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Conceptual design and dimensional optimization of the linear delta robot with single legs for additive manufacturing

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Abstract

Because of remarkable characteristics such as superior speeds and accelerations, high stiffness and good dynamic performance, parallel robots are being increasingly adjusted to different task requirements in the manufacturing field. Their parallel structures made by closed-loop kinematic chains are better suited to develop new curved and multidirectional fabrication strategies in Additive Manufacturing. Based on this application, the conceptual design and dimensional optimization of a new structure of the linear delta parallel robot for Additive Manufacturing (three-dimensional printing) is presented. The new structure uses an innovative concept of delta mechanism with single legs and rotational joints, which consists of 12 links (three single parallel legs), three prismatic joints, and 11 revolute joints. A particular feature of the proposed mechanism is that it contains a joint common to all the kinematic chains instead of a mobile platform. Quality function deployment is used as a methodology for conceptual design. Then the kinematics of the mechanism is described in detail, including mobility analysis, inverse and direct kinematics, and a study of dimensional optimization. A method of efficient optimization based on genetic algorithms is used to find the minimum dimensional parameters of the robot, considering the maximization of the useful workspace as main performance index. Finally, a prototype of the robot is developed to validate the design concepts and functionality of the machine.

Keywords

Optimal design, linear delta robot, parallel kinematics, Additive Manufacturing, genetic algorithm

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Introduction

Parallel kinematic robots are basically made of two or more closed-loop mechanisms linking a movable platform and other one fixed. This kind of robots exhibits important advantages such as superior speeds and accelerations, high stiffness, improved accuracy, and an optimal ratio between moving mass and payload, for which they have gained more and more interest from both industry and research community. Numerous configurations of parallel robots are known today;[1–4] all these architectures are really fabulous, but the Delta configuration proposed by Clavel[3] has been perhaps the most celebrated and recognized for demonstrating high-performance to complete specific tasks that include mainly pick and place operations and machining tool-path movements.

The Clavel Delta parallel robot[3] is a mechanical structure which has the advantage of parallel robots and ease of serial robots modeling. The solutions for complete modeling of the Delta parallel robot (direct and inverse kinematics, inverse statics, inverse dynamics), presents few arithmetic and trigonometric operations. It consists of three identical parallel kinematic chains between the fixed base and moving platform, where each chain is constituted by the arm being rotationally actuated connected to a links parallelogram through spherical joints. However, the Delta parallel robot presents an inherently small working area.[5]

According to Pierrot et al.,[5] the Clavel Delta Robot is interesting in three ways: it is designed with a parallel structure presenting few moving masses and a high dynamic potential; it is a symmetrical structure involving low-cost building; it is a simple structure minimizing modeling complexity.

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Meanwhile, the Linear Delta robot format is a variation of the original architecture of the Clavel's Delta, where the actuated rotationally arms are discarded and replaced by linear actuators.[6]

This kind of architecture has been successfully applied in many manufacturing processes that include pick and place operations, packaging, welding, painting, and computerized numeric control (CNC) operations. Additive Manufacturing[7] (AM) is an innovative manufacturing process that has exploded in popularity recently, shifting design and manufacturing paradigms, and it has become a technological mega-trend in the new global era of the fourth industrial revolution.[8] A clear reason is its unique ability to create three-dimensional (3D) physical objects with complex geometries, multimaterials, and multicolored, directly from the virtual 3D-model data. As a popular AM technique characterized by its affordability and simplicity, Fused Deposition Modeling[9] (FDM) consists of extruding a filament of thermoplastic material layer by layer onto a build platform, where positioning the extruder in the 3D space is achieved through a CNC FDM machine.

FDM machines usually have a cartesian serial nature, which presents severe limitations regarding speed, acceleration, and dynamic performance, since an actuated axis must load the high masses of the following one(s). Therefore, the kinematic configuration of the FDM machine can result in a crucial factor to improve the manufactured part quality and shorten build times.[10–12]

In this sense, FDM machines with parallel kinematic offer the noteworthy advantages considered above and, furthermore, enables the development of new material deposition strategies for AM/FDM such as spiral curve and multidirectional strategy.[10–12] For example, they are suitable to be applied in Curved Layer Fused Deposition[13] (CLFD), where the fused material deposition is made along a curved and non-horizontal path in contrast to conventional planar paths. This FDM method results in parts with better surface finish, structurally stronger and higher accuracy.[13],[14] Linear Delta robot brings favorable characteristics for CLFD, since mobile mass is equitably divided among the three chains of the robot and, kinematic and dynamic performance is identical in the three motorized axis.

Despite these remarkable advantages, the uppermost problem of the parallel robots is undoubtedly their reduced and complex-shape workspace. This cumbersome subject continues to challenge researchers and designers when defining the best relationship between the size of the robot structure and the useful workspace. Minimizing the dimensions of the structure and links of the robot not only reduce material costs but also decreases the operation energy expenditure (compliant with the green technologies concept) in order to make processes more profitable. However, this minimization should not be exaggerated considering the maximization of the workspace as the main performance criteria of the robot, which is sensitive to its kinematics parameters.

In this article, a methodology of design and dimensional optimization of the Linear Delta robot for AM (LDr-AM) task is presented. The conceptual design of Linear Delta robot is considered in the product vision through the Quality Function Deployment (QFD) method in order to systematize the defining design requirements. The proposition of a novel kinematic configuration of the Linear Delta mechanism, which is broadly detailed in a later section, that uses single legs (without parallelogram links and movable platform) and revolute joints is also presented. The dimensional optimization method based on genetic algorithms considers the useful workspace as a performance index of the objective function. The inclusion of this index is essential to obtain the optimal kinematic parameters of the new Linear Delta configuration being proposed.

This work complements another work by the authors,[15] where some initial results about the Linear Delta Robot are presented, which allowed the validation of the proposed approach in a simulated way without actually assembling the Robot. In the current work, the actual results obtained and the Linear Delta Robot are presented.

The outline of the article is as follows. In the section "Related work," a literature review is done on machines' dimensional optimization of parallel structures. In the "Conceptual design" section, the conceptual design of the linear delta robot through a methodology based on QFD is described. The "Description of the Linear Delta Robot architecture" section then investigates the dimensional synthesis of the proposed mechanism, including the inverse kinematic analysis of the linear delta mechanism and the results optimization of the kinematic parameters of the robot. Finally, conclusions are drawn.

Related work

As was highlighted above, the major shortcoming of parallel robots is their reduced workspace. To address this issue, many researchers[16–20] have pointed to the dimensional optimization of the robot kinematic structure that is understood as the determination of minimum dimensional parameters necessary to pose the robot end-effector at any point within a given workspace.

Merlet[16] has been among the first to address the issue, who proposed an algorithm to determine the possible kinematic parameters of Gough-type 6-DOF (Degrees Of Freedom) parallel robots for the desired workspace.

Boudreau and Gosselin[17] presented a procedure to find the optimal parameters of parallel robots in order to obtain a workspace as similar as possible to a prescribed one. They applied this procedure to two planar three-DOF parallel manipulators, one with prismatic joints and one with revolute joints.

Kosinska et al.[21] proposed an algorithm of designing optimal parameters of a Delta-4 parallel manipulator for a prescribed workspace denoted by a set of points.

