International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 1 Issue 2 ǁ December. 2013ǁ PP-31-40

Some Studies On Normal and Non-Normal Process Capability Indices

Ajit Goswami[1] & Harendra Narayan Dutta[2]
[1] *Ph.D Scholar, Department of Statistics, Dibrugarh University, Dibrugarh, Assam, India*
[2] *Professor, Department of Statistics, Dibrugarh University, Dibrugarh, Assam, India*

ABSTRACT:*Nowadays there is no organisation that can operate on the market without constant improvement of its performance focused on quality of products and services. The challenge in today's competitive market is to be on the leading edge of producing high quality products at minimum costs. This can be done with a systematic approach by using Statistical Process Control (SPC) tools which can be used to improve the quality in mass production process including process and product design prior to manufacturing in quantifying process variability. It is obvious that customer's satisfaction plays a key role for success of any business. So, in analysing this variability relative to product requirements or specifications, one has to need to eliminate or greatly reduce this variability by the method of Process Capability Analysis (PCA). The Process Capability Indices (PCIs) provides a measure of how a process fits within the specification limits.Since 1980s, theories of PCIs have been developed to analyse the capability of processes. During the past few decades, various PCIs have been developed both for normal as well as for non-normal process data. In this paper, a brief review of past works on process capability indices dealing with both normal & non-normal process data will be studied and some of the PCIs would be discussed for both the cases.*
Key Words: PCA, SPC, PCIs, Normal and Non-normal

I. INTRODUCTION:

It is noteworthy that, in today's competitive world market, the manufacturing industry is facing intensive competition. Therefore, both the cost and quality aspect of the product have become very important issues among producers. Thus producers thrive on providing economical processes which are also capable of meeting the customer's quality requirements. To achieve the goal for high quality with low cost, Process Capability Indices (PCI) are developed both for on-line and off-line quality management system. The Process Capability Analysis (PCA) is a technique that has application in many segments of the product cycle including product and process design, vendors sourcing, production or manufacturing planning and manufacturer parts within the tolerance limits and engineering values. Quality engineering experts found that Process Capability indices are powerful tools of monitoring the process ability for manufacturing a product to meet the particular specifications. It is note-worthy that certain assumptions are essential to use Process Capability Indices to have a meaningful interpretation of the Process data.Process capability measures the variability of a process relative to its specification limit based on three assumptions, namely, (i) the process is itself in control, (ii) target value and specifications of a quality characteristics are specified, and (iii) the process measures quality characteristics that follow a normal distribution.

Definition of Process Capability: The capability of a process is the ratio of the distance from the process mean to the nearest specification limit divided by measure of process variability.

Mathematically, it can be expressed as:

$$\text{Process capability} = Min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right) \qquad \ldots (2.1)$$

Where μ and σ are the process mean and standard deviation of the measured characteristic of interest. USL and LSL are the upper and lower specification limits respectively. Usually these values are estimated from the data collected from the production process. There are several definitions of process capability as put forwarded by the quality scientist in recent times and more may be offing in future. Here we mention a few definitions for interested readers in this area.Sinha and Willborn [1] defines that process capability provides a quantified measures of adequacy and is learned indirectly by quality control done on actual product; while Juran and Gryana [2] refers that process capability is the minimum variation that a process can achieve. Wasserman et.al [3] states that -Process capability is the ability of the production process of producing items within specified tolerances". Pyzdek [4] concluded that "Process Capability Analysis is conducted to compare a controlled process to measurement. It is an analytical study which attempts to estimate future events. Lester, Enrich and

Motley [5] defines process capability as the quality that a process can produce with operations a normal level under practical conditions with company's economic means. Montgomery [6] more specifically defines Process Capability Analysis as an engineering study to estimate process capability. The estimate of process capability may be in the form of a probability distribution having a specified shape, center (mean) and spread (standard deviation). In most recent times, Sagbas [7] defines process capability analysis as the technique applied in many stages of the product cycle- including process, product design, manufacturing and manufacturing planning, since it help to determine the ability to manufacture parts within the tolerance limits and engineering values.

II. REVIEW OF PAST WORKS ON PROCESS CAPABILITY INDICES:

Gogoi [8] made an extensive review of the literature on process capability measures developed up to 2003. He had of course mainly studied the past works on PCA dealing with normally distributed product. In recent times there are several papers on PCA for normally as well as no-normally distributed product. In this section, we shall only highlight briefly the **recent works** on PCA for normally distributed product and in next section we will deal with non-normally distributed product.

