MIT 2.810 Manufacturing Processes and Systems

Homework Solution 2a – Machining

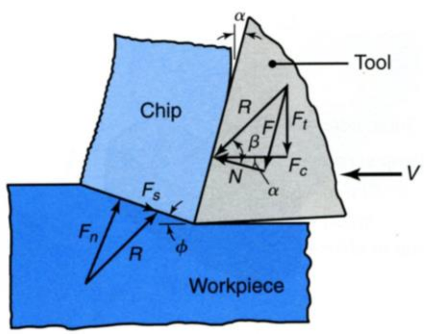
October 2, 2013

Problem 1

a.) For orthogonal cutting, please verify that the following expression for shear stress holds:

$$\tau = \frac{F_c \sin\phi \cos\phi - F_t \sin^2\phi}{t_0 w}$$

Answer: The shear stress, τ, is equal to the shear force divided by the shear area. The shear force is $F_s = F_c*\cos\phi - F_t*\sin\phi$. The shearing takes place along a length $t_0/\sin\phi$, along the entire width, w, of the cut. Substituting these values in the expression for τ, we get the desired expression.



b.) Assuming that the shear angle adjusts to minimize the cutting force, derive the following expression:

Answer:

$$\phi = 45^o + \frac{\alpha}{2} - \frac{\beta}{2}$$

$$\tau = \frac{1}{t_0 w}(F_c \sin\phi\cos\phi - F_c \tan(\beta-\alpha)\sin^2\phi)$$

$$\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\sin\phi\cos\phi - \tan(\beta-\alpha)\sin^2\phi\right) = 0$$

$$-\sin^2\phi + \cos^2\phi - \tan(\beta-\alpha)\times 2\sin\phi\cos\phi = 0$$

$$\tan(2\phi) = \frac{1}{\tan(\beta-\alpha)}$$

But,

$$\tan(2\phi) = \cot(\pi/2 - 2\phi)$$

Therefore,

$$90 - 2\phi = \beta - \alpha$$

$$\phi = 45 + \alpha/2 - \beta/2\,.$$

This result, the so called Merchant equation, gives one a good sense for how the shear angle would vary with changes in rake angle and friction. However, because the actual shear strength of a material may vary (e.g. with strain rate and temperature) rather then remain constraint, this equation should be viewed as an approximate relationship, rather than an exact one.

c.) Cutting forces were measured during the machining of a steel bar and found to be F_c = 520 N and F_t = 189 N. The rake angle on the tool is 10º. Based on this information, calculate the coefficient of friction at the tool-chip interface. Also calculate the shear angle.

Answer:

Ft = Fc*tan(β-α) = Fc*(tanβ – tanα)/(1 + tanβ*tanα).

Substituting Fc, Ft and α, we get μ = tanβ = 0.47, β = 25º.

The, substituting α and β in Merchant's equation, we get, ϕ = 37.5º.

Problem 2

a.) An 8 in. long 304 stainless steel rod of diameter 0.5 in. is being machined on a lathe to a final diameter of 0.48 in. A collet grips the rod so that a 6 in. length can be turned. The spindle speed is 400 RPM and the tool travel rate is 8 in/min. Please calculate the depth of cut d, the feed f, the MRR and the cutting time.

$$d = \frac{0.5 - 0.48}{2} = 0.01in \qquad F = \frac{v}{\Omega} = \frac{8in/\min}{400rev/\min} = 0.02\frac{in}{rev}$$

$$MRR = d \cdot f \cdot \pi D_{ave} \cdot \Omega = 0.01 \times 0.02 \times \pi \times 0.49 \times 400 = 0.12\frac{in^3}{\min}$$

b.) Also calculate the power and the cutting force (the component tangent to the circumference of the rod.)

$$Power = 1.47\frac{hp.\min}{m^3} \times 0.12\frac{in^3}{\min} = 0.18hp$$

$$Note\frac{1hp.\min}{in^3} = 396,000psi$$

$$F_c = u_s \cdot f \cdot d = 396,000 \times 1.47 \times 0.01 \times 0.02 = 116lbs$$

c.) We are also interested in possible deflections caused by the cutting forces acting on the rod. Estimate the elastic twist angle when the tool engages at the very tip of the cantilevered workpiece. Assume the shear modulus G is 10.8 X 10^6 psi.

$$\theta = \frac{TL}{GI_z} = \frac{28.5 \times 6}{10.8 \times 10^6 \times 0.05} = 0.0003 radians$$

$$T = 116 lb \times \frac{0.49}{2} = 28.5 lb.in; I_z = \frac{\pi D^4}{32} = 0.05 in^4$$

d.) Will these forces cause elastic bending of the workpiece? Of the tool?

