulate in the buffer.

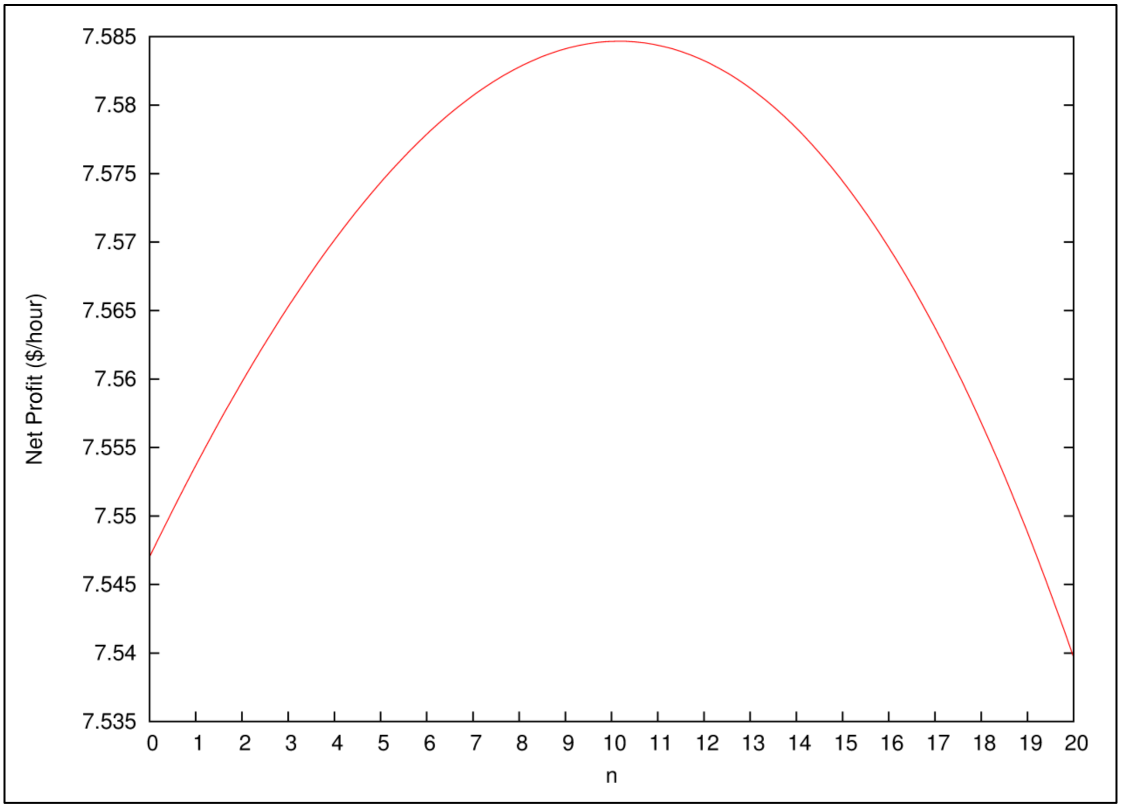

Figure 2: The net profit as a function of n

d. Inventory costs

As the market matures, we are looking to cut our production costs. Inventory costs which were so far not considered in our profit calculations are now considered important. Inventory cost per part per hour is $0.01. You have estimated the production rates and average inventory in the system for various sizes of buffers. These are given in the table below.

Buffer size	Production rate	Average inventory
1	0.7866	0.611
5	0.8006	3.077
10	0.8127	6.218
20	0.8274	12.74
50	0.8449	34.345
100	0.8522	76
200	0.8545	170
400	0.8547	370

1. Considering the inventory costs, find the buffer size (out of the options in the table) which maximizes your profit.
2. What answer would you get for the optimum buffer size using Gershwin's Approximation?

Answer:
1.

Buffer size (A)	Prod rate (B)	Avg. inventory (C)	Profit not considering inventory costs (D = B*10 - $1/hr)	Inventory costs based on $0.01/part/hr (E = 0.01*C)	Net Profit ($/hr) (F = D – E)
1	0.7866	0.611	6.866	0.00611	6.85989
5	0.8006	3.077	7.006	0.03077	6.97523
10	0.8127	6.218	7.127	0.06218	7.06482
20	**0.8274**	**12.74**	**7.274**	**0.1274**	**7.1466**
50	0.8449	34.345	7.449	0.34346	7.10554
100	0.8522	76	7.522	0.76	6.762
200	0.8545	170	7.545	1.7	5.845
400	0.8547	370	7.547	3.7	3.847

Based on the inventory costs, we find that a buffer of size 20 gives us the maximum net profit.

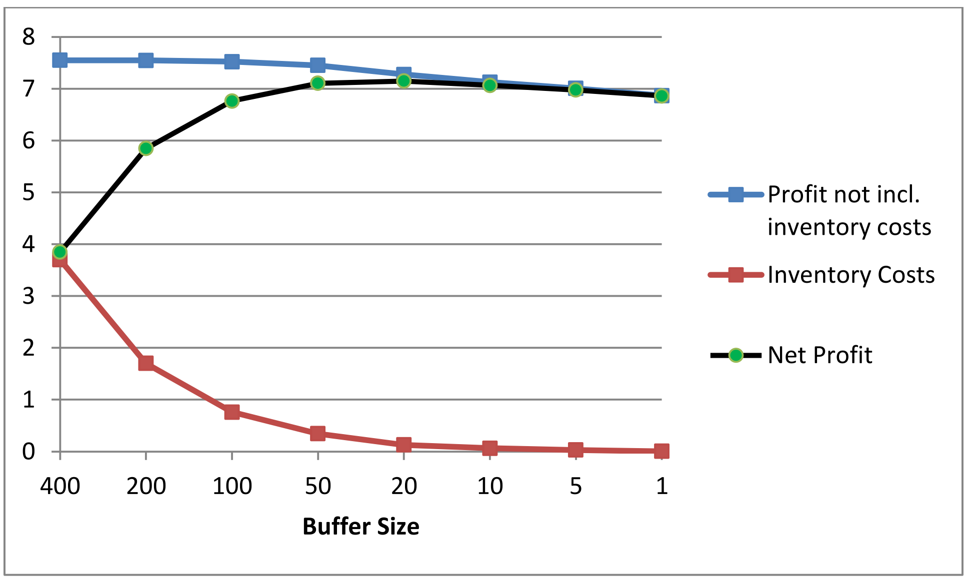

Figure 3: Inventory costs and net profit

Note: Here, we have the first three-machine line producing parts at a faster rate than the second three-machine line. Recall from the "Time and Rate" lecture [slide 37-38], that for an M/M/1 queue with $\lambda > \mu$, the inventory in the system with an **infinite buffer** tends to infinity over time. In this part of the problem, we are considering finite buffers. Here, we witness the phenomenon called **blocking**. When the buffer gets full, the upstream three-machine line gets "blocked" and can only make parts at the rate at which the downstream three-machine line can process parts from the buffer. Thus, we expect the upstream three-machine line to be idle quite often. Because of the assumption of operation-dependent failures, the upstream three-machine line fails **less frequently** than it would without blocking.

2. Let us now use Gershwin's Approximation to find the optimum buffer size. As per the approximation,

$$N^{*} = 2\ to\ 6 * \overline{MTTR} * \mu$$

   Here, $MTTR_1 = 1$ and $MTTR_2 = 1$. Thus, the mean MTTR is 1. Also, μ for each three-machine line is 1. Thus, we get, N* to be between 2 to 6.