3(A) PCI for normally distributed data:

Vannman, and Albing [9], presented a graphical method useful for analysing process capability for a process having one-sided specification limits. Under the assumption of normality, estimated process capability plots are used to assess process capability at a given significance level. They suggested that graphical approach is helpful to determine the variability, the deviation from target, or both which is required to be reduced to improve the capability. Albing [10] gave emphasis on process capability indices when specification limits are one-sided. Albing and Vannman [11] discussed the process capability in a situation with non-negative process data having a skew distribution with a long tail towards large values. Urdhwereshe [12] discussed about measurement system analysis by comparing ratio of two values, namely Process tolerance and Process variation. He has also analysed capability indices with a case example.Guevera, et.al [13] discussed process capability indices for dependent data i.e., when the data are auto-correlated. Though a simulation study they showed that higher the auto-correlation level, the lower is the capability index value. Jeang, et.al [14] studied process capability indices for off-line application of production life cycle. The product quality is represented by quality loss function and production cost is expressed by tolerance cost function. They observed that an economical and quality of product designed and process planning can be achieved during off-line application of process capability indices.

Czarski, [15] emphasized for correct determination of process capability indices, identification of distribution shapes and quality improvement method like Statistical Process Control (SPC) tools to be understood by the users of capability analysis before using the process capability tools.Ali, et.al [16] summarised the characteristics of the process capability indices for both real and simulated data. In their paper they discussed normal and exponential distribution with different values of sample sizes to ascertain sample size effect on process capability indices.Sagbas, A [7] carried out a process capability analysis in the machining line of a medium size company that produces machine and spare parts. He suggested that in order to satisfy the process capability measures, it is necessary to improve the quality level by shifting the process mean to the target value and reducing the variations in the process.Flaig J.J. [17] proposed a new metric which differs from the commonly used engineering metrics. The metric consists of two economic capability measures the expected profit and the variation in profit of the process. This dual economic metric offers a number of significant advantages over other engineering or economic metric used in Process Capability Analysis. Lundkvist et.al. [18] discussed about the comparison of decision methods using the process capability index C_{pk} when data are autocorrelated. He has done it through a case study followed by a simulation study and found that two methods appeared to be better than the others.

3(B) PCI for non- normally distributed data:

Pan & Wu [19] investigated the process capability indices for non-normal data which is divided into three categories i.e. bilateral specifications; unilateral specification with target value and unilateral specification without target value. Pearn & Chen [20] introduced C_p (u,v) which includes the four basic PCI's (C_p, C_{pk}, C_{pm} and C_{pmk}) and they have generalised C_p (u,v) as C_{Np} (u,v) and C'_{Np} (u,v) for non-normally distributed data and made a comparison between C_{Np} (u,v) and C'_{Np} (u,v) and found that C_{Np} (u,v) are superior to C'_{Np} (u,v) in measuring process capability. Bittanti [21] addresses the problem of defining and computing reliable estimates for process capability indices (and particularly for Cpk) for nonnormal processes; in particular, a curve-fitting approach to the estimation problem is taken and the problem of providing confidence intervals for the estimates of PCI's is considered. Finally, some application examples are also presented.

Abbasi, et.al [22] measures Process Capability Indices (PCIs) for bivariate non-normal process using the bivariate Burr distribution. They have used univariate Burr distribution to improve the accuracy of estimates of PCIs for univariate non-normal distribution. They also estimated the PCIs of bivariate non-normal distributions using the bivariate Burr distribution.Casalino & Rotondo [23] proposed the multivariate process incapability index MCpp (q) for non-normal data. A case study on the comparison between two different cold extrusion processes has been presented to demonstrate how the proposed index can be applied to real data.Ahmad et.al [24] reviews the performances of the Clements [25] non-normal percentile method, the Burr based percentile method and Box-Cox [26] method for non-normal cases. They also conducted a simulation study using Weibull, Gamma and Lognormal distribution.Albing [27] proposed a class of capability indices that are useful when the quality characteristic of interest has a skewed, zero-bound distribution with a long tail towards large values and there is an upper specification with a pre-specified target value, T=0. He has investigated the process capability indices for Weibull distribution. Czarski [28] discussed the comparative analysis of capability indices using Clement's method and exact method.Lovelace & Swain [29] proposed a process capability index estimation methodology for Cp and Cpk for the case of non-normal, zero-bound process data using the delta distribution, a variant of the lognormal distribution. This approach utilizes quantile estimates derived from a proposed modification of lognormal quality control charts.

Ahmed [30] discussed the estimation of *Cp an*d *Cpk* , commonly used process capability indices (PCI), in case of nonnormal data using the characteristics of Weibull distribution. Quantiles are estimated by probability plotting technique and then control limits are obtained to determine whether the process is in statistical control or not. Percentage points of the fitted distribution have been used to compute under the assumption of Weibull distribution. Kantam et.al. [31] studied the point estimates of process capability index as suggested by Clements through simulation when the underlying distribution is Half Logistic distribution. They also used Moment estimate, maximum likelihood estimate (MLE) and best linear unbiased estimate (BLUE) .