Stock and Miller[6] developed an optimal kinematic design method based on an exhaustive search minimization algorithm considering the workspace as a performance index, which they have applied to a Linear Delta robot. Liu et al.[18] introduced the maximum inscribed workspace concept in the designing of a Linear Delta manipulator considering a specified workspace and swing range of its spherical joints.

Affi et al.[22] highlighted a methodology for the synthesis and optimization of the workspace of a 3-translational-DOF in the parallel manipulator. The platform of the robot studied is composed of a classic three variable-length legs active structure (a classical Stewart platform) and two other kinematics chains, with passive joints, that enable them to eliminate the three rotations of the platform. The particularity of this article is to propose an optimization of the passive workspace for a given active workspace that allows obtaining the optimized workspace of the robot.

Laribi et al.[19] presented an approach of dimensional synthesis of the Delta robot that uses an objective function based on the concept of the power of a point, which reflects the position of a point with respect to the boundary of the prescribed workspace.

Yuan et al.[23] developed an optimal design method to determine the kinematic parameters of the Linear Delta robot based on the concept of performance chart in order to obtain a prescribed cuboid dexterous workspace.

Courteille et al.[24] proposed an approach for design optimization of parallel robots considering multiple global stiffness objectives.

Zhang et al.[25] presented a dynamic dimensional synthesis of the Linear Delta robot using the transmission angle constraints. Besides the classical dimensional parameters optimization for achieving good kinematic and dynamic performances in the entire workspace, the study includes two other parameters to minimize: the inertial and centrifuge/Coriolis components of the driving torque.

Kelaiaia et al.[20] proposed a methodology of multi-objective dimensional optimization of parallel robots using genetic algorithms that consider several criteria of performance such as stiffness, kinematic and dynamic performances, and the prescribed regular dexterous workspace.

Zhao[26] employed the velocity transmission index as an objective function of dimensional synthesis of the Delta robot.

Fiore et al.[27] worked on kinematic synthesis of a 5-DOF parallel robot for AM using genetic algoritms.

Liu et al.[28] used an optimization method based on genetic algorithms and sequential quadratic programming to optimize all design variables of a 3-DOF Delta mechanism haptic device respect to the desired cube workspace.

Many other authors also addressed the dimensional optimization of different configurations of parallel robots.[29–36] However, to date, there is a lack of detail about the design requirements and optimization of the kinematic parameters of the LDr-AM.

Conceptual design

The LDr-AM can be considered as a mechatronics product since it involves three technological domains: mechanical, electro-electronic, and computational.[37] This kind of products have a complex and interdisciplinary nature; therefore, it is recommended to adopt structured design methodologies in order to support and systematize decision making during the design process.[37] A design methodology that considers the user's preferences and requirements can be very convenient. QFD is one of the most known and used methods to define the attributes or characteristics of a product based on the user's requirements (URs).

QFD has been successfully applied on the design of different mechatronics products including agriculture robots,[38,39] underwater robots,[40] service robots,[41] and 3D printers,[42,43] among others. Further detailed information about QFD can be found in the following references.[44,45]

In this work, QFD method is applied to design process of the LDr-AM, described through the following steps:

Step 1: identifying the users

This step deals with identifying the users of the product/LDr-AM. In this sense, it is possible to rank the FDM machines in three categories: low-cost machines, professional machines, and industrial machines. The LDr-AM tries to fit into the first category, which is mainly used in small businesses, universities, fablabs, or at personal homes. That is why in this work a group of 12 possible users was consulted, including some small business owners, fablabs technicians, and university students and professors.

Step 2: defining the URs

Defining the URs was carried out based on the experience of the users consulted and literature review. This survey considered the idea of obtaining a high-performance and low-cost product. In this way, 23 requirements were consolidated grouped into five categories (Capacity, Operation, Design, Economy, and Reliability) to facilitate analysis and average. The requirements are listed in Table 1.

Step 3: prioritizing URs

To identify and prioritize the RUs that represent the most important within the product design, it is necessary to attribute to them a relative importance and

Table 1. User requirements grouped in five categories.

Categories	User requirements (WHATs)
1. Capacity	1.1 Printing quality
	1.2 Rapid movement
	1.3 Long operating time
	1.4 Wide workspace
	1.5 Cross-platform system
	1.6 Print with different types of materials
2. Operation	2.1 Easy to operate
	2.2 Quick and silent machine
	2.3 Easy to service
3. Design	3.1 Compact machine
	3.2 Robust structure
	3.3 Light weight
	3.4 Minimum number of parts
	3.5 Easy to assemble
	3.6 Good appearance
	3.7 Easy to add technology
4. Economy	4.1 Low manufacturing and assembly cost
	4.2 Low operating cost
	4.3 Low energy consumption
	4.4 Low cost control system
	4.5 Low maintenance cost
5. Reliability	5.1 Safety operation
	5.2 Low failure probability

weight. In this step, a Mudge diagram was used, which compares each requirement with its peers, where relative importance is computed as the sum of the score of each UR obtained in the comparison process. Similarly, the relative weight is calculated by dividing the relative importance of the UR by the sum of all relative importance.

The result of the analysis with Mudge diagram can be understood through the Pareto diagram presented in Figure 1. According to this, the most important URs in the design of the LDr-AM are printing quality, ample workspace, rapid movement, long operation time, print with different types of materials, low manufacturing and assembly cost, and low energy consumption. On the contrary, the less important URs are compact machine, lightweight, and good appearance.

According to Pareto diagram analysis, to achieve an 80% compliance of the URs in the robot design, URs with relative importance above 18 must be strictly complied with. In this way, the first part of the QFD matrix with the list of RUs and their respective weights and relative importance on the left side is prepended, as shown in Figure 2. In the QFD language, the URs can also be called "WHATs."[44]

Step 4: selected design parameters

This step consists of deploying the URs into design parameters (DPs), which represent physical concepts of the product that can be better understood by the designer. This stage is the first decision on the product design, which defines definitive parameters that will meet the established requirements.

The DPs were selected based on the experience of the design team and literature review. The 23 selected DPs are listed in Table 2 and placed on top of the QFD matrix in Figure 2. The desired tendency is established for each DP of the robot, which can be crescent, decrescent, or target as shown in Figure 2. In QFD language, DPs are usually called "HOWs."[44]

Step 5: relationship between URs and DPs

The process of identifying relationships between URs and DPs is performed subjectively by the designer. The degree of relationships is set to three levels as follows: strong relation = 9, medium relation = 3, and weak relation = 0. As a result of this analysis, the URs scores are obtained on the right side of the QFD matrix in Figure 2.

It is observed that the requirements *low cost of manufacturing and assembly, robust structure, low failure probability*, and *low maintenance cost* with scores 114, 118, 110, and 99, respectively, are the most influenced URs from the selected DPs. On the contrary, the lowest relationship URs according to this analysis were *cross platform system, rapid movement*, and *quick and silent machine*, with scores of 37, 35, and 26, respectively. Further detailing of the relationships between URs and DPs of the LDr-AM is reported in a previous work.[37]

Step 6: DPs' correlations

The correlation between DPs is located on the ceiling of the QFD matrix in Figure 2, where each DP is compared to its peers. The degree of correlation can be positive (+) or negative (–). A positive correlation implies that an increase in the DP results in a positive effect on its pair. On the contrary, if the correlation is negative, a satisfactory balance is sought between the DPs being compared.