Answer: The cutting force and transverse force could cause deflection of the part as well as deflection of the tool (e.g. when delicate tools are used such as in micromachining). The analysis is complicated by the fact that the deflect reduces the load. This is a “statistically in determinant” problem and requires a more complicated analysis than just using elementary beam deflection theory.

Problem 3

A steel alloy bar 4 in. in diameter is being turned on a lathe at a depth of cut, d = 0.050 in. The lathe is equipped with a 10-hp electric motor and has a mechanical efficiency of 80%. The spindle speed is 400 rpm. Estimate the maximum feed that can be used before the lathe begins to stall.

Answer:

Since the lathe has a 10-hp motor and a mechanical efficiency of 80%, 8 hp are available for the cutting operation. For steel alloys the specific power required is 3.4 hp min/in3. Therefore, the maximum metal removal rate is

MRR = 8 [hp] / 3.4 [hp min/in3] = 2.3 in3/min

The removal rate is also given by

MRR = πD_{ave} d f N

Therefore, the maximum feed f is

f = 2.82 / π*3.95*0.05*400 = 0.0095 in/rev.

Problem 4

Two tools are being considered for turning a steel rod. The first is a high-speed steel tool and the second is a coated carbide tool. The Taylor Tool Life properties for machining steel are given in table 1.

Properties	**High-Speed Steel**	**Coated Carbide**
n	0.125	0.25
C (ft/min)	200	2200
Cutting Speed, *V* (ft/min)	100	1200

Table 1: Properties for the high-speed steel and coated carbide tools for machining steel

a.) Compare the tool life for these two materials.

$$HSS:\quad (2)^{1/0.125} = 256\,\text{min}$$

$$CC:\quad \left(\frac{2200}{1200}\right)^{1/0.25} = 11.3\,\text{min}$$

b.) Assuming a depth of cut of 0.1 in. and a feed of 0.01 in. for each of the tools, calculate the MRR for each tool.

$$\text{MRR} = d \cdot f \cdot V, \quad \text{where} \quad V = \text{surface speed}$$

$$\text{HSS} \quad \text{MRR} = 0.1 \times 0.1 \times 100 \times 12 = 1.2\text{in}^3/\text{min}$$

$$\text{CC} \quad \text{MRR} = 0.1 \times 0.1 \times 1200 \times 12 = 14.4\text{in}^3/\text{min}$$

c.) Now consider the costs associated with using the two different tool materials. Assume that the HSS tool cost $30 each, and the carbide tool cost $150 each, and it takes 5 minutes to change a tool. If you have an operation, as described above, that requires removing 300 in3 of material and the machine and operator costs are $60/hr which tool results in a lower cost? How about if the machine and operator cost $120/hr.

During one tool life, each tool can cut:

$$\text{HSS} \quad 1.2\frac{\text{in}^3}{\text{min}} \times 256\,\text{min} = 307in^3$$

$$\text{CC} \quad 14.4\frac{\text{in}^3}{\text{min}} \times 11.3\,\text{min} = 163in^3$$

In order to remove 300 in^3 you would need:

HSS 250min machining + 1 tool + 1 tool change

CC (11.3 min + 9.5 min) machining + 2 tools + 2 tool changes

Case 1

HSS 255 min × \$1/min + \$30 = \$285

CC 30.8 min × \$1/min + \$ 130 × 2 = \$291

Case 2

HSS 255 min × \$2/min + \$30 = \$540

CC 30.8 min × \$2/min + \$260 = \$322

We see the lowest cost tool changes depending upon the machine + operator cost.