III. VARIOUS APPROACHES FOR PROCESS CAPABILITY ANALYSIS:

We have already stated that Process Capability Analysis is a technique that has application in many segments of the product cycle, including product and process design, vendor sourcing, production on manufacturing planning and manufacturing. There are three primary techniques that can be used to study the Process Capability Analysis. These are:

(a) Histogram or probability plots
(b) Control Charts
(c) Designed of Experiments.

Jeang et.al [14], observed that in today's competitive World Market, the manufacturing industry is facing intensive competition. Therefore, both the cost and quality aspect have become very important issues among the producers. Thus producers thrive on providing economical processes which are also capable of meeting the customer's quality requirements. To achieve the goal for high quality with low cost, Process Capability Indices (PCI) are developed for on-line as well as off-line quality management system. The PCI value is typically defined as the ability to carryout the intended goal. The frequently use Process Capability Ratios (PCR) are C_p, C_{pk} and C_{pm} which can be formulated as follows-

(A) On-line Process Capability Indices:

(i) The Process Capability Ratio C_p

This can be expressed as:

$$C_p = \frac{USL - LSL}{6\sigma} \qquad \dots(4.1)$$

where USL and LSL are Upper and Lower specification limits respectively.

In practical application, the Process standard deviation σ is almost always unknown and must be replaced by an estimate of σ. To estimate σ we typically use either the sample standard deviation s or $\frac{\bar{R}}{d_2}$ (when variables control charts are used in the capability study).

For one-sided specification we define one-sided PCRs as follows-

$$C_{pu} = \frac{USL - \mu}{3\sigma} \qquad \text{(upper specification only)} \qquad \dots (4.2)$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma} \qquad \text{(lower specification only)} \qquad \dots (4.3)$$

It may be noted that during the production process, the process mean can be changed other than its target value. If the process variance σ^2 is constant then C_p value would remain unchanged. This was the major defect of C_p owing to the fact that the deviation of the process mean cannot be reflected in the measurement but only the spread of the process. These are the main reason to develop the C_{pk}.

(ii) The Process Capability Ratio Cpk:

This index can be expressed as:

$$C_{pk} = Min(\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}) \qquad \ldots (4.4)$$

$\approx Min(C_{pu,} C_{pl})$ where C_{pu} and C_{pl} are given in equation (4.2) and (4.3) respectively.

The magnitude of C_{pk} relative to C_p is a direct measure of how off-center the process is operating. More generally we usually say that C_p measures potential capability of the process, whereas C_{pk} measures actual capability.

(iii) The Process Capability Ratio C_{pm}:

There is still a deficiency which is not measured by the above two P.C.I s namely C_p or C_{pk} since in a process, the mean and variance are generally changed.

To measure such simultaneously changes, C_{pm} has been developed. The formula of C_{pm} is given as

$$C_{pm} = \frac{USL-LSL}{6\tau} \qquad \ldots (4.5)$$

Where τ is the square root of expected squared deviation from target

$$T = \frac{1}{2}(USL + LSL)$$

$$\tau^2 = E[(X-T)^2]$$

$$= E[(x-\mu)^2] + (\mu - T)^2$$

$$= \sigma^2 + (\mu - T)^2 \qquad \ldots (4.6)$$

which can finally be expressed as

$$C_{pm} = \frac{C_p}{\sqrt{1+\xi^2}} \qquad \ldots (4.7)$$

Where

$$\xi = \frac{\mu - T}{\sigma}$$

Chan et.al. [32] and Boyels [33] discussed detail about C_{pm} and its usefulness in measuring process centering.
The interpretation of C_p and C_{pk} and C_p relative to C_{pm} can be summarized into following ways:

1. If $C_p = C_{pk}$, then it implies that the process centered at the midpoint of the specification.
2. If $C_{pk} < C_p$, then it implies that the process is off-center. The user should try to center the process.
3. If $C_{pk} = 0$, then the process is exactly equal to the one of the specification limit.
4. If $C_p < 0$, then the process mean is outside the specification limits.
5. If $C_p = 1$, it implies that the process is centered. For a normally distributed product $C_p = 1$ implies that 2700 parts per million (ppm) are non-conforming i.e., fall out rate of 2700 ppm for two sided specifications.
6. If $C_p < 1$, it implies that the process is not fully capable.
7. If $C_p > 1$, capable of meeting the specification
8. $C_{pk} < -1$ implies that entire process lies outside the specification limit. Usually C_{pk} is non-negative. Similarly,
9. If $C_{pm} \geq 1$, implies that process mean μ lies within the middle third of the specification band. If $C_p = C_{pk} = C_{pm}$ then μ = Target value, but all the PCI decreases if move away from the target value.

IV. PCI FOR NON-NORMAL DATA:

Non-normally distributed processes are not uncommon in practice. Combining this fact with the misleading results of applying basic PCIs to non-normal processes while treating them as normal distribution forced academicians and practitioners to investigate the characteristics of process capability indices with non-normal data.

