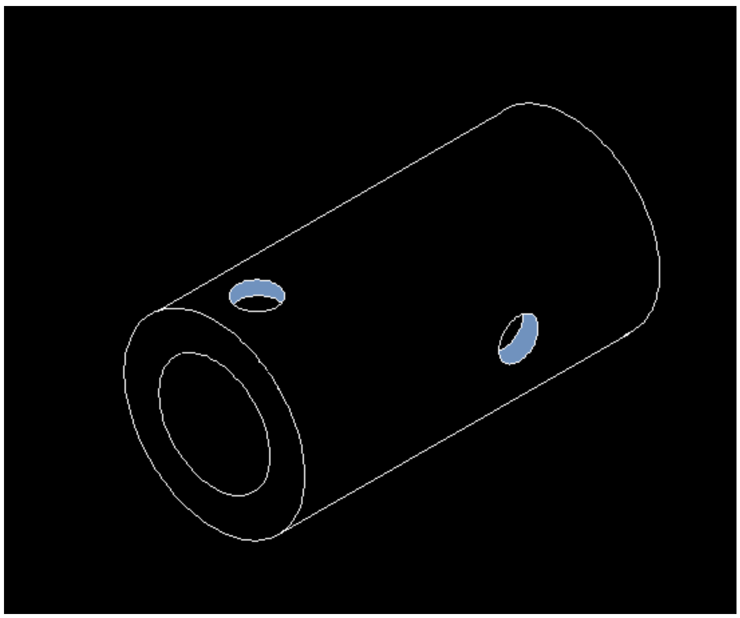
2.810 Homework # 1
Solutions
Revised September 2013

Total points: 91

1.1 Understanding Engineering Drawings

1.1a) **(4 points)** These are explained using the lecture #1 handout "Geometric Tolerancing" on the class website.

1.1b) **(2 points)** Isometric Sketch



1.1c) **(2 points)** Heat treatment to obtain the hardness requirement, and possibly grinding, and honing of the center hole or possibly reaming of that hole while the part is solid to meet the tolerances required.

1.1d) **(2 points)** Raw stock is a steel rod (high speed steel). After machining the part needs to be heat treated to obtain Rockwell hardness 48-52C. (Read Section 4.8 in Kalpakjian and Schmid)

1.1e) **(3 points)** The basic steps would be
1. Preparing the materials
2. Fixturing or locating the bread
3. Applying the peanut butter and the jelly
4. Closing the sandwich

If you have to do a lot of sandwiches there are lots of options. Consider for example:

1. The simplest and most costly is to simply add more capacity without changing the process.

2. One could automate the application of PB & J using a tube, roller application, doctor blade etc. perhaps applying both, PB and J one after the other on the same size of the bread. This will simplify handling.

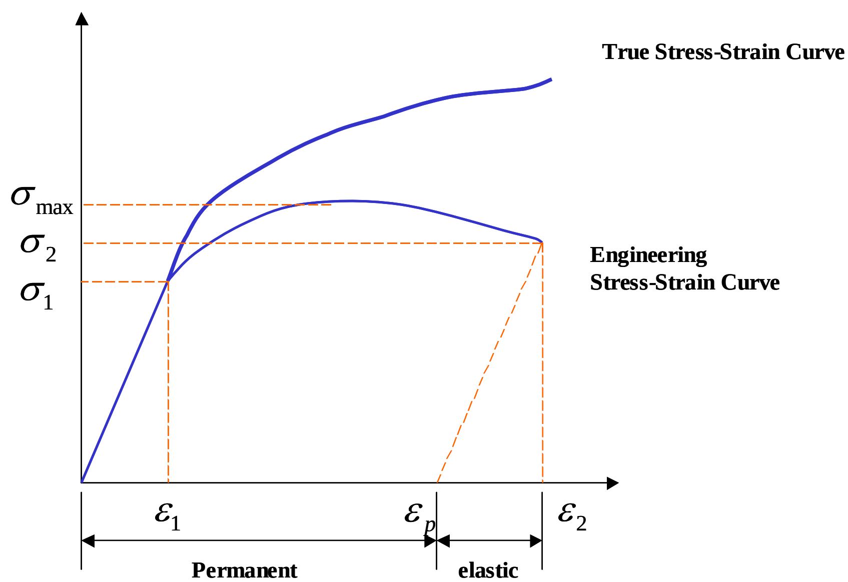
3. The application could also be simplified by mixing the PB & J, but some people may not like the taste. Also, it may be less flexible if you plan to use different jelly flavors.