The total cost of the machine is affected by the design solutions selected to guarantee a machine that is accurate, fast, and with good print quality. Therefore, it is necessary to seek a balance between these DPs. Similarly, it happens with the cost of maintenance since expensive solutions imply high maintenance costs. The workspace volume adversely affects the maximum dimensions of the machine, as the increase in the workspace increases the dimensions of the robot. In this case, the equilibrium can be treated with the dimensional optimization of the kinematic parameters of the robot as it has been thought in this work. Further detailing of the correlations between DPs of the LDr-AM is reported in a previous work.[37]

Step 7: defining goal specifications

As a result of applying the QFD method, a set of goal specifications of the LDr-AM are obtained, which are placed on the bottom of the QFD matrix in Figure 2. Each DP is associated with a goal specification. So,

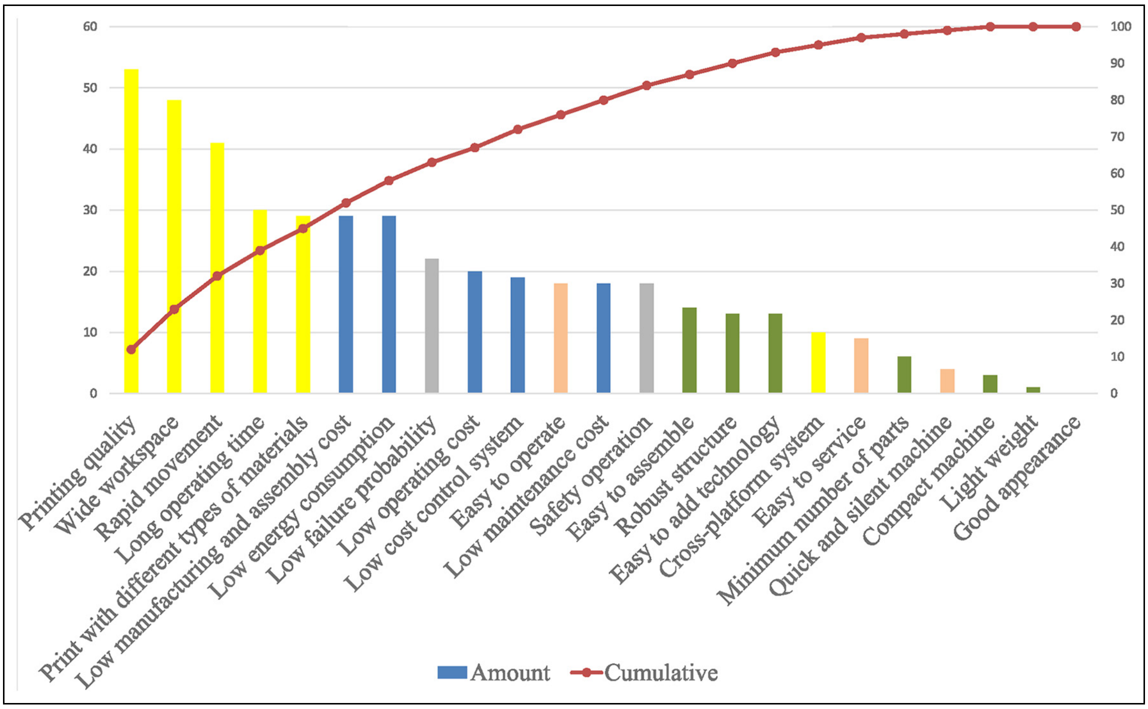

Figure 1. Average importance ratings for the user requirements shown in the Pareto chart.

main goal specifications defined for the LDr-AM are positioning precision of 50 μm; maximum speed of 160 mm/s; overall cost less than \$ 2000 USD; temperature range of 0°C–300°C; simple software; print cylindrical volume of diameter 250 mm × height 300 mm; total size of 500 × 500 × 1000 mm^3; power supply of 110–220 V 350 W; aluminum structure; nozzle with diameter less than 0.4 mm; noise less than 80 dB; support for windows, linux, and macOS; weight less than 40 kg, less than 300 parts, finished manufactured parts, and open software, among other.

Description of the linear delta robot architecture

The linear delta parallel robot has three linear actuators located vertically on a fixed base and spaced symmetrically forming an angle of 120° between each actuator. This robotic configuration is characterized by providing high stiffness in the vertical direction thanks to the arrangement of its actuators. The actuators produce linear motion through the screw mechanism that converts rotational motion into linear motion by providing torque and accuracy. Figure 3 shows the configuration of the parallel linear delta robot with the basic components of the architecture.

Figure 4 presents a comparison of the mechanisms, traditional linear delta, and proposed linear delta with single legs. The conventional linear delta (Figure 4(a)) is based on the concept of parallelogram links that connect the mobile platform with the linear actuators through spherical joints (S). However, this type of joints tends to be expensive, hardly affordable, and cumbersome to assemble. Besides, commercial spherical joints usually have a limited work angle, which implies movement restrictions of the mechanism and plays against workspace maximization.

However, looking for a more economical solution that offers greater performance in terms of workspace capacity, the proposed linear delta (Figure 4(b)) is constituted by three kinematic chains using single links and revolute joints. Thus, the new LDr-AM has a $PRRR - PRRR\ddot{R} - PRRRR$ topology, where P is prismatic joint, R is revolute joint, and $\ddot{R}$ is a revolute joint with 2-DOF that is common to the three chains instead to use mobile platform. This fact represents a particular feature of the new mechanism being proposed. The linear actuators transmit their displacement to the end-effector coupled to the common-joint through the links that locate it on any point in the workspace guided by the articular position of each actuator.

Mobility analysis

The Chebychev–Grübler–Kutzbach criterion[46] is utilized to analyze the mobility of LDr-AM (Figures 3 and 4). According to this criterion, the mobility of a system M formed from n moving links and j joints, each with freedom f_i, is given by equation (1)