The situation, where the distribution of the monitored feature of quality does not correspond with normal distribution can in principle be addressed in the following ways:

a) Transformation of data to a variable coming from normal distribution
b) Use of different theoretical model of distribution

Transformation of data to normally distributed variable

In the case of data transformation, the original data, which do not correspond to normal distribution, use suitable transformation function to be transformed into data, whose distribution is most closely approximating normal distribution. The next step will verify the normality of the transformed data using suitable numerical tests. If the normality of data is confirmed it is followed by the transformation of tolerance limits using the identified transformation function, and the process capability analysis can be carried out in accordance with the procedure for normal distribution.

Box- Cox data transformation:

George E.P. Box and David Cox have created a transformation that allows approximation of selection distribution to normal distribution with respect to skewness and kurtosis. It is a very commonly used transformation, mainly due to its simplicity. Box-Cox transformation follows the relations:

$$y = \begin{cases} \dfrac{x^{\lambda} - 1}{\lambda} \\ \ln(X) \end{cases} \quad \text{for} \quad \lambda \neq 0 \text{ and } \lambda = 0 \qquad \text{... (5.1)}$$

Johnson data transformation:

Johnson transformation chooses one of three types of equations, depending on whether the random variable is bounded (SB), log-normal (SL) or non-bounded (SU). The equations are:

Type SB-bounded random variable:

$$y = a + b.\ln\frac{(x+c)}{(d-x)} \quad \text{for } b>0;\ -c< x <d \qquad \text{... (5.2)}$$

Type SL- log-normal distribution of random variable:

$$y = a + b.\ln(x+c) \quad \text{for } b>0;\ -c<x \qquad \text{... (5.3)}$$

Type SU-non-bounded random variable:

$$y = a + b.A\sinh\frac{(x-c)}{d} \quad \text{for } b>0;\ d>0 \qquad \text{... (5.4)}$$

The use of another theoretical model of distribution

Another way of solution of the problem of non-normality of the monitored characteristics of quality is based on finding another suitable probability distribution to describe the distribution of characteristics.Perhaps the most widely used method of evaluation of process capability using a different model of distribution is ISO method. Similarly as in the case of normal distribution, where the values of μ-3σ and μ+3σ correspond to the quantiles for which the distribution function reaches the values of 0.00135 and 0.99865, another probability model uses quantiles corresponding to these values of distribution function. The corresponding values of capability indices can be calculated as:

$$C_p = \frac{USL - LSL}{x_{0.99865} - x_{0.00135}}$$

$$C_{pk} = \min\left\{\frac{x_{0.5} - LSL}{x_{0.5} - x_{0.00135}};\frac{USL - x_{0.5}}{x_{0.99865} - x_{0.5}}\right\} \qquad \text{... (5.5)}$$

where:

$X_{0.00135}$ -0.135 % quantile of appropriate distribution
$X_{0.99865}$ -99.865 % quantile of appropriate distribution
$X_{0.5}$- median of appropriate distribution

Clements Percentile PCI method: Clements method is popular among quality practitioners in industry. Clements proposed that 6σ in equation (4.1) by replaced by the length of the interval between the upper and lower 0.135 percentage points of the distribution of X. Therefore, the denominator in equation (4.1) can be replaced by (U_p-L_p), i.e.

C_p = (USL-LSL) / (U_p-L_p) … (5.6)

where U_p is the upper percentile i.e. 99.865 percentile of observations and L_p is the lower percentile i.e. 0.135 percentile of observations. Since the Median "M" is the preferred central value for a skewed distribution, so C_{pu} and C_{pl} as follows:

C_{pu} = (USL-M) / (Up-M) … (5.7)
C_{pl} = (M-LSL) / (M-Lp) … (5.8)
and C_{pk} = min {C_{pu}, C_{pl}} … (5.9)

Clements approach uses the standard estimators of skewness and kurtosis that are based on 3rd and 4th moments respectively, and may not be reliable for very small sizes.

An Empirical Study of Normal PCA (In a Chemical Industry)
Problem: The specification of formalin product of a chemical factory is set up as 37± 0.5, by weight of Formaldehyde content. If the weight of formaldehyde gas in Formalin is below 36.5%, the customers do not accept it and if the same is above 37.5%, it is not affordable to manufacturer so far its cost-benefit margin are concerned. The data for thirty days has been collected by taking 5 samples from the recorded data sheet from the factory.

Following section 3, we shall apply the primary techniques like Histogram, probability plot and control charts (X-bar-R) to the formalin data in our next section for studying process capability analysis.