4. Redesign the bread to be long slices, cover the whole thing and then cutting to size.

5. …what else?

1.2 Relating Processes Behavior to Engineering Fundamentals

1.2a) **(4 points)** Solids



1. Stiffness; E-Modulus: $E = \sigma_1/\varepsilon_1$

2. Yield Strength: σ_1. Not clear from the diagram by convention it is usually obtained by construction of a line parallel to the linear elastic region but off-set, starting at $\sigma = 0$, $\varepsilon_1 = 0.002$.

3. Ultimate Strength: σ_{max}

4. Permanent Extension: $\varepsilon_p = \varepsilon_2 - \sigma_1/E$

5. **(2 points)** True Stress-Stress Curve (see diagram)

6. **(4 points)** In general, sand cast aluminum parts have a larger grain structure compared to parts machined from wrought stock. This leads to a more brittle behavior with a lower strain to failure in the cast part. The stiffness of the two materials however, would be identical. Forging causes metal flow and grain structure development which usually leads to improved strength and toughness. Again the stiffness of a forged and machined part of the same basic material (aluminum) would be the same.

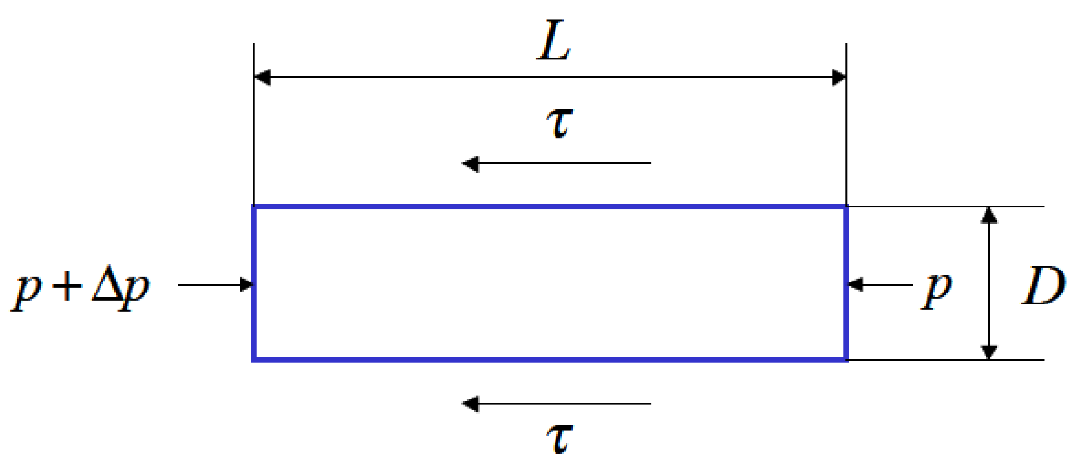
1.2b) Fluids

1. **(2 points)** F/A = μ V/h, hence μ = Fh/VA

2. **(2 points)** In general viscosity follows an Arrhenius temperature dependence of the type $\mu = Ae^{E/RT}$

3. **(2 points)** Hence, the change is more than linearly

4. **(3 points)** Consider the free body diagram of a "chunk" of fluid in a tube.