$$M = 6n - \sum_{i=1}^{j}(6 - f_i) \quad (1)$$

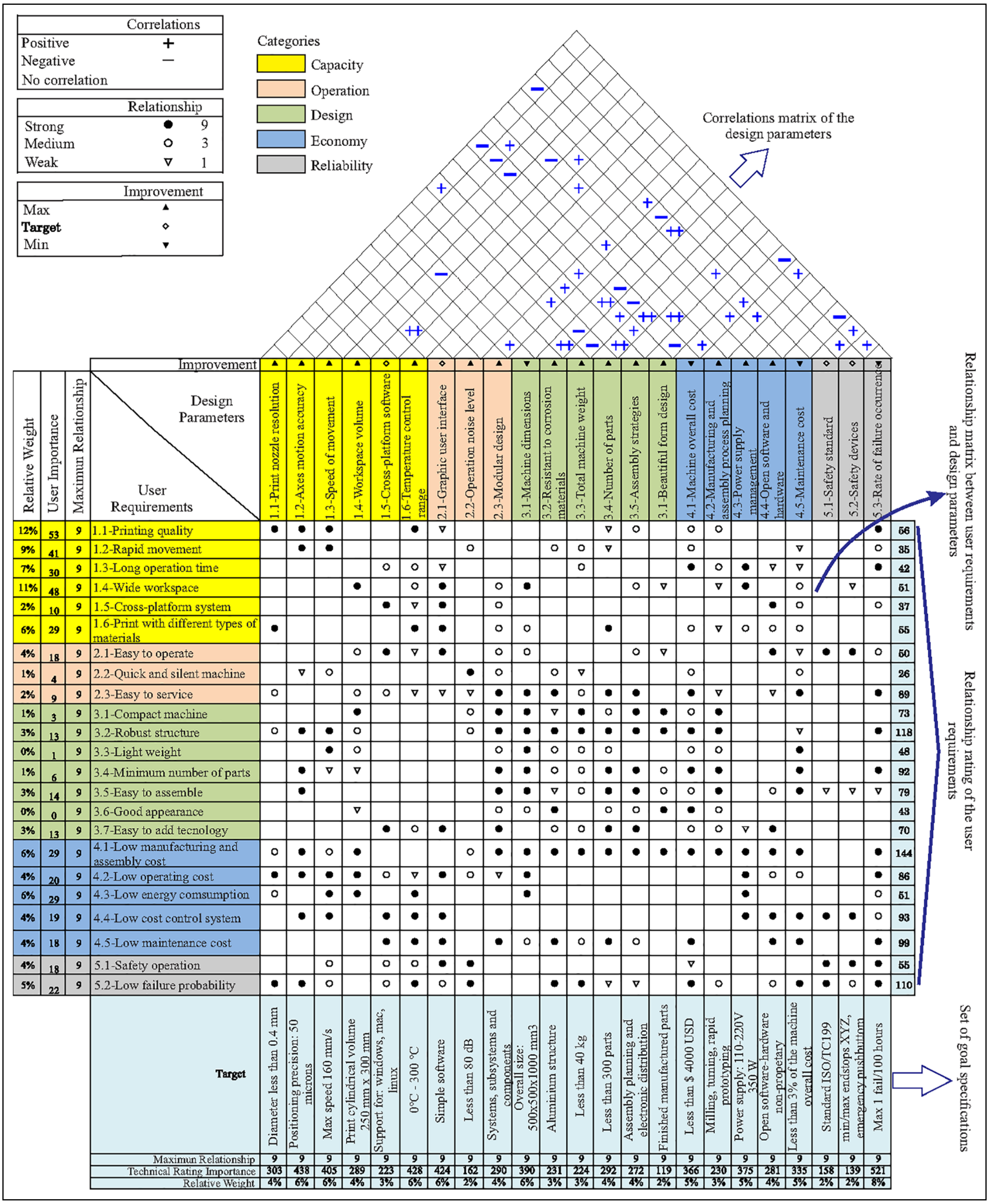

Figure 2. Analysis of the relationships between user requirements and design parameters.

From equation (1), it is possible to determine the mobility of the system described previously. There are 11 revolute joints, three prismatic joints, and 12 moving links. It should be noted that the revolute common-joint has two DOF. Substituting these values into equation (1) (as in equation (2)), we have that the mobility of the Linear Delta mechanism is thus three DOF. This mobility analysis enables to validate the system for a three-axis FDM machine

$$M = 6(12) - \sum_{i=1}^{13}(6-1) - \sum_{i=1}^{1}(6-2) \tag{2}$$

$$M = 72 - 13(5) - 1(4) = 3 \tag{3}$$

Table 2. Design parameters grouped in five categories.

Categories	Design parameters
1. Capacity	1.1 Print nozzle resolution
	1.2 Axes motion accuracy
	1.3 Speed of movement
	1.4 Workspace volume
	1.5 Cross-platform software
	1.6 Temperature control range
2. Operation	2.1 Graphic user interface
	2.2 Operation noise level
	2.3 Modular design
3. Design	3.1 Machine dimensions
	3.2 Resistant to corrosion materials
	3.3 Total machine weight
	3.4 Number of parts
	3.5 Assembly strategies
	3.6 Beautiful design
4. Economy	4.1 Machine overall cost
	4.2 Manufacturing and assembly process planning
	4.3 Power supply management
	4.4 Open software and hardware
	4.5 Maintenance cost
5. Reliability	5.1 Safety standard
	5.2 Safety devices
	5.3 Rate of failure occurrence

Inverse kinematic

The problem of computing the possible actuated joints coordinates of a robot given the end-effector pose is called inverse kinematic problem. The resolution of this problem is essential for position control of the LDr-AM. The kinematic model and geometric parameters of the Linear Delta robot are shown in Figure 3, where the reference coordinate system O is located on the printing platform on the center of the fixed base A-B-C, with the z-axis normal to the printing platform and the x-axis in the A direction. The actuated prismatic joints are denoted by $Q_i(x_i, y_i, d_i)$, with $i = 1, 2, 3$, which are located at a distance R_b from the reference system O and spaced 120° apart from each other (Figure 3). The common-joint pose respect to the reference system is denoted by $P(x,\ y,\ z)$. The aim is to determine the (x_i, y_i, d_i) coordinates for the actuated joints.

From Figure 3, the expressions in equation (4) can be deduced to obtain the (x_i, y_i) coordinates of each actuated joints Q_i

$$x_i = R_b \cos\theta_i \quad y_i = R_b \sin\theta_i \quad i = 1,2,3 \tag{4}$$

where

$$\theta_i = \frac{2i-2}{3}\pi \quad i = 1,2,3 \tag{5}$$

It is assumed that all the three legs of the Linear Delta robot are identical in length (L_e). The (x_i, y_i) coordinates are used to compute z_i by using Pythagoras through equation (6)

$$d_i = z \pm \sqrt{L_e^2 - (x - x_i)^2 - (y - y_i)^2} \quad i = 1,2,3 \tag{6}$$

In equation (6), the notation $\pm$ means that there are two possible solutions for the Linear Delta robot pose. However, in this work, only the solution with sign $+$ that results in the configuration of the robot shown in Figure 3 is considered, thus completing the procedure for the inverse kinematics of the Linear Delta robot.

Direct kinematic

The forward kinematics model of the Linear Delta robot can be represented as the intersection of three spheres as illustrated in Figure 5(a). The centers of the three spheres are given by the prismatic joints points $Q_i(x_i, y_i, d_i)$ (with $i = 1,\ 2,\ 3$) and the radius of each sphere is equivalent to the length of the corresponding leg. The solution(s) for the Linear Delta robot end-effector pose can be obtained from the point(s) of intersection between the three spheres. Obtaining the solution(s) is subject to three possible situations: (1) when a sphere is a tangent to the intersection of the other spheres, resulting in a unique solution; (2) when there are two points of intersection between the spheres resulting in two possible solutions; and (3) when there is no intersection between the spheres resulting in an unsolvable system.

A fourth situation could arise when the centers of two or more spheres coincide with the same point, and the system has infinite solutions. However, in the case of the Linear Delta robot, this situation is impossible.

Resolution for the forward kinematics problem of the Linear Delta robot by using the spheres intersection has been shown in Stock and Miller[6] and Laribi et al.[19] The relationship between the coordinates of each actuated prismatic joint and the coordinates of the common-joint of the Linear Delta robot can be described by equation (7), which represents the equation of the sphere corresponding to each kinematic chain of the robot

$$(x - x_i)^2 + (y - y_i)^2 + (z - d_i)^2 = L_e^2 \quad i = 1,2,3 \tag{7}$$

In order to find the point(s) of intersection between the three spheres, the Trilateration method has been used which allows for the determination of the absolute or relative locations of the points by the measurement of distances, using the geometry of circles, spheres, or triangles. To compute the intersection of three spheres by using the Trilateration method, we must consider that

- All three centers are in the plane $z = 0$;
- One sphere center is at the origin;
- One other is on the x-axis.