Histogram and Normal Probability Plot:
Probability plotting is a graphical method for determining whether the sample data conform to a hypothetical distribution (Normal distribution in our case) based on subjective visual examination of the data. Figure 1 below presents a normal probability plot (Minitab Version) of the observational data. As the plotted points is falling approximately along a straight line, so it gives no indication that formalin data is non-normal. However, for severe departure from normality assumption may need to examine other statistical test, such as non-parametric test which is outside the preview of our analysis. The histogram and capability analysis is depicted in our figure 2.

By comparing the values of C_p = 0.67 and C_{pk}= .38, we found that C_p < 1 i.e., 0.67 < 1 so the process is not fully capable and C_{pk} < C_p i.e., 0.38 < 0.67 implies that the process is off-center. Hence, the user should try to center the process.

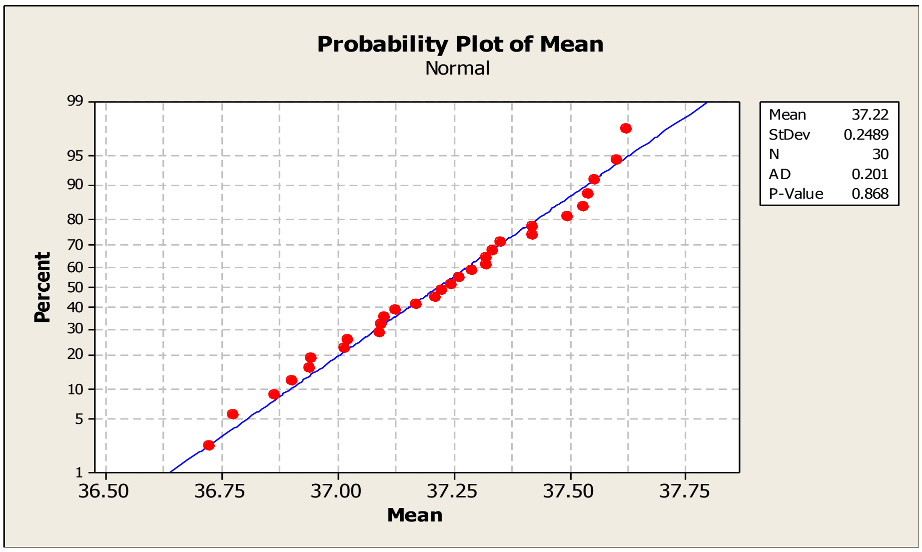

Fig 1. Normal Probability plot

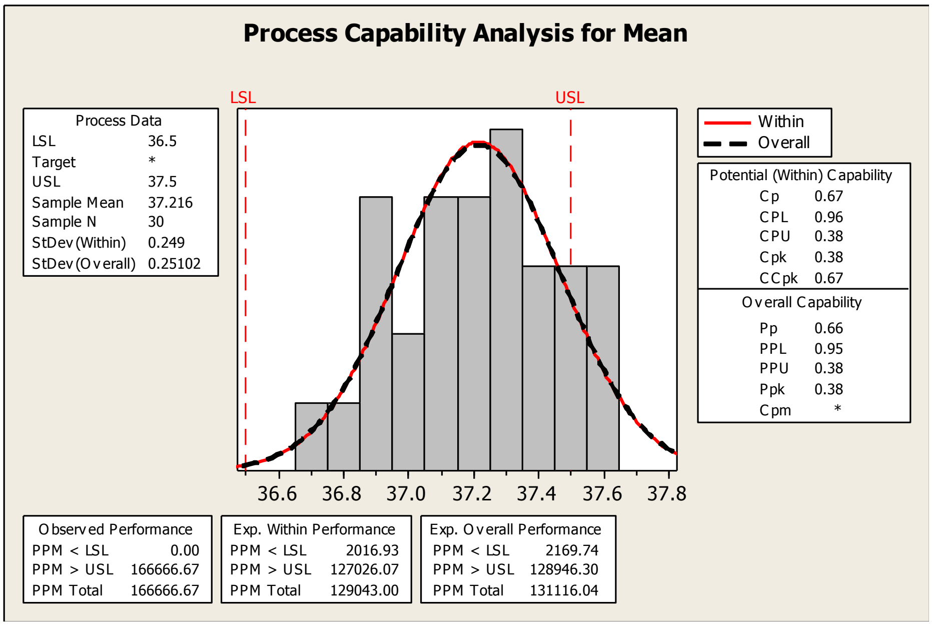

Fig. 2. Histogram of analyzed data

Control Chart:

If the process is under statistical control, then we are able to reliably use our estimates for spread and location. In order to assess whether or not a process is under statistical control, which are sensitive to large shifts in the process parameters we may use X-bar-R chart or X-bar-S chart. We shall depict here the X-bar-R chart (Minitab version) in Figure-3 below which shows that process is under statistical control and thus we can reliably use our estimates to measure the spread and location of the Formalin data for analysing process capability.