In steady state, viscous behavior, a force balance gives

$$\Delta P \frac{\pi D^2}{4} = \tau \pi D L$$

Hence, $\Delta P = 4\tau \frac{L}{D}$

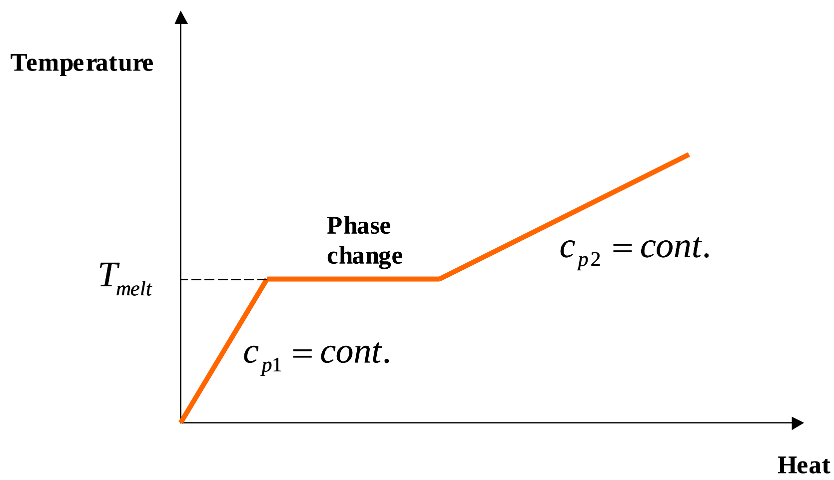
From part 1 above we know that $\frac{F}{A} = \mu \frac{V}{h}$ for a thin gap, this translates to $\tau \sim \mu \frac{\overline{V}}{D}$ for our case. Substitution above yields

$$\Delta p \sim \mu \frac{\overline{V} L}{D^2}$$

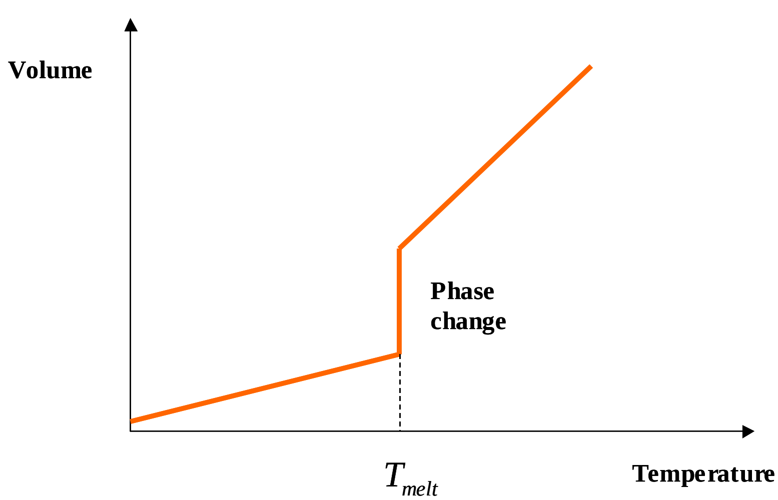
Hence, for a similar flow condition (average velocity $\overline{V}$ is constant, μ is constant and L is a constant) the pressure drop is inversely proportional to the square of D.

1.2c) Heat

1. **(2 points)** Conceptual Temperature vs. Heat Impact Diagram for a simple metal.



2. **(2 points)** Volume vs. Temperature Diagram for a simple metal.