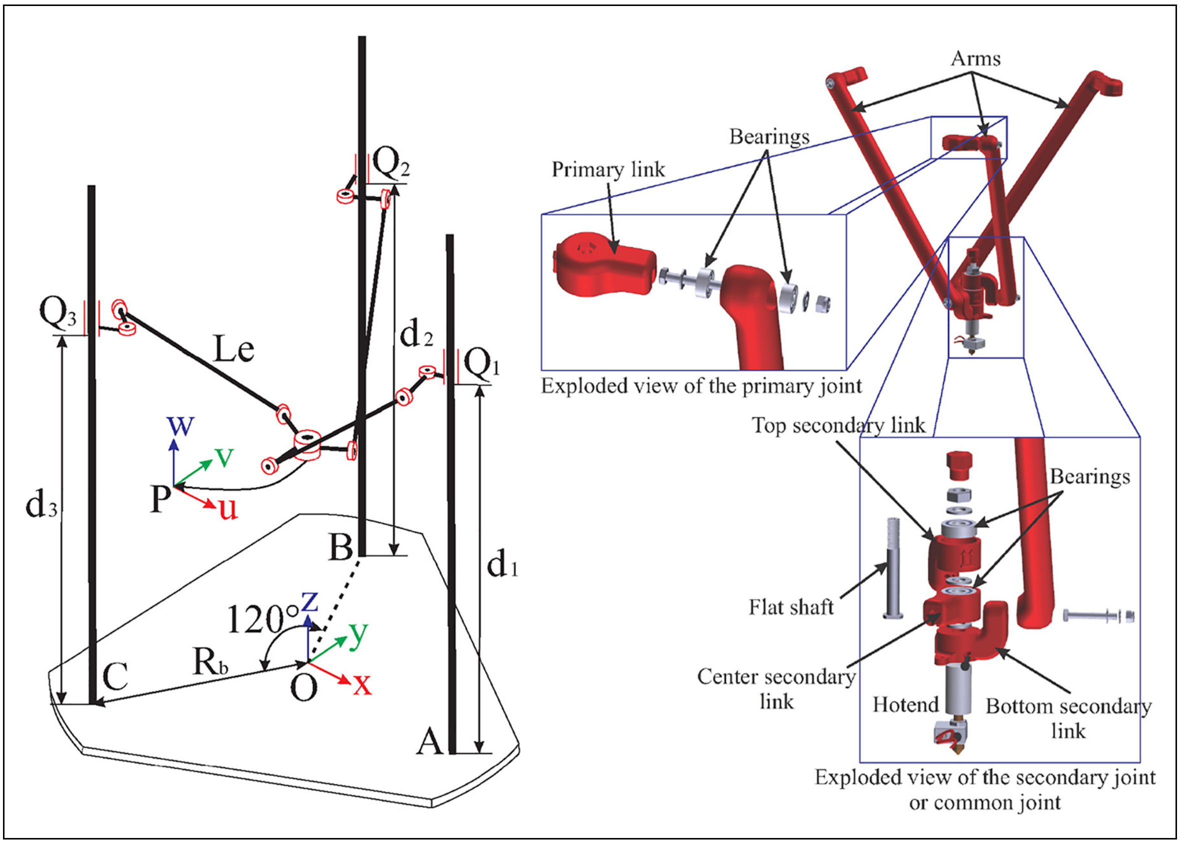

Figure 3. Linear Delta Parallel Robot kinematic schematic and details legs.

Following this approach, the coordinate system of reference O, now denoted by O', has been moved to one center sphere Q_2 as shown in Figure 5(b). Therefore, the equations of the spheres with respect to the new reference system can be rewritten as

$$\begin{aligned} L_e^2 &= x'^2 + y'^2 + z'^2 \\ L_e^2 &= (x' - d)^2 + y'^2 + z'^2 \\ L_e^2 &= (x' - k)^2 + (y' - j)^2 + z'^2 \end{aligned} \quad (8)$$

The equation set that allows computing the unknowns $x,'$ y', and z' can be obtained by reducing the system in equation (8). Then we have

$$x' = \frac{1}{2}d \quad (9)$$

$$y' = \frac{k^2 + j^2}{2j} - \frac{k}{j}x \quad (10)$$

$$z' = \pm\sqrt{L_e^2 - x^2 - y^2} \quad (11)$$

where k is the signed magnitude of the x component of the vector from Q_2 to Q_3, d is the distance between the centers Q_2 and Q_1, and j is the signed magnitude of the y component of the vector from Q_2 to Q_3. These values can be computed from equations (12)–(14)

$$k = \hat{e}_x \cdot (Q_3 - Q_2) \quad (12)$$

$$d = ||Q_1 - Q_2|| \quad (13)$$

$$j = \hat{e}_y \cdot (Q_3 - Q_2) \quad (14)$$

Q_1, Q_2, and Q_3 are treated as vectors in the original coordinate system O. Similarly, $\hat{e}_x$ and $\hat{e}_y$ are unit vectors, the first in the direction Q_2 to Q_1 and the second in the y-direction, both in the original coordinate system. These unit vectors can be calculated through equations (15) and (16), respectively

$$\hat{e}_x = \frac{Q_1 - Q_2}{||Q_1 - Q_2||} \quad (15)$$

$$\hat{e}_y = \frac{Q_3 - Q_2 - k\hat{e}_x}{||Q_3 - Q_2 - i\hat{e}_x||} \quad (16)$$

The third unit vector can be obtained by using the cross-product properties as $\hat{e}_z = \hat{e}_x \times \hat{e}_y$.

With this, the system solution(s) represented by the intersection of the three spheres, with respect to the original coordinate system, can be obtained from equation (17)

$$\vec{p}_{1,2} = Q_2 + x\hat{e}_x + y\hat{e}_y \pm z\hat{e}_z \quad (17)$$

Usually, the intersection of the three spheres results in two solution points which, in this case, describe two possible positions of the end-effector with respect to the base. For this reason, the notation $\pm$ appears in equation (17). In this work, we opt for the solution with sign $+$ that describes the configuration of the robot shown in Figure 5(a).