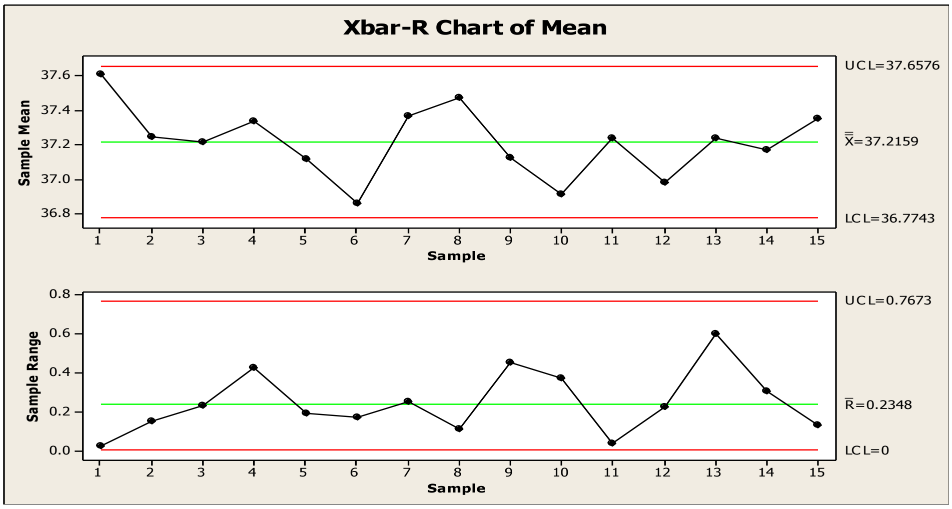

Fig.3. X-bar- R Control Chart

Numerical example for Non-normal PCA:

To illustrate the use of above described method for non-normal PCA, 25 subgroups with a subgroup range of 5 for Gamma and Weibull distribution were generated by computer software. For Gamma distribution with parameters of α=4.75 and β=5.24, the tolerance limits have been set as follows: LSL= 5 and USL = 70. The character of distribution of the monitored feature and its position in relation to the tolerance limits are shown in figure 4.

The values of C_p and C_{pk} determined using different methods is presented in Table 1. (for Gamma distribution)

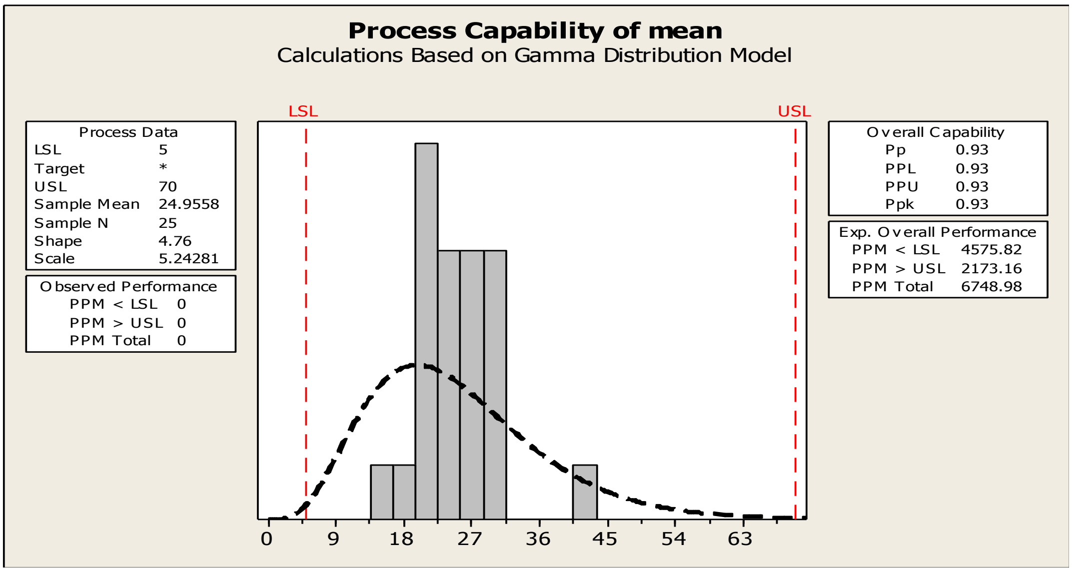

Fig.4 Histogram of analyzed data

Table 1: Capability indices calculated by means of different methods

Method	Cp	Cpk
Box-Cox transformation	2.16	1.72
Johnson transformation	2.05	1.26
"ISO" method	0.93	0.93
Clements	2.55	2.13

Similarly, For Weibull distribution with parameters of α=4 and β=1, the tolerance limits have been set as follows: LSL= 0.2 and USL = 2. The character of distribution of the monitored feature and its position in relation to the tolerance limits are shown in figure 5.The values of C_p and C_{pk} determined using different methods is presented in Table 2. (for Weibull distribution)