1.2d) **(15 points)** Process Classification (see Paper by David Hardt)

Process	**Type**	**Energy Source**	**Serial or Parallel**
(a) extrusion	deformation	mechanical	serial
(b) sintering	addition	thermal	parallel
(c) stereo lithography	addition	chemical	serial
(d) swaging	deformation	mechanical	parallel
(e) arc welding	solidification	electrical	serial
(f) die casting	solidification	thermal	parallel
(g) forging	deformation	mechanical	parallel
(h) thermoforming	deformation	thermal/mech.	parallel
(i) injection molding	solidification	thermal	parallel
(j) reaming	removal	mechanical	serial

1.2e) **(10 points)** Energy Calculation

Order of magnitude estimate of the energy per unit volume required to remove material by:

1. Assuming pure, constant shear, plastic deformation requires plastic work on the order of $\int \tau \, \gamma \; \mathrm{dVol}$. At yielding $\tau \cong \dfrac{Y}{2}$ and γ can be of order 3 to 6 for turning. Hence a rough estimate (using the larger value) of the work per unit volume in machining is 3Y which, it turns out is approximately equal to the material hardness expressed in the units of stress. (This derivation will be reviewed again when we discuss machining).

2. Local melting
Performing an energy balance on a unit volume

$$u_m = \int_{T_o}^{T_m} \rho c dT + \rho \Delta H_m = \rho c (T_m - T_o) + \rho \Delta H_m \text{, for c} \approx \text{constant}$$

where,

$\int_{To}^{Tm} \rho c \, dT$ = energy required to raise temperature to T_m

ΔH_m = energy required for phase change (i.e., latent heat of melting)

3. Local vaporization
As in (b):

$$u_v = \int_{T_o}^{T_m} \rho c_s dT + \rho \Delta H_m + \int_{T_m}^{T_v} \rho c_l dt + \rho \Delta H_v = \rho c (T_v - T_o) + \rho \Delta H_m + \rho \Delta H_v$$

Comments:

For simplicity we assume $c_s = c_l$ = constant (in order to do an order of magnitude calculation)

$\rho \Delta H_v$ = energy required for phase change (i.e. latent heat of vaporization/unit vol.)

Using these relations the following order of magnitude estimates can be made:

Specific Energy (J/mm^3)

Material	u_s	u_m	u_v
Steel	6	8	62
Aluminum	1	2.6	37

As expected the vaporization processes U_v are much more energy intensive compared to shearing (plastic work) U_s and melting U_m.

The energy cost per unit weight can be found using the relation:

$$C_e = \left(\frac{specific\ energy}{density} \right) \bullet Cost\ of\ energy$$

Substituting the given data and converting into requested unit yields:

Energy Cost per Unit Weight ($/lb)

Material	**Plastic Def.**	**Local Melt**	**Local Vap.**
Steel	0.0094375	0.01258	0.0975
Aluminum	0.00466	0.01211	0.1724

Where it was assumed that the cost of energy is $0.10/kW-hr. The costs of mild steel and aluminum are $0.36/lb and $2.27/lb. respectively. Note that in all three categories, the material costs far outweigh the energy costs of each of the processes.

COMMENT

Specific energy calculations:

The assumption of a constant specific heat, c, is dependent on the material under consideration. For our order of magnitude estimates with steel and aluminum, however, a constant cp should be a reasonable approximation. The values of c_p for steel used in these calculations varied from 0.473 kJ/kg-K to 0.561 kJ/kg-K (i.e. less than 20% of the mean value) over the range of temperatures from T_o to T_m. In contrast, equivalent typical data for a ceramic sample may vary by 80%.

1.3 Basic Understanding of Time, Rate, Cost, Quality and Flexibility

1.3a) **(2 points)** Serial vs. Parallel Processes

with Little's Law: $L = \lambda W$; process rate: λ time in system: W

Serial process: $W = 2t$; $L = 2$ => $\lambda = 1/t$
Parallel process: $W = t$; $L = 2$ => $\lambda = 2/t$

1.3b) **(2 points)** Parallel Process w/ n Die Cavities

$W = t$; $L = n$ => $\lambda = n/t$

1.3c) **(2 points)** Little's law suggests that to increase λ_f we can a) decrease the lead time W_f by making operations more efficient, or b) increase the inventory L_f e.g. by adding multiple identical production units, each with $W = W_f$.