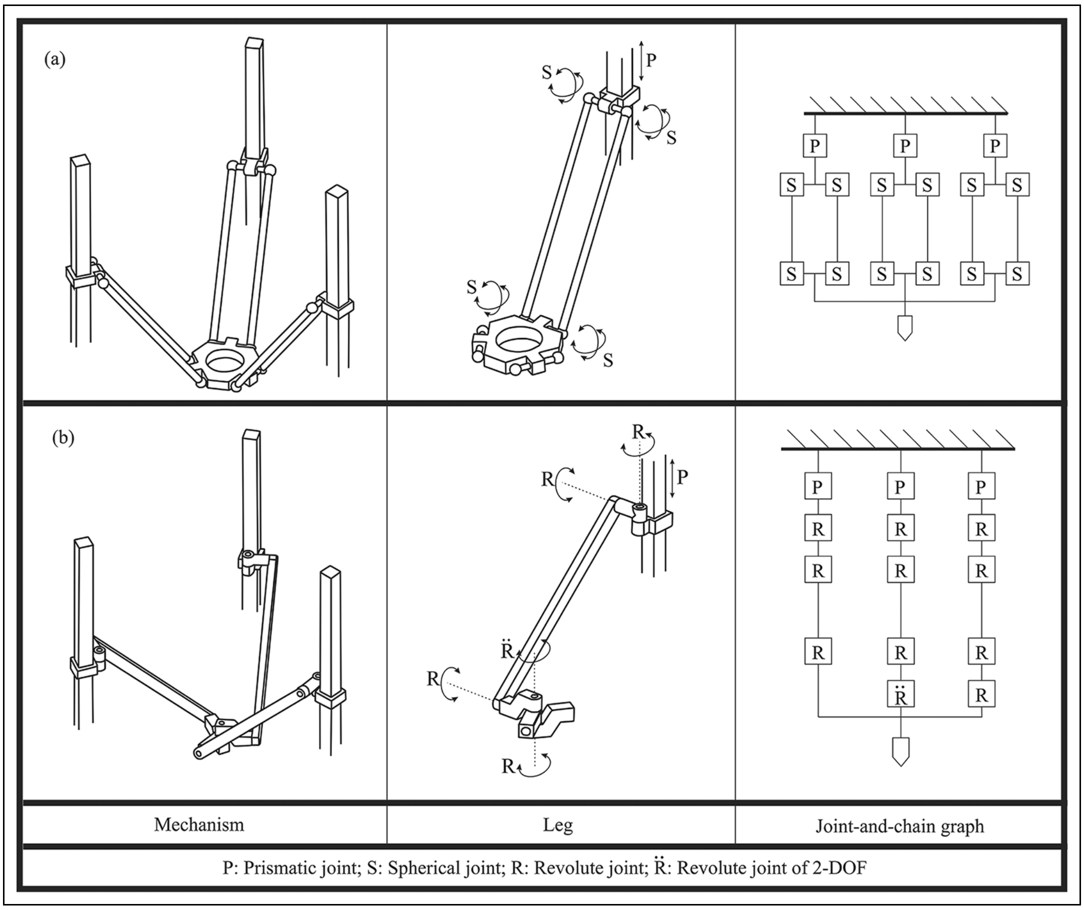

Figure 4. Topology description: (a) conventional linear delta and (b) proposed linear delta with single legs.

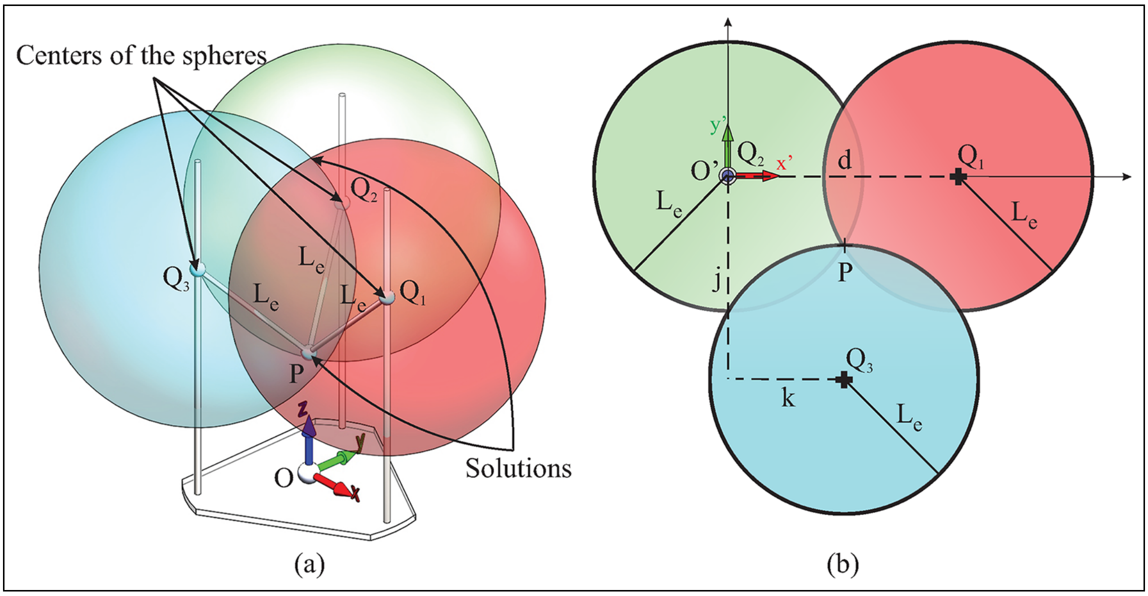

Figure 5. Forward kinematics problem: (a) the Linear Delta robot represented as the intersection of three spheres and (b) translation of the coordinate reference system based on the Trilateration concept.

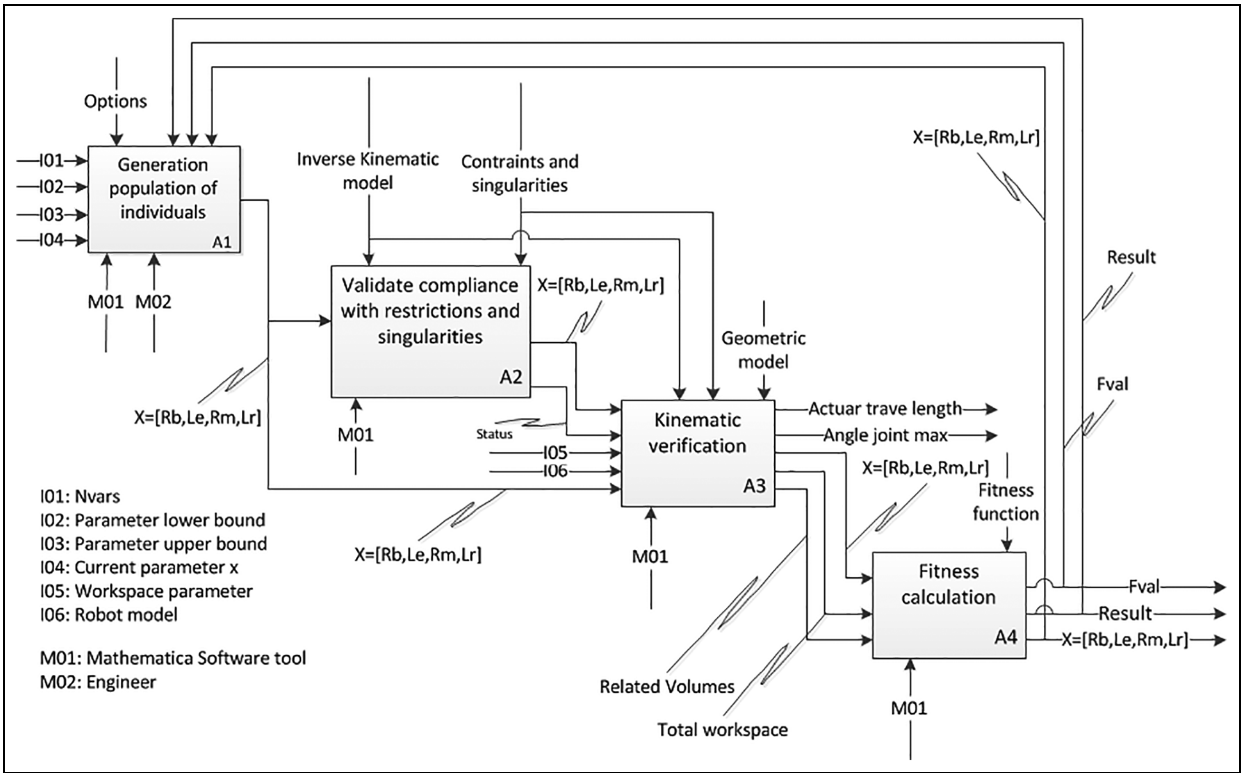

Figure 6. IDEF0 model: activities A1–A4.