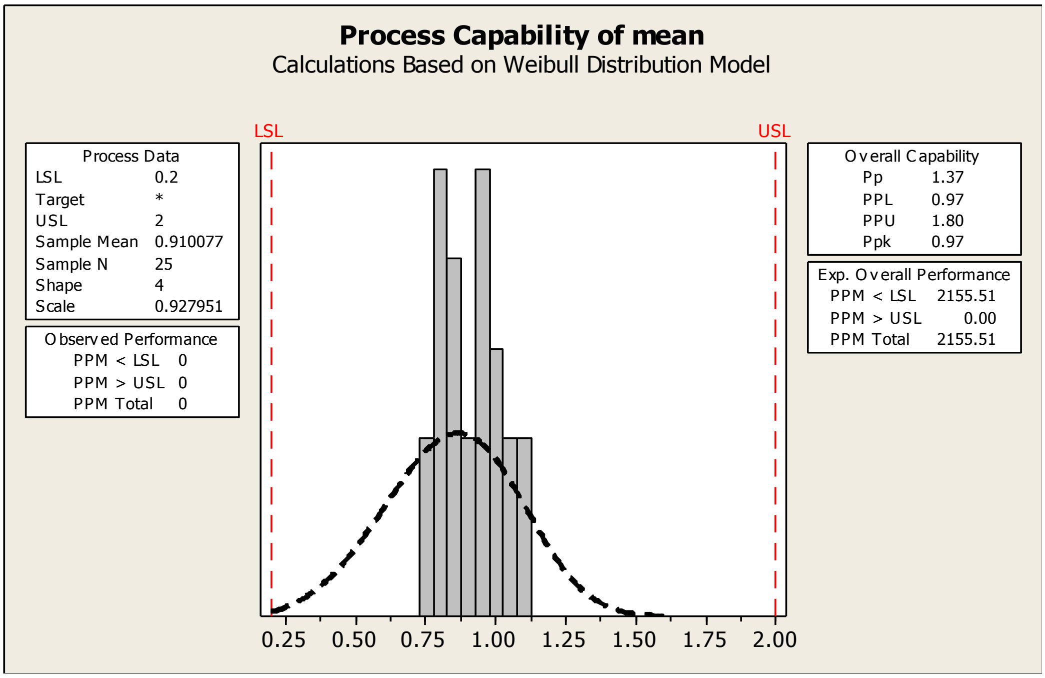

Fig 5. Histogram of analysed data

Table 2: Capability indices calculated by means of different methods

Method	Cp	Cpk
Box-Cox transformation	6.48	1.77
Johnson transformation	2.81	2.21
“ISO” method	1.37	0.97
Clements	4.71	3.91

V. CONCLUSION:

The tabulated results of process capability indices in Table 1 and 2 above clearly indicate that the process capability analysis of data for non-normal distribution may significantly differ depending on methods of analysis of process capability.It is also evident that, the obtained indices from the different approaches reflect that the process that produces the data considered in this study are off-centered because here $C_{pk} < C_p$ for all the cases except ISO method though they are capable to meet the specification. If the process is off-centered, the user should try to adjust the process.

REFERENCES:

[1] M.N. Sinha and W.O. Willborn : “Quality Control”, 2nd Edition. Prentice Hall. New Jersey. NJ, USA. 1985

[2] M. Juran, Joseph and M. Gryana, Frank. : “Juran’s Quality Control Hand Book”, Fourth Edition, 1988, Mc Graw-Hill Book Company.

[3] G.S. Wasserman, H.A. Mohsen, and L.A. Franklin : “A programme to calculate bootstrap confidence intervals for process capability index CPK”, Communication in Statistics Simulation and Computation, 1991, 20, (2/3), 397-510.

[4] T. Pyzdek: “Process Capability Analysis using Personal Computers”. *Quality Engineering, 1992,* 4(3), 419-440.

[5] D.H. Lester, N.L. Enrich and H.E. Motely : “Quality Control for Profit”, Industrial Press, New York, NY USA.1997

[6] D.C. Montgomery: “Statistical Quality Control- A modern introduction”, Six Edition by John Willey & Sons, Inc., 2011

[7] A. Sagbas : “Improving the Process Capability of a turning operation by the application of statistical Techniques”, *Materials and Technology, 2009,* 43 (1), pg. 55-59.

[8] R. Gogoi : “Process Capability Analysis in a working chemical manufacturing industry.” *Unpublished Ph.d Thesis, 2003,* Department of Statistics, Dibrugarh University (Assam).

[9] K. Vannman , and M. Albing : “Process Capability Indices for One sided Specification limits and Skew zero-bound Distribution” Luela University of Technology, Department of Mathematics, 2005, *Research Report 11.SE-97187* Lulea, Sweden

[10] M. Albing. : “Process Capability Analysis with focus on Indices for One-sided specification limits”, Department of Mathematics, 2006, Lulea University of Technology, SE-97187 Lulea, SWEDEN.