1.3d) **(2 points)** Takt Time Calculation

Def: Takt Time = Avail. Time / Required Units
Takt Time = (3 months * 22 days * 8 hrs./day * 60min/hr.)/5000 units
Takt Time = 6.4 units / minute

1.3e) **(2 points)** Little's Law: fluid flow analogy
For example: average mass flow through a system = volume of the system residence time in the system.

1.3f) **(2 points)** Unsteady conditions lead to an accumulation or depletion of units in the system.

1.3g) **(4 points)** Costs

Total costs: $C = F + V * N$

Here: C_{steel} = (\$1E6/N) + (2 * \$25/hr. * 1/360 hrs.) + (\$0.35/lb. * 7lbs.)

$C_{comp.}$ = (\$0.5E6/N) + (1 * \$25/hr. * 1/60 hrs.) + (\$1/lb. * 7 lbs.)

For N = 50,000, we get,

C_{steel} = \$22.58,
$C_{comp.}$ = \$17.41.

And, for N = 300,000, we get,

C_{steel} = \$5.92,
$C_{comp.}$ = \$9.08.

Thus, we see that for larger volumes, stamped steel is the more economical choice.

1.3h) **(2 points)** Quality Assurance Calculation: Part 1

For a centered specification band Cp is defined as:

Cp = (USL – LSL / 6σ , hence for Cp = 0.5 => spec. band = 3 σ

The probability for parts being in the interval [-1.5 σ, + 1.5 σ] is
$P[-z, +z] = 2 * \Phi(z) - 1$

For parts, which are out of spec:

$P_{out}[-z, +z] = 1 - P[-z, +z] = 2 - 2 * \Phi(z)$

Hence for $P_{outCp0.5} = 2 - 2 * 0.9332 = 13.36\%$

$P_{outCp0.9} = 2 - 2 * 0.9965 = 0.7\%$

1.3i) **(2 points)** Quality Assurance Calculation: Part 2

For non-centered specific bands Cpk is defined as:

Cpk = min. (USL – σ, σ – LSL) / 3 σ, hence Cpk = 1/3

In order to obtain the desired value we integrate the from –∞ to z = 1.
=> $P_{OUT} = 1 - \Phi(Z) = 15.9\%$

The selection of Cp does not make any sense.

Note: the standard normal curve areas given in the table are for the area *Φ*.
The cumulative area Φ = *Φ* + 0.5.

1.3j) **(4 points)** Machining Costs

$Cost_1 = C_{su} + N * V + (N * T * I)/2$

$Cost_2 = 2 * C_{su} + 2 * (N * V)/2 + 2 * ((N/2 * T/2 * 1) / 2)$

$= 2 * C_{su} + N * V + (N * T * I) / 4$

In general, where n = number of batches

$Cost_n = n C_{su} + N * V + (N * T * I) / 2n$

Find minimum: $dC_n/dn = C_{su} - (N * T * I) / 2n^2 = 0$

$n = \text{square root } (N * T * I) / 2\, C_{su}$) and with batch size $B = N/n$

$\Rightarrow B_{best} = \text{square root } ((2 * C_{su} * N) / (T * I)$

1.5 Process Control

a) **(2 points)** Control Regimes as defined by David Hardt:

Regime 1 Sensitivity and Parameter Optimization: minimize $\partial Y / \partial \Delta\alpha$

Regime 2 Minimum of Disturbances: minimize $\Delta\alpha$

Regime 3 Feedback Control of Processes

b) **(4 points)** Classification of Control Events

1)	inspection of incoming material,	Regime 2
2)	monitoring for tool breakage	Regime 2
3)	controlling the ambient temperature in the machine shop,	Regime 2
4)	warranty repair	Regime 3
5)	poke-yoke inspection,	Regime 2
6)	SPC,	Regime 2
7)	process simulation, and	Regime 1
8)	design of experiments parameter optimization	Regime 1