Dimensional optimization

In the search to reduce the manufacturing cost of the robot without losing functionality, a method based on genetic algorithms is proposed to determine the optimal values of the most critical dimensional parameters of the LDr-AM. The optimization based on genetic algorithms is included within the method, to explore a set of given possibilities within a range of values with characteristics of the desired solution. The problem is summarized in finding the values of the DPs of the robot that best adapt to the kinematic model and allow covering a predefined cylindrical workspace of diameter 250 mm × height 300 mm. Finding the value of the optimal parameters allows generating a design that uses fewer resources in its development and construction.

Figure 6 shows the different functions that make up the dimensional optimization method. To analyze and present the functional perspective of the algorithm, the IDEF0 (Integration Definition for Function Modeling) method is used, which allows organizing initially the different functions used in the search of values. IDEF0, used as an analysis and modeling tool, helps to identify the necessary functions and define the inputs and outputs of each function within the algorithm.

The method is composed of four main activities A1, A2, A3, and A4 as seen in Figure 6. The search starts with the functional block A1 destined to execute the genetic algorithm. Block A1 is responsible for generating a population of individuals with a solution characteristic. In each interaction of the algorithm, the individuals generated in block A1 are subject to two types of verification. The first verification occurs in the functional block A2 which is responsible for determining if the geometric model of the robot can be built with those values and if the values are within the admissible parameters that do not produce singularities and overcome the restrictions.

The second verification is kinematic and the A3 functional block occurs. The solution equation of the inverse kinematics of the parallel linear Delta robot given in equation (6) is used to implement the kinematic verification function. The kinematic verification is done with the geometric model of the robot and with a point cloud that represents the predefined workspace. For each point of the cloud in the cylindrical workspace, the joint position that allows positioning the effector on that point is calculated. If for some particular reason this cloud point cannot be reached by the current kinematic model, the values of these individuals are discarded.

From the kinematic verification, additional information can be extracted as the maximum linear travel of the actuators and maximum opening angles of the joints and a set of volumes associated with the current values of dimensional parameters. The last functional block A4 receives the values of the parameters that satisfy both the conditions and restrictions and the kinematic verification. These values are quantified by using a objective function, and weight dependent on their

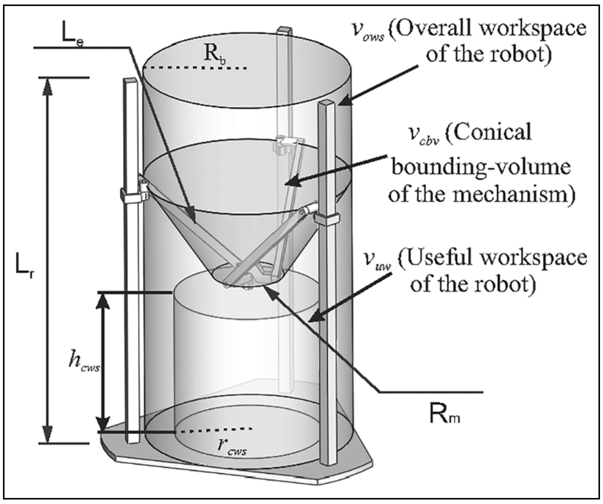

Figure 7. Volumes associated with the parameters of the fitness function.

performance is assigned. These values are classified as individuals with solution characteristic and used within the genetic algorithm to generate new populations that tend to minimize the value of each parameter.

The applied method gives an optimal solution that evolves in each interaction of the genetic algorithm, causing the dimensions of the robot to decrease without violating the geometric restrictions of the Linear Delta robot. Figure 7 shows the Delta linear robot configuration where it is possible to observe the four important parameters that define the geometric model of the robotic structure. The parameters of the geometric model of the robot subjected to optimization are

- R_b: Radius of the base of the robot;
- L_e: Length of links;
- R_m: Radio of the mobile platform;
- L_r: Length of the linear actuator measured from the base.

Mathematical optimization problem

The result of the robot sizing method depends directly on the formulation of the optimization problem. The mathematical formulation is associated with a minimization problem related to three volumes derived from the geometric model of the robot v_{cbv} (Conical bounding-volume of the mechanism), v_{ows} (Overall workspace of the robot), and v_{uw} (Useful workspace of the robot). The volume of the workspace is predefined with 125 mm radius (r_{cws}) and 300 mm height (h_{cws}) and is the reference for the calculation of the objective function. The problem seeks to minimize the value of f, a function that contains the values of the parameters of the geometric model of the optimized robot (R_b, L_e, R_m, L_r). The problem is focused on finding $x = (R_m, L_r, R_m, L_r)$ to minimize the function

Table 3. Result of Dimensional Calculation based on workspace.

Fval (mm^3)	R_b (mm)	L_e (mm)	R_m (mm)	L_r (mm)
1.17x10^8	135.0442	202.5768	35.0000	505.5150

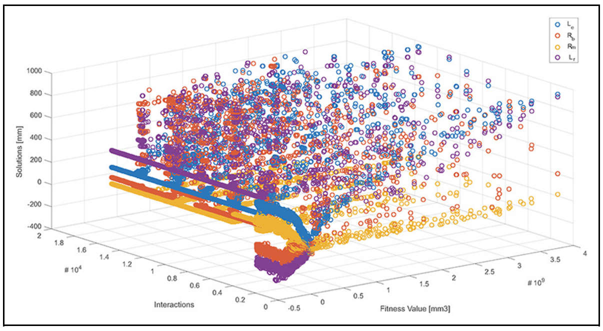

Figure 8. Convergence criteria to obtain the optimal solution.

$$f(x) = \sum_{i=1}^{n} v_{uw}(v_{cbv_i} v_{ows_i}) \tag{18}$$

where

$$v_{uw} = \pi h_{cws} r_{cws}^2 \tag{19}$$

$$v_{cbv_i} = \frac{1}{3}\pi \left(\sqrt{L_{(i,e)}^2 - (R_{(i,b)} - R_{(i,m)})^2}\right) \left(R_{(i,b)}^2 + R_{(i,m)}^2 + R_{(i,b)}R_{(i,m)}\right), \quad i = 1, 2, \ldots, n \tag{20}$$

$$v_{ows_i} = L_{(i,e)}\pi R_{(i,b)}^2, \quad i = 1, 2, \ldots, n \tag{21}$$

Optimization result

The results obtained in the determination of the values for the dimensional parameters of the robot are shown in Table 3, which presents the values (specified in millimeters) obtained for the optimization parameters $R_b, L_e, R_m,$ and L_r. The genetic algorithm required creating 53 generations and 1080 interactions to converge in an optimal solution. The method provides a solution with optimal values for the dimensional parameters of the robot.