[11] M. Albing and K. Vannman : “Process Capability Indices for One sided Specification limits and Skew zero-bound Distribution” Luela University of Technology, Department of Mathematics, 2006a, *Research Report 11.SE*-97187 Lulea, Sweden.

[12] H. Udhayasee: “Assessing Initial Process Capability” Published in *Quality World* Feb. 2007.

[13] D.R. Guevera, and J.A. Vargas : “Comparison of Process Capability Indices under Autocorrelated data”. *Revista Colombian de Estdistica*, 2007, Vol.30, No.2. pp. 301-316.

[14] A. Jeang, C.P. Chung , H.C. Li and M.H. Sung : “Process Capability Index for off-line Application of Product life cycle”. *Proceeding of the World Congress on Engineering* 2008, Vol.55, WCE 2008, July 2-4, London, U.K.

[15] A. Czarski : “Estimation of process capability indices in case of distribution unlike the normal one”. *Archieves of Materials Science and Engineering, International Scientific Journal,2008,* Published by World Academy of Materials and Manufacturing Engineering, Vol.34, Issue-1, Pg. 39-42.

[16] S.M. Ali, M.A. Sarwar and A. Sultana: “Finding Characteristics of Process Capability Index Cpk with different distribution and sample size”. *Journal of Quality and Technology Management*, 2008, Vol. IV, Issue 11, Pg. 5-11.

[17] J.J. Flaig J.J.: “A new approach to Process Capability Analysis”, *Quality Engineering, 2009,* 9(2), 205-211

[18] Lundkvist: “A Comparison of Decision Methods for Cpk when data are autocorrelated”, *Quality Engineering*, 2012, 24: 460-472.

[19] J.N. Pan & S.L. Wu : “Process Capability Analysis for Non-normal Relay Test Data”, *Microelectron. Reliab*., 1996, Vol.37, No. 3, pp. 421-428

[20] W.L. Pearn, & K.S. Chen : “Capability Indices for Non-normal distribution with an application in Electrolytic Capacitor Manufacturing”, *Microelectron. Reliab*., 1997, Vol.37, No. 12, pp. 1853-1858

[21] S. Bittani, M. Lovera & L. Moiraghi : “Application of Non-Normal Process Capability Indices to Semiconductor Quality Control”, *IEEE TRANSACTION ON SEMICONDUCTOR MANUFACTURING, 1998,* Vol. 11, No. 2

[22] B. Abbasi, M. Ahmad , M. Abdollahian & P. Zeephongsekul : “Measuring Process Capability for Bivariate Non-Normal Process Using the Bivariate Burr Distribution”, *WSEAS TRANSACTIONS on BUSINESS and ECONOMICS*, 2007, Issue 5, Vol.4, Pg.70-77

[23] G. Casalino, & A. Rotondo : “Multivariate Process Incapability Index for Non-normal Data: A case Study”, *DAAAM International Scientific Book*, 2007, pp- 071-086, Chapter 07.

[24] S. Ahmad, M. Abdollahian, and P. Zeephongsekul : “Process Capability Estimation for non-normal quality characteristics: a comparison of clement, Burr and Box-Cox method”, *ANEIAM, 2008,* J.49 (EMAC 2007). Pp. 642.

[25] J.A. Clements: “Process Capability Calculations for non-normal distribution”. *Quality Progress*, 1989 22: 95-100.

[26] G.E.P Box and D.R. Cox : “An analysis of transformation”, *J Roy Stat Soc, 1964, B* 26: 211-252.

[27] M. Albing: “Process Capability Indices for Weibull Distribution and Upper Specification Limits”, *Quality and Reliability Engineering International*; 2009, 25:317-334.

[28] A. Czarski : “Analysis of CLEMENTs’ Method for Capability Indices Estimation”, *Metallurgy and Foundry Engineering*, 2009, Vol. 35, No.1

[29] C.R. Lovelace , & J.J. Swain : “Process Capability Analysis Methodologies for Zero Bound, Non-Normal Process Data”, *Quality Engineering*, 2009, 21: 190-202

[30] E. Ahmed : “Process Capability Analysis for Non-normal data”, *Pakistan Business Review.2010*

[31] R.R.L. Kantam, K. Rosaiah and R. Subba Rao : "Estimation of Process Capability Index for Half Logistic Distribution", *International Transactions in Mathematical Sciences and Computer*, 2010, Vol. 3, No. 1, pp-61-66

[32] L.K. Chan, S.W. Cheng, and F.A. Spiring : "A New Measure of Process Capability: Cpm", *Journal of Quality Technology*, 1988, vol. 20(3), pp.160-175.

[33] R.A. Boyles: "The Taguchi Capability Index", *Journal of Quality Technology*, 1991, Vol.23 (2), pp.107-126