Figure 8 shows the behavior of the objective function that managed to converge into a solution of minimum values for the robot that allows it to cover the predefined cylindrical workspace, as well as the random behavior and the heuristic search of the values that through the verification functions are evaluated and classified. It can also be observed that although in the objective function, the parameters to be optimized are interrelated, the parameters maintain independence that makes it easier for the algorithm to tend toward a solution.

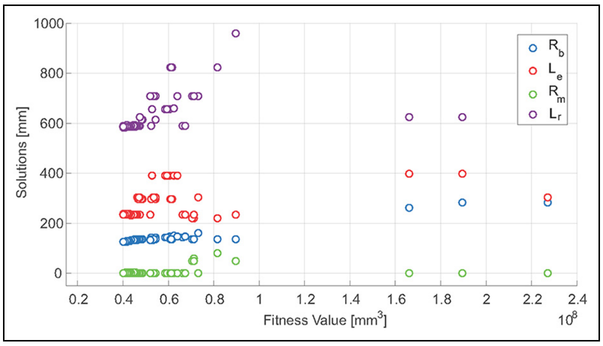

Figure 9. Performance of the genetic algorithm using the concept of penalty.

The concept of penalty performs a vital role in reducing time spent searching for the solution. The optimization problem formulated with the penalty concept allows correcting the jump in the evaluation of performance that occurs when the values of the parameters approach an optimum value because a small variation can result in a singularity and in case of absence of the penalty those values close to the solution would be discarded. The penalty function allows that close values of an optimal solution but that incur a violation of the restrictions is penalized with an additional percentage value. This penalty allows the generation of new individuals from these solutions and preserves some of their characteristics for future generations to overcome the restrictions found.

Figure 9 shows from another perspective the performance of the genetic algorithm using the concept of penalty. It is observed that although some values of the parameters are found with values close to the solution, there is a possibility that at least one is out of that optimal value producing a poor performance in the evaluation. However, the values are not entirely discarded and allow creating new generations with their characteristics. The values close to the optimal solution are located on the left side of Figure 9 and on the right the values with undesired performance.

Figure 10 shows the geometric model of the robot with the optimal parameters obtained and the point cloud representing the cylindrical workspace. The simulation is executed to verify that the robot reaches each of the points located within the cloud that represents the predefined workspace. This favorable result in the simulation validates the values to be used in the design and manufacture of the robot.

In order to validate the design feasibility of the proposed LDr-AM with optimized dimensional proportions, a prototype has been developed. Figure 11 shows different views of the prototype achieved with aluminum structure. Besides, Table 4 presents a comparison of some of the characteristics, mainly related to workspace capacity and structure, of the LDr-AM with those of other commercial 3D printers such as Kossel 3D Printer,[47] Atom 2.5 FX,[48] and RepRap Rostock.[49]

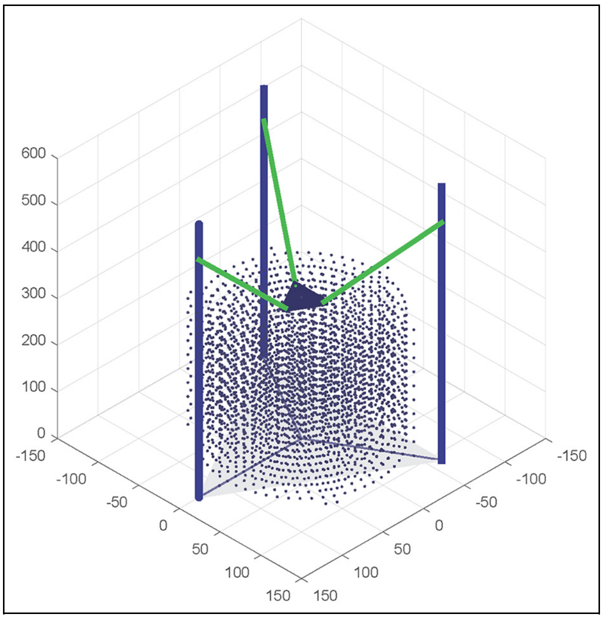

Figure 10. Geometric model of the robot with the optimal parameters.

Conclusion

In this work, the conceptual design and dimensional optimization of a robot with linear delta parallel kinematics for AM were presented. The ~~robot~~ Delta ~~Linear~~ for AM (~~rDL-AM~~) has as main differential the use of an innovative concept of delta mechanism with single legs and rotational joints. This solution is also different from the conventional delta architectures that have a mobile platform, while the ~~rDL-AM~~ has a common-joint where the end-effector is directly coupled.[AQ: 1]

The inverse and direct kinematic problems of the robot were solved. The approach based on the concept of Trilateration allowed solving the direct kinematic problem of the proposed delta mechanism.

The satisfactory results in the determination of the optimal values of the dimensional parameters of the robot were due to the combination of search methods based on genetic algorithms and kinematic verification methods. The correct definition of the joint restrictions implemented in the verification algorithms facilitated the obtaining of the optimal parameters avoiding considering parameters that provoke in singularities. Due to the non-linear nature of the optimization problem and the large restrictions derived from the complex geometric model associated with the parallel robotic configuration, the method based on genetic algorithms allowed us to meet the objectives set. The methodology presented in this document can be easily adapted to optimize the dimensions of other robotic configurations.

Figure 11. The fabricated aluminum prototype of the LDr-AM.

Table 4. Comparison between the LDr-AM and some commercial 3D printers.

	Workspace (Diameter × Height) (mm^2)	Overall size (mm^3)	Resolution (mm)	Aluminum structure	Type of Delta mechanism
This ~~rDL-AM~~	250 × 300	500 × 500 × 1000	0.01	Yes	Single Legs
Kossel 3D Printer[47]	170 × 240	300 × 300 × 600	0.03	No	Conventional
Atom 3D Printer[48]	220 × 345	420 × 420 × 930	0.01	No	Conventional
RepRap Rostock[49]	200 × 400	300 × 350 × 1000	0.1	No	Conventional

LDr-AM: Linear Delta robot for Additive Manufacturing; 3D: three-dimensional.

Based on the optimization parameters, a prototype was developed, which allowed the validation of the requirements defined by QFD, as well as delivering the product's target specifications.

In order to validate the design of the proposed rDL-AM, a prototype of the machine has been developed. Figure 11 shows the prototype of the ~~rDL-AM~~ with aluminum base structure achieved after the validation of the first prototype with wooden base structure. The URL https://tinyurl.com/ydxufqdz presents a video of the Robot Linear Delta in operation.

When comparing the proposed Linear Delta Robot printer workspace with other commercial printers, for example, Kossel printer (conventional linear delta topology), it is verified that they do not present significant difference since the volume of work is a cylinder and the dimensions of the mechanism are optimized to meet a particular workspace, and cylinder diameter and height, which vary the dimensions of the links and length of linear transmission.

In future works, the aim is to study the robot's kinematic reconfiguration and perform uncertainty and clearances analysis of links and joints. Also, there is the idea to use the robot as a platform for the implementation of a controller based on the new STEP-NC numerical control standard.[8]

Declaration of conflicting